

## 12 Uncertainty and Insurance

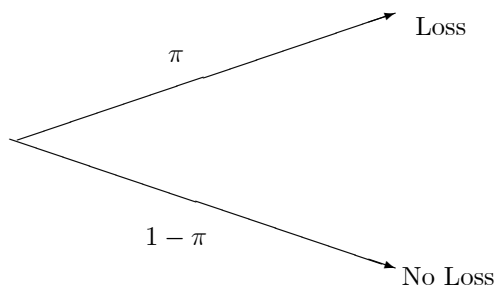


Figure 1: Possible Outcomes

For simplicity we will think of a situation with two possible *outcomes*.

- With probability  $\pi$ , the consumer suffers a “loss”
- With probability  $1 - \pi$ , the consumer suffers “no loss”.

Observe that  $\pi$  is a number between 0 and 1 and has nothing to do with areas of circles (I’m just conforming with Varians choice of notation). Thus, one example is  $\pi = 0.5$  (fair coinflip). Also, the language “loss” versus “no loss” I use only because of the particular application in mind. It could be any situation where there is uncertainty about something the consumer cares about. Examples:

- House burns down-House does not burn down
- Loose job-keep job
- Win state lottery-don’t win state lottery (or any other gamble)

However which example you prefer we will think of the “state” occurring with probability  $\pi$  as the “bad state” and the “state” occurring with probability  $1 - \pi$  as the “good state” and we assume that for some reason the disposable income is lower in the bad than in the good state. Write

- $m_b$  for the income in the bad state (probability  $\pi$ )
- $m_g$  for the income in the good state (probability  $1 - \pi$ )
- $\Rightarrow$  “Loss” =  $m_g - m_b$

Now, we will assume that the consumer can purchase insurance, so the consumption *need not* be  $m_b$  or  $m_g$ . We denote actual consumptions by

- $c_b$  for the consumption in the bad state
- $c_g$  for the consumption in the good state

In terms of how the consumer evaluates different “bundles” of  $(c_b, c_g)$  the *interesting question is how these are evaluated before the consumer knows whether she suffered a loss or not*. Just think about it, the decision on how much insurance you buy or how to invest in a risky portfolio *must be done before the uncertainty is resolved*. We will assume that the consumer evaluates different “bundles” (ex ante) according to

$$U(c_b, c_g) = \pi u(c_b) + (1 - \pi) u(c_g),$$

which is called an EXPECTED UTILITY function. What is particular about this is that the utility function is:

1. Linear in probabilities
2. The same *function*  $u(c)$  is used for the “ex post evaluation” of consumption in both “states”.

There are VERY GOOD REASONS to think that the expected utility representation is a good way to model choice under uncertainty. However, this is rather deep and advanced material—you’ll have to take on faith that this is reasonable.

Note that given these preferences we can draw indifference curves in the usual way. The marginal rate of substitution will as usual be the slope of the indifference curves and this is

$$MRS(c_g, c_b) = -\frac{\frac{\partial U(c_b, c_g)}{\partial c_g}}{\frac{\partial U(c_b, c_g)}{\partial c_b}} = -\frac{(1 - \pi) u'(c_g)}{\pi u'(c_b)}$$

Now, how the indifference curves will look will clearly depend on the choice of  $u(c)$  and the shape of this function turns out to have a natural interpretation.

**12.0.1 Example:**  $u(c) = ac + b$

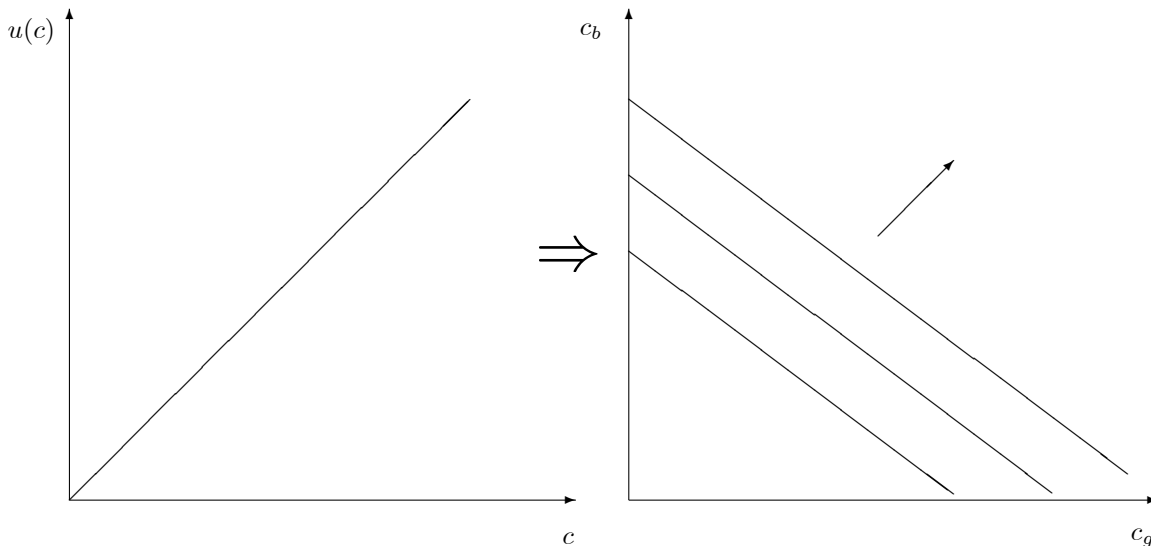


Figure 2: Indifference Curves for  $u(c) = ac + b$

In this case, the slope of the indifference curve is  $-\frac{1-\pi}{\pi}$ , so we have the indifference curves as in Figure 2. The slope is constant and equal to the negative of the ratio of the respective probabilities. Preferences of this sort are called *risk neutral preferences*. To see why note that

$$\pi c_b + (1 - \pi) c_b$$

is the *expected value of consumption* and that

$$\begin{aligned} & \pi (ac_b + b) + (1 - \pi) (ac_g + b) \\ &= a (\pi c_b + (1 - \pi) c_b) + b, \end{aligned}$$

so that the consumer is indifferent between all bundles (or contingent plans) that has the same expected consumption.

**12.0.2 Example:**  $u(c) = \sqrt{c}$

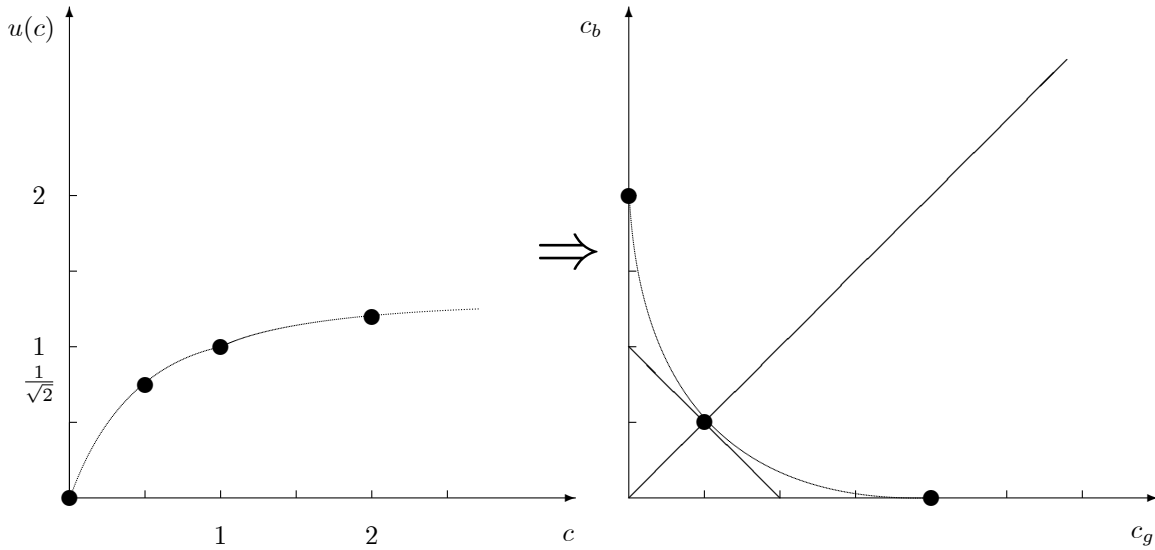


Figure 3: Indifference Curves for  $u(c) = \sqrt{c}$

In this case, the slope of the indifference curve is

$$-\frac{(1 - \pi)\sqrt{c_g}}{\pi\sqrt{c_b}}$$

To actually depict them, let  $\pi = \frac{1}{2}$  and consider the curve through  $(1, 1)$ .

- $\frac{1}{2}\sqrt{1} + \frac{1}{2}\sqrt{1} = 1$ , gives  $k = 1$  for the curve going through  $(1, 1)$
- $\frac{1}{2}\sqrt{4} + \frac{1}{2}\sqrt{0} = 1 \Rightarrow (4, 0)$  and  $(0, 4)$  on curve

Thus, the indifference curves look like in Figure 3.

Hence, the example generates nice convex preferences, which is true for all functions  $u(c)$  that have a slope which is decreasing in  $c$  (that is concave functions  $u$ )

## 12.1 Interpretation of Convex Preferences in Applications with Uncertainty

In the canonical model when we think of convex preferences we motivate it by saying that it may be natural for agents to “prefer a little bit of each” rather than going for extremes.

In this particular application we can put a bit more flesh and bones on the story.

We say that consumers with the kind of preferences graphed in the last example (or any other  $u(c)$  with decreasing slope) are *risk averse*—they rather “mix their portfolio” than put all the eggs in the same basket. To see this in a little bit of a different way inspect Figure 4

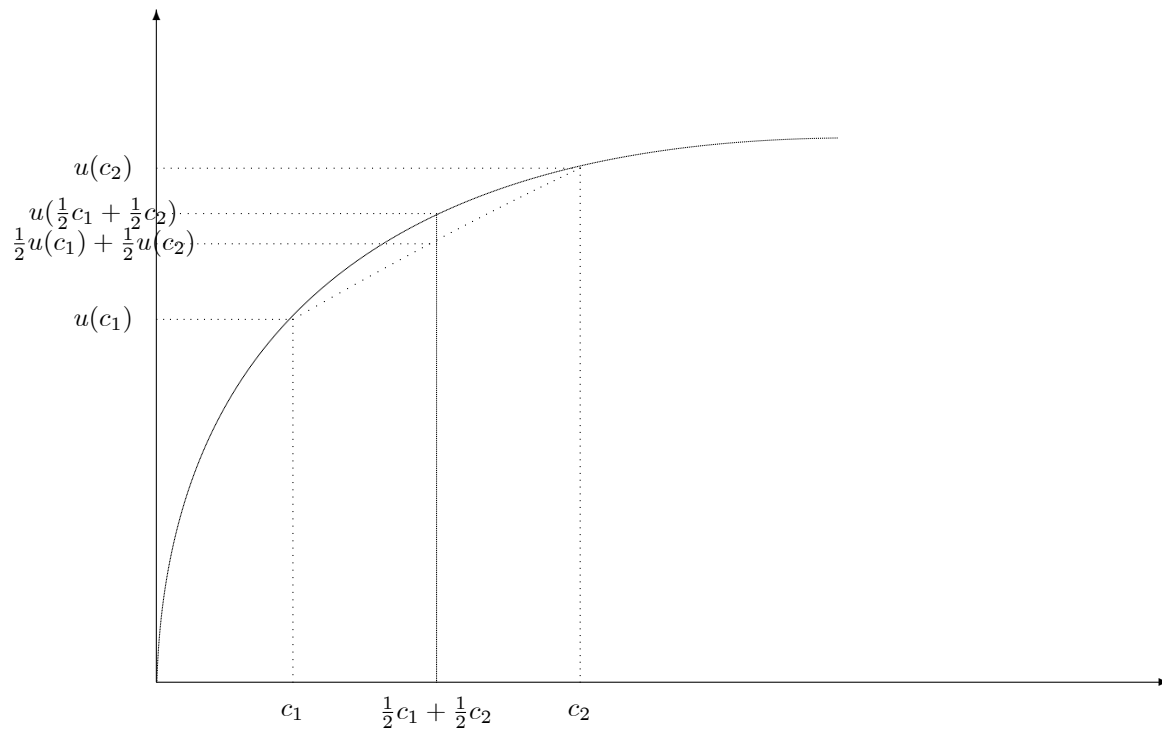


Figure 4: Comparing Utility of Expected Value with Expected Utility of Gamble

**THOUGHT EXPERIMENT:**

Take all your savings, would you

- a.** rather flip a coin and double savings if heads and loose everything if tails?
- b.** keep your savings for sure?

If answer is b)  $\Rightarrow$  “risk averse”. (a. would be “risk loving” and indifference “risk neutral”).

Economists usually assume agents are risk averse (otherwise there wouldn’t be any rationale for a market for insurance) or risk neutral. In particular large firms are often modelled

as risk neutral (idea-lot's of independent risks  $\Rightarrow$  firm can pool risks and thereby eliminate them).

## 12.2 Affordable Consumption Plans in the Presence of Insurance

Now suppose that consumer can buy insurance:

- Let  $z$  be quantity insurance (units paid to consumer in the event of a loss)
- Let  $p$  be the price per unit of insurance

$\Rightarrow$

- $c_b = m_b + z - pz$  is the consumption in bad state
- $c_g = m_g - pz$  is the consumption in good state

Now, we can eliminate  $z$  from this to get

$$\begin{aligned} c_b &= m_b + z(1 - p) = \\ &= m_b + \frac{(m_g - c_g)}{p}(1 - p) \end{aligned}$$

or, equivalently

$$c_g + \frac{p}{1 - p}c_b = m_g + \frac{p}{1 - p}m_b$$

or

$$(1 - p)c_g + pc_b = (1 - p)m_g + pm_b$$

AS IN THE INTERTEMPORAL PROBLEM-LIKE A STANDARD “APPLE & BANANA” SETUP WITH  $c_g$  instead of  $x_1$ ,  $c_b$  instead of  $x_2$ , prices  $\left(1, \frac{p}{1-p}\right)$  and income  $m_g + \frac{p}{1-p}m_b$ . The budget set is depicted in Figure 5

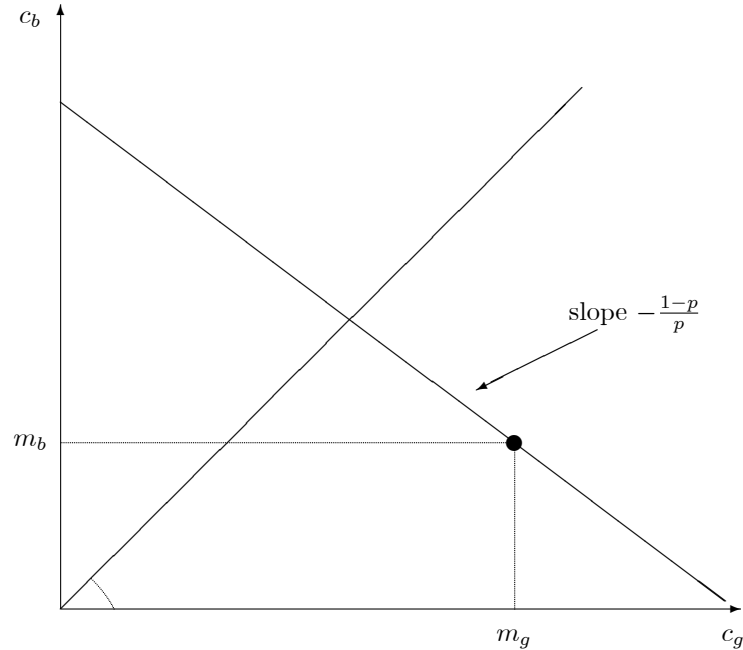


Figure 5: The Budget Set

### 12.3 The Choice Problem

Graphically, given convex preferences, the solution will be characterized in the usual way as a nice tangency between (the highest possible) indifference curve and the budget set. Note that, at the diagonal line (the “certainty line”) the slope of the indifference curve is

$$-\frac{1 - \pi}{\pi}$$

to be compared with the slope of the budget line

$$-\frac{1 - p}{p},$$

so if  $p > \pi$ , then  $\frac{1-p}{p} < \frac{1-\pi}{\pi}$ , meaning that *at the diagonal* the indifference curves must be steeper than the budget line meaning that the tangency must occur somewhere below the diagonal as in Figure 6.

Characterizing the solution using calculus it is convenient to keep  $z$  as the choice variable (although you may use  $c_g$  or  $c_b$  if you want).

$$\max_z \pi u(m_b + z - pz) + (1 - \pi) u(m_g - pz)$$

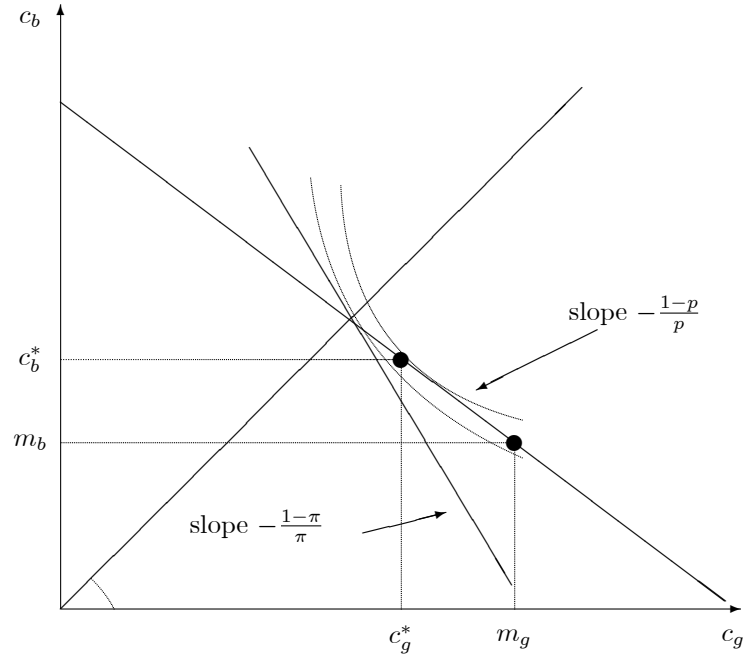


Figure 6: Graphical Solution with  $p > \pi$

FOC is

$$\pi u' \left( \underbrace{m_b + z - pz}_{c_b} \right) (1 - p) + (1 - \pi) u \left( \underbrace{m_g - pz}_{c_g} \right) (-p) = 0$$

$$\Downarrow$$

$$\frac{1 - p}{p} = \frac{(1 - \pi) u' (c_g^*)}{\pi u' (c_b^*)},$$

which we sort of knew already from the picture since this just says that the slope of the indifference curve must equal the slope of the budget line. However note that:

1. If  $p = \pi$ , then

the consumer pays  $pz = \pi z$

the insurance company gives the consumer  $z$  with probability  $\pi$

the insurance company gives the consumer 0 with probability  $1 - \pi$

$\Rightarrow$  Expected profit for insurance company  $pz - \pi z = 0$

This is called a “fair premium” since the insurance company breaks even on average. Note that the solution in this case is  $c_g^* = c_b^*$ , since  $u'(c)$  is a decreasing function. The conclusion is clear: if a risk averse consumer can buy insurance at a fair price the consumer will fully insure.

2.  $p > \pi \Rightarrow$ partial insurance (or no insurance).

3.  $p < \pi \Rightarrow$ overinsurance.

## 13 Adverse Selection and Insurance; The Case with a Monopoly

- Suppose that there are two “types” of consumers. Call them  $\{H, L\}$
- Type  $H$  has a probability of an accident given by  $\pi_H$
- Type  $L$  has a probability of an accident gives by  $\pi_L$
- For both types, the endowment is  $\{m_G, m_B\}$ , where  $m_G > m_B$  (i.e., “state  $B$ ” is when loss occurs).
- Risk neutral monopolist selling insurance;

### 13.1 Benchmark; Observable Types

If the monopolist knows which types consumer he/she deals with one may be inclined to proceed as follows. Let  $p$  be the per unit price of insurance. Let  $D_J(p)$  be type  $J$ s demand for insurance as a function of the price, that is

$$D_J(p) = \max_z \pi_J u(m_B + z(1-p)) + (1 - \pi_J) u(m_G - z)$$

Then, the monopolist should solve

$$\max_p D_J(p) (p - \pi_J).$$

One **could** analyze this problem, but in general the monopolist could do better! The reason is that allowing the consumer to buy any number of units at the same unit price necessarily leaves some consumer surplus to the consumer. That is, we know (assuming risk aversion) that  $p = \pi_J$  in order for the consumer to fully insure. But that would give no profit to the monopolist. Hence, whatever the profit maximizing unit price would be, it must involve under insurance.

Suppose instead that the monopolist proceeds as follows. The consumer may get either full insurance and consume  $x_J$  units (regardless of whether an accident occurs or not) or get no insurance at all, where

$$u(x_J) = \pi_J u(m_B) + (1 - \pi_J) u(m_G)$$

The expected profit of this arrangement is

$$\pi_J (m_B - x_J) + (1 - \pi_J) (m_G - x_J)$$

**Proposition 1** *There is no insurance contract that both gives the monopolist a higher profit and makes the consumer willing to buy insurance. I.e., a profit maximizing monopolist fully insures the consumer and extracts all the consumer surplus.*

To see this, suppose that  $x_B, x_G$  are the consumptions for the consumer in a better contract. If  $u$  is concave/consumer is risk averse we have that

$$u(\pi_J x_B + (1 - \pi_J) x_G) \geq \pi_J u(x_B) + (1 - \pi_J) u(x_G)$$

If the expected profit is higher from  $(x_B, x_G)$  than from  $(x_J, x_J)$  then

$$\begin{aligned} \pi_J (m_B - x_J) + (1 - \pi_J) (m_G - x_J) &< \pi_J (m_B - x_B) + (1 - \pi_J) (m_G - x_G) \\ &\iff \\ x_J &> \pi_J x_B + (1 - \pi_J) x_G \end{aligned}$$

But  $u$  is strictly increasing, so

$$\begin{aligned} \pi_J u(m_B) + (1 - \pi_J) u(m_G) &= u(x_J) > u(\pi_J x_B + (1 - \pi_J) x_G) \\ &\geq \pi_J u(x_B) + (1 - \pi_J) u(x_G), \end{aligned}$$

meaning that the consumer is better off buying no insurance at all.

**Remark 1** *Contract specifying consumption in each state is without loss of generality. This is called a “revelation principle”. The idea is that if the monopolist designs any sort of contract, say, where the price is a highly non-linear function of how much insurance is bought, when the optimal choice is eventually made the agent ends up with some CONSUMPTION in each state. We can always replicate this by removing from the choice set all levels of insurance that are not purchased (except 0 since we take the view that the consumer must be willing to buy).*

## 13.2 Non-Observable Types (Private Information)

Again, for the same reasons as above, an insurance contract can be viewed as two numbers  $(x_B, x_G)$ . From these numbers we may define concepts that may be more familiar in real world insurance.

$$\text{Premium} = P = m_G - x_G$$

$$\text{Benefit} = B = x_B + P - m_B = x_B + m_G - x_G - m_B$$

Notationally it is simpler to perform analysis in terms of  $(x_B, x_G)$ , but it is equivalent with maximizing over  $(P, B)$ .

The crucial aspect when the monopolist cannot see who is who is that  $L$  must be willing to pick contract designed for  $L$  and  $H$  must be willing to pick contract designed for  $H$ . This yields the following problem. The monopolist designs two contracts,  $(x_B^H, x_G^H)$  and  $(x_B^L, x_G^L)$

to solve

$$\begin{aligned} \max_{x_B^H, x_G^H, x_B^L, x_G^L} & \underbrace{\alpha \left[ \pi_L (m_B - x_B^L) (1 - \pi_L) (m_G - x_G^L) \right]}_{\text{expected profit if type is } L} & (1) \\ & + (1 - \alpha) \underbrace{\left[ \pi_H (m_B - x_B^H) + (1 - \pi_H) (m_G - x_G^H) \right]}_{\text{expected profit if type is } H} \end{aligned}$$

$$\pi_J u(x_B^J) + (1 - \pi_J) u(x_G^J) \geq \pi_J u(m_B) + (1 - \pi_J) u(m_G) \quad (2)$$

$$\pi_L u(x_B^L) + (1 - \pi_L) u(x_G^L) \geq \pi_L u(x_B^H) + (1 - \pi_L) u(x_G^H) \quad (3)$$

$$\pi_H u(x_B^H) + (1 - \pi_H) u(x_G^H) \geq \pi_H u(x_B^L) + (1 - \pi_H) u(x_G^L) \quad (4)$$

We will be able to use graphs for most of the analysis. But, to get to this point we need to be able to compare slopes of the indifference curves for type  $L$  and  $H$ .

### 13.3 Reminder on Slopes of Indifference Curves

*You can skip this Section if you are comfortable with slopes of indifference curves. There are several ways to understand this. I don't care which way you understand it, as long as you can figure out what the slope of a particular indifference curve is in some way.*

Fix a point  $(x_B^*, x_G^*)$ . The indifference curve that for type  $L$  that goes through this point is then defined as all  $(x_B, x_G)$  that satisfies

$$\begin{aligned} \pi_L u(x_B) + (1 - \pi_L) u(x_G) &= \pi_L u(x_B^*) + (1 - \pi_L) u(x_G^*) \\ &\Leftrightarrow \end{aligned}$$

$$\pi_L [u(x_B) - u(x_B^*)] + (1 - \pi_L) [u(x_G) - u(x_G^*)] = 0$$

As long as  $x_B \neq x_B^*$  (so that we avoid dividing by zero) we may write this as

$$\begin{aligned} 0 &= \pi_L \left[ \frac{u(x_B) - u(x_B^*)}{x_B - x_B^*} \right] + (1 - \pi_L) \left[ \frac{u(x_G) - u(x_G^*)}{x_B - x_B^*} \right] \\ &= \pi_L \left[ \frac{u(x_B) - u(x_B^*)}{x_B - x_B^*} \right] + (1 - \pi_L) \left[ \frac{u(x_G) - u(x_G^*)}{x_G - x_G^*} \right] \left[ \frac{x_G - x_G^*}{x_B - x_B^*} \right] \end{aligned}$$

Now,

1. Assuming differentiability

$$\lim_{x_B \rightarrow x_B^*} \frac{u(x_B) - u(x_B^*)}{x_B - x_B^*} = u'(x_B^*)$$

$$\lim_{x_G \rightarrow x_G^*} \frac{u(x_G) - u(x_G^*)}{x_G - x_G^*} = u'(x_G^*)$$

2. Interpretation: If  $x_B$  is close to  $x_B^*$  and  $(x_B, x_G)$  on same indifference curve as  $(x_B^*, x_G^*)$  we have that

$$0 \approx \pi_L u'(x_B^*) + (1 - \pi_L) u'(x_G^*) \left[ \frac{x_G - x_G^*}{x_B - x_B^*} \right]$$

or

$$\left. \frac{\text{change in consumption with accident}}{\text{change in consumption with no loss}} \right|_{\text{utility fix}} = \frac{x_B - x_G^*}{x_G - x_B^*} \approx \frac{(1 - \pi_L) u'(x_G^*)}{\pi_L u'(x_B^*)}$$

3. Can derive the same thing by letting the indifference curve be described by a function  $f_L(x_G)$  that solves

$$\pi_L u(f_L(x_G)) + (1 - \pi_L) u(x_G) = \pi_L u(x_B^*) + (1 - \pi_L) u(x_G^*)$$

for every  $x_G$  (in some interval around  $x_G^*$ ). Taking derivatives we get

$$\begin{aligned} \frac{d}{dx_G} [\pi_L u(f_L(x_G)) + (1 - \pi_L) u(x_G)] &= \pi_L u'(f(x_G)) \frac{df_L(x_G)}{dx_G} + (1 - \pi_L) u'(x_G) \\ &= \frac{d}{dx_G} [\pi_L u(x_B^*) + (1 - \pi_L) u(x_G^*)] = 0 \\ &\Leftrightarrow \end{aligned}$$

$$\text{Slope of indifference curve for type } L = \frac{df_L(x_G)}{dx_G} = -\frac{(1 - \pi_L) u'(x_G)}{\pi_L u'(f(x_G))}$$

Finally, evaluate at  $x_G = x_B^* \Rightarrow f(x_G^*) = x_B^*$ ;

$$\text{Slope of indifference curve for } L \text{ at } (x_G^*, x_B^*) = \frac{df_L(x_G^*)}{dx_G} = -\frac{(1 - \pi_L) u'(x_G^*)}{\pi_L u'(x_B^*)}$$

Obviously, we can do same thing for type

$$\text{Slope of indifference curve for } H \text{ at } (x_G^*, x_B^*) = \frac{df_H(x_G^*)}{dx_G} = -\frac{(1 - \pi_H) u'(x_G^*)}{\pi_H u'(f(x_G^*))}$$

### 13.4 The Low Risk Type Has Steeper Indifference Curves Everywhere

Now, just comparing the slopes at any point  $(x_B^*, x_G^*)$  we have that

$$\begin{aligned} \frac{\text{Slope of indifference curve for } L \text{ at } (x_B^*, x_G^*)}{\text{Slope of indifference curve for } H \text{ at } (x_B^*, x_G^*)} &= \frac{\frac{df_L(x_G^*)}{dx_G}}{\frac{df_H(x_G^*)}{dx_G}} = \frac{\frac{(1-\pi_L)u'(x_G^*)}{\pi_L u'(x_B^*)}}{\frac{(1-\pi_H)u'(x_G^*)}{\pi_H u'(x_B^*)}} \\ &= \frac{\frac{(1-\pi_L)}{\pi_L}}{\frac{(1-\pi_H)}{\pi_H}} = \frac{(1-\pi_L)\pi_H}{\pi_L(1-\pi_H)} > 1 \end{aligned}$$

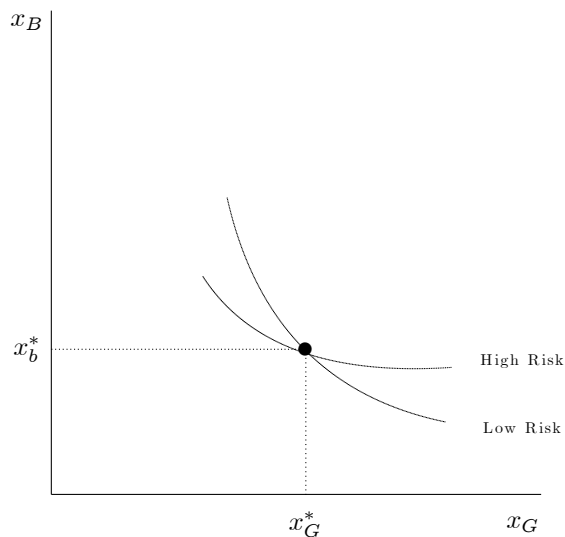


Figure 7: Relative Slopes of Indifference Curves

## 14 Monopolist “Indifference Curves” (Isoprofits)

Suppose that the monopolist sells contract  $(x_B, x_G)$  to a consumer with low risk. Then, the expected profit is

$$\pi_L (x_B - m_B) + (1 - \pi_L) (x_G - m_G),$$

where we usually would have that  $x_B - m_B < 0$  and  $x_G - m_G > 0$ . An “isoprofit” is then simply a line with constant profits, that is, solutions to

$$\begin{aligned} \pi_L (x_B - m_B) + (1 - \pi_L) (x_G - m_G) &= k \\ x_B &= -\frac{1 - \pi_L}{\pi_L} x_G + \frac{k + \pi_L m_B + (1 - \pi_L) m_G}{\pi_L} \end{aligned}$$

I.e., straight lines with slope  $-\frac{1 - \pi_L}{\pi_L}$ . Similarly, the relevant isoprofit lines for a high risk individual are

$$x_B = -\frac{1 - \pi_H}{\pi_H} x_G + \frac{k + \pi_H m_B + (1 - \pi_H) m_G}{\pi_H}$$

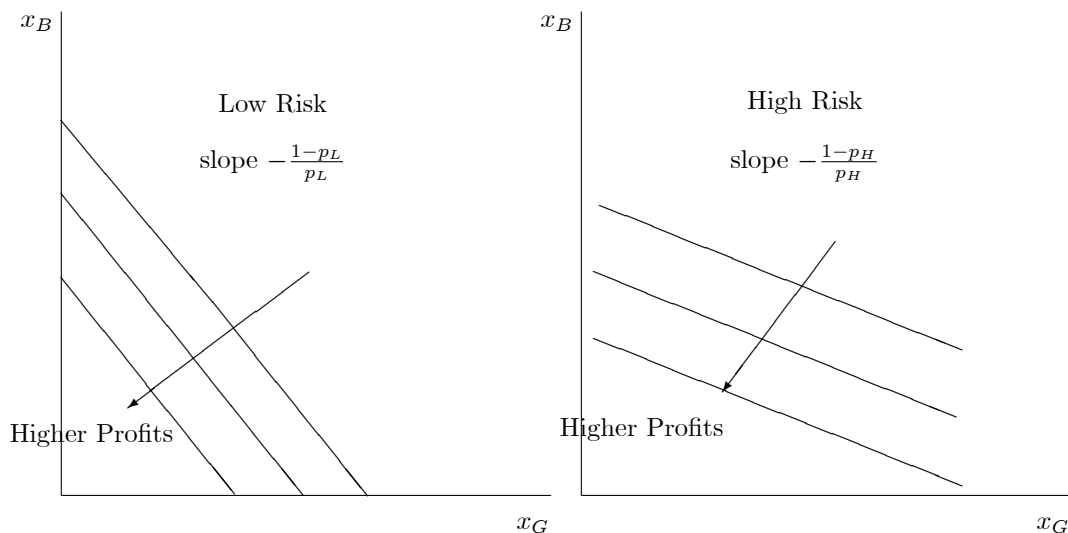


Figure 8: Constant Profit Loci (Isoprofits) for Low and High Risk Type

## 15 The Profit Maximizing Contract

**Step 1** *If the low risk type isn't insured (eg., if  $(x_B^L, x_G^L) = (m_B, m_G)$ ), then optimal contract fully insures the high risk type at a premium that extracts all the consumer surplus from the high risk type.*

**Proof.** See Picture. The straight line is the isoprofit when selling to the low risk type only that goes through the full insurance point at indifference curve through endowment. Any

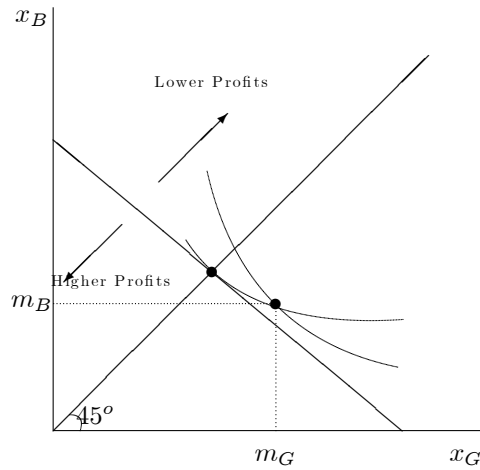


Figure 9: Why High Risk Type is Fully Insured if No Insurance Sold to Other Type

Plan that gives a higher profit therefore violates the Individual rationality constraint for the high risk type. ■

**Step 2**  $x_G^L \geq x_G^H$

**Proof.**

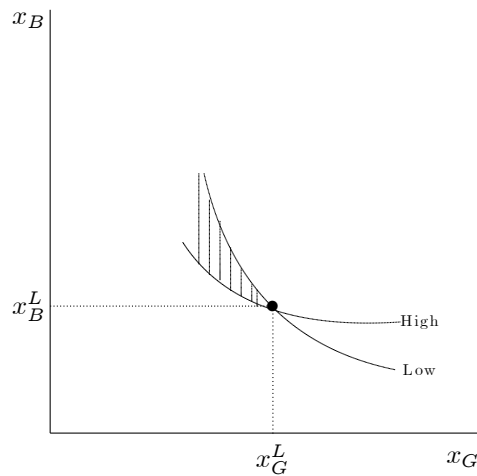


Figure 10: Why  $x_G^L$  is at least as large as  $x_G^H$  in Optimal Solution to Contracting Problem

See the Figure. Fix  $(x_B^L, x_G^L)$  arbitrarily. For IC-L to be satisfied (i.e., for  $L$  to be better off with  $(x_B^L, x_G^L)$  than with  $(x_B^H, x_G^H)$ ) it must be that  $(x_B^H, x_G^H)$  is below the indifference

curve for the low risk type. For IC-H to hold (i.e., for  $H$  to be better off with  $(x_B^H, x_G^H)$  than with  $(x_B^L, x_G^L)$ ) it must be that  $(x_B^H, x_G^H)$  is above the indifference curve for the high risk type. Hence, only the shaded area in the Figure remains, which proves the claim. ■

**Step 3**  $x_G^L \leq m_G$  (no “anti-insurance”).

**Proof.** Suppose that  $x_G^L > m_G$ . Consider Fig 11, where  $l$  is the hypothetical optimal contract for type  $L$  and point  $A$  is the point where the indifference curve through the endowment for the high risk type intersects the indifference curve for the low risk type through point  $l$ . To satisfy IR-H and IC-L it is therefore necessary that the contract for type  $H$  is in the wedge beginning at point  $A$ . Note then that;

1) If (as in Figure) type  $L$  is at a higher indifference curve than the one through the endowment, then it is possible to reduce the consumption for  $L$  in (say) the bad state and keep everything the same. This increases the expected profit for monopolist. If instead type  $L$  is at the same indifference curve as the endowments (redraw the picture) point  $A$  coincides with the endowment. Moving type  $L$  to the endowment will keep IC-H satisfied. Since this is a movement in the direction of increased insurance along a given indifference curve the monopolist will increase its profit.

**Step 4** *IC-H binds*

**Proof.** See Figure. If the incentive constraint for the high risk type is not binding the monopolist may reduce the consumption in one state of the world for the high risk type and keep everything else the same. Because of Step 3, point  $l$  in the graph is at least as good as the endowment for the high risk type, so the movement from  $h$  to  $h'$  (which corresponds to reducing the consumption for  $H$  in case of accident) will satisfy both IR-H and IC-H. Obviously this increases the profits for the monopolist.

*Notice that this argument uses the result that the low risk type doesn't get “anti-insurance” in Step 3 to rule out the possibility that point  $l$  is worse for  $H$  than the endowment, in which case  $h'$  would not satisfy IR-L. ■*

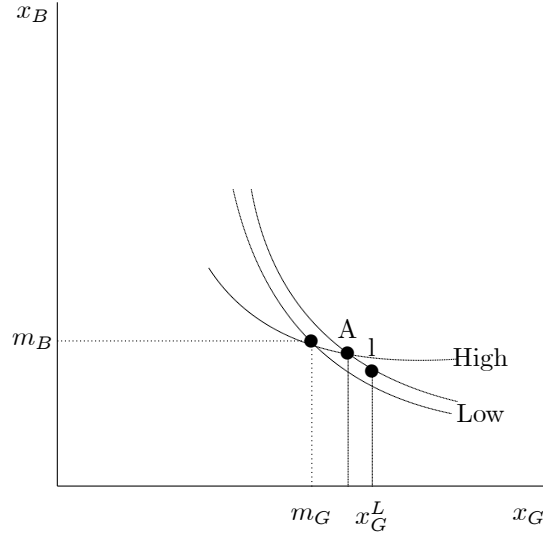


Figure 11: No Anti-Insurance at the Optimum

Then, we realize that moving the high risk contract to the endowment increases profits on the high risk type (profits increasing in direction of full insurance). For the low risk type, moving the contract down to the point on indifference curve that goes through the endowment increases profits (you give less in case of a loss and keep consumption constant when there is no accident). Finally, moving the low risk to the endowment increases the profits further (profits increasing in direction of full insurance). ■

**Step 5** *IR-L binds*

**Proof.** See Picture. Fixing the contract for type  $L$  (point  $l$  in graph) we know that the contract for  $H$  must be in the “wedge”. Call that contract  $h$ . Now, offer  $l'$  to type  $L$ , where the only difference is that the consumption in the case of an accident is reduced to make IR-L binding (consumption when no accident is unchanged). This is obviously better for monopolist, but could possibly upset IC-H. However, by simultaneously reducing the consumption in case of an accident for type  $H$  by moving from  $h$  to  $h'$  we see that both incentive constraints will hold, and, again, reducing the consumption in case of an accident and keeping it constant when there is no accident is better for the monopoly provider.

**Step 6** *L is not over insured.*

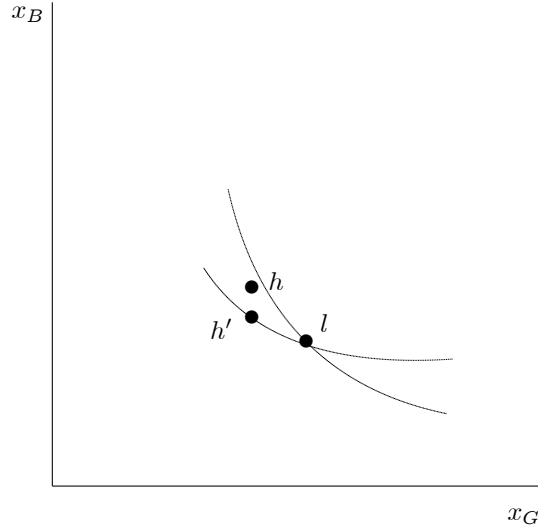


Figure 12: IC-H Binds

**Proof.** See Figure. If  $L$  is over insured it must be that he gets a contract like the point  $l$ . Since IC-H binds,  $H$  gets a contract on the indifference curve through  $l$  and to the left of  $l$ . Hence, moving from  $l$  to  $l'$  doesn't change the utility for  $L$  and the incentive constraint for  $H$  remains satisfied. The picture is a bit bad, but the straight line is supposed to be the isoprofit for the firm (when selling to  $L$ ), which is tangent to the indifference curve at point  $l'$  where  $L$  gets full insurance. Hence,  $l'$  gives a higher profit than  $l$  (since it is a movement on an indifference curve in the direction of full insurance). ■

**Step 7** *Full insurance for High Risk Type*

**Proof.** Draw a Picture! Fix  $(x_B^L, x_G^L)$  anywhere between full insurance point and endowment point on indifference curve going through the endowment. Highest profit along indifference curve for high risk type is full insurance (just like the reasoning in previous picture). ■

Hence we have;

**Proposition 2** *The optimal contract has the following features.*

1. *High risk type fully insured*

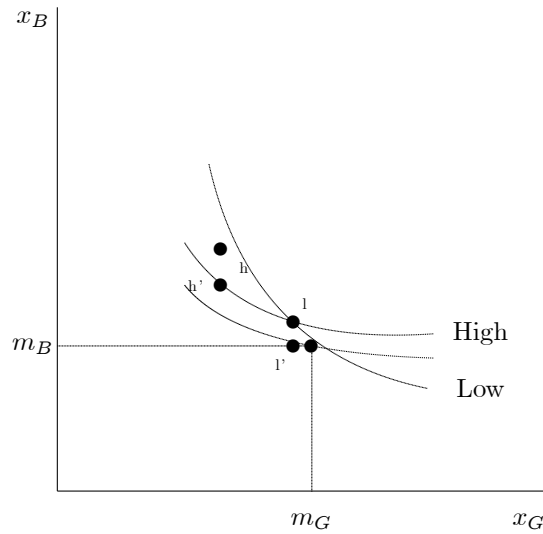


Figure 13: IR-L holds with Equality

2. *High risk type indifferent between his and low risk type contract*

3. *Low risk type at reservation utility level.*

Remarks;

1. High risk type earns informational rents. Can get some of gains from trade due to informational advantage.
2. Trade-off for monopolist. Efficiency gains of full insurance versus how much surplus can be extracted from High risk type.
3. Example of price discrimination/non-linear pricing
4. Also note; any observable variable that would be correlated with risk or willingness to pay should be used by monopolist. In example, no such observable variables exist.

## 16 Adverse Selection in Insurance and Competition

We maintain the assumptions;

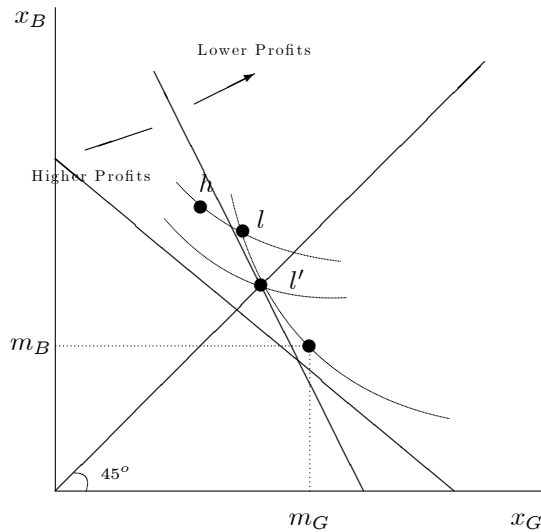


Figure 14: Monopolist can improve on contract  $L$  is overinsured

- Suppose that there are two “types” of consumers. Call them  $\{H, L\}$
- Type  $H$  has a probability of an accident given by  $\pi_H$
- Type  $L$  has a probability of an accident gives by  $\pi_L$
- For both types, the endowment is  $\{m_G, m_B\}$ , where  $m_G > m_B$  (i.e., “state  $B$ ” is when loss occurs),

from the analysis of a monopoly. However, instead of assuming that there is a risk neutral monopoly provider of insurance, we assume that there is a “competitive market”. Exactly how to think about a “competitive market” is a bit up for grabs. Rothschild and Stiglitz (which is the most well-known contribution and which is what this is based on) defines a competitive equilibrium as follows:

**Definition 1** *An equilibrium in a competitive insurance market is a set of insurance contracts such that;*

1. *Every contract in the equilibrium set is at least as good as all other contracts for at least some type of consumer (i.e., all contracts in the set are chosen by somebody).*

2. *No contract in the equilibrium set makes negative expected profits given that consumers choose contracts to maximize expected utility.*
3. *There exists no contract outside the equilibrium set that, if offered, would make a strictly positive profit.*

This is somewhat blurry (and the blurriness has led to a bit of a literature), but the idea is roughly that there are two stages;

**Stage 1** Firms simultaneously offer contracts. Once offered, there is no possibility for firms to withdraw contract offers.

**Stage 2** Consumers pick whatever contract they want (by assumption, a consumer can only buy a single insurance contract).

More recently, the game theory behind this has been clarified, but I will avoid that (at least for now).

## 16.1 Benchmark: A single Type (or type being observable)

Suppose that there is a single type with accident probability  $\pi$  (the analysis also applies to the case where accident probabilities  $\pi_L$  and  $\pi_H$  are known). Suppose that  $(x_B^*, x_G^*)$  is in the equilibrium set (which means that the consumer must be willing to buy). We know that  $(x_B^*, x_G^*)$  must be weakly profitable (this is part 2 of the definition-the rationale is that offering no contract would be better otherwise). Suppose that the contract is strictly profitable, that is

$$\Pi(x_B^*, x_G^*, \pi) = \pi(m_B - x_B^*) + (1 - \pi)(m_G - x_G^*) > 0.$$

Then, for  $\varepsilon$  sufficiently small we have that

$$\begin{aligned} \Pi(x_B^* + \varepsilon, x_G^* + \varepsilon, \pi) &= \pi(m_B - x_B^* - \varepsilon) + (1 - \pi)(m_G - x_G^* - \varepsilon) \\ &= \Pi(x_B^*, x_G^*, \pi) - \varepsilon > 0, \end{aligned}$$

which means that  $(x_B^* + \varepsilon, x_G^* + \varepsilon)$  makes a strictly positive profit. If  $(x_B^* + \varepsilon, x_G^* + \varepsilon)$  is part of the equilibrium set  $(x_B^*, x_G^*)$  would not be chosen, so we conclude that *any contract in the equilibrium set must make zero expected profits*.

Next, we observe that, if the consumer is *strictly* risk averse, then the only contract in the equilibrium set is full insurance at an actuarially fair rate:

**Proposition 3** *Suppose that  $u(\cdot)$  is strictly concave (=strict risk aversion= $u''(x) < 0$  for every  $x$ ). Then*

$$x_B^* = x_G^* = \pi m_B + (1 - \pi) m_G$$

*is the only contract in the equilibrium set.*

In class we proved this by picture (which is OK).

An algebraic proof is as follows. Any contract in the equilibrium set must make zero profits, that is, satisfy

$$\pi x_B + (1 - \pi) x_G = \pi m_B + (1 - \pi) m_G$$

If  $x_B \neq x_G$  we have that

$$u(\pi m_B + (1 - \pi) m_G) = u(\pi x_B + (1 - \pi) x_G) > \pi u(x_B) + (1 - \pi) u(x_G),$$

which implies that 1) if  $x_B^* = x_G^* = \pi m_B + (1 - \pi) m_G$  is offered alone it satisfies the definition of equilibrium, and; 2) if any other contract is offered, then there exists a full insurance contract that makes a strictly positive profit. This second part follows since if

$$u(\pi m_B + (1 - \pi) m_G) > \pi u(x_B) + (1 - \pi) u(x_G)$$

we can always find an  $\varepsilon > 0$  such that

$$u(\pi m_B + (1 - \pi) m_G - \varepsilon) > \pi u(x_B) + (1 - \pi) u(x_G).$$

## 16.2 Two Types

Really, there would be some work to establish that it is not possible to create equilibria with multiple contracts designed for the same group. At the intuitive level it is pretty clear though; if one type buys two different contracts (i.e., you may think of it as the type randomizing) it must be that;

1. Both contracts break even, and;
2. Both contracts are on the same indifference curve.

Given strict concavity this is impossible. Now, to really work this out is a bit messy and I will leave it to the interested to deal with these details and take for granted that at most two contracts are offered in the equilibrium. Then there are three distinct possibilities:

**Definition 2** *An equilibrium is said to be a pooling equilibrium if both groups buy the same contract; that is  $(x_B^L, x_G^L) = (x_B^H, x_B^H)$*

**Definition 3** *An equilibrium is said to be a separating equilibrium if H and L purchase different contracts: that is  $(x_B^L, x_G^L) \neq (x_B^H, x_B^H)$ .  $(x_B^L, x_G^L) = (x_B^H, x_B^H)$*

**Definition 4** *An equilibrium is said to be a hybrid (semi-pooling, semi-separating) equilibrium if one group picks a contract for sure and the other group randomizes between the contracts.*

**Lemma 1** *There cannot be a pooling equilibrium.*

**Proof.** To understand this, let  $\alpha$  be the proportion of high risk agents. In a pooling equilibrium, the profit on the equilibrium contract  $(x_B^*, x_G^*)$  must be

$$\begin{aligned}
 & \alpha [\pi_H (m_B - x_B^*) + (1 - \pi_H) (m_G - x_G^*)] + (1 - \alpha) [\pi_L (m_B - x_B^*) + (1 - \pi_L) (m_B - x_B^*)] \\
 = & [\alpha \pi_H + (1 - \alpha) \pi_L] (m_B - x_B^*) + [\alpha (1 - \pi_H) + (1 - \alpha) (1 - \pi_L)] (m_G - x_G^*) \\
 = & \hat{\pi} (m_B - x_B^*) + (1 - \hat{\pi}) (m_G - x_G^*) = \Pi (x_B^*, x_G^*, \hat{\pi}),
 \end{aligned}$$

where

$$\widehat{\pi} = \alpha\pi_H + (1 - \alpha)\pi_L.$$

Just like in the case with a single type it has to be that profits are zero in equilibrium (the argument is identical), that is

$$\Pi(x_B^*, x_G^*, \widehat{\pi}) = \widehat{\pi}(m_B - x_B^*) + (1 - \widehat{\pi})(m_B - x_B^*) = 0.$$

Now, recall (go back to the monopoly case) that the slope of the indifference curve at  $(x_B^*, x_G^*)$  is

$$\begin{aligned} & -\frac{(1 - \pi_L)u'(x_G^*)}{\pi_L u'(x_B^*)} \text{ for type } L \\ & -\frac{(1 - \pi_H)u'(x_G^*)}{\pi_H u'(x_B^*)} \text{ for type } H \end{aligned}$$

Because of these differences in the slopes, for every  $\varepsilon > 0$  there exists a contract  $x' = (x'_B, x'_G)$  such that (see the Figure to convince yourself...the crucial fact is that the slope of the “zero profit line with pooling” must be in between the slope of the indifference curves for  $L$  and  $H$ . One way to realize this is to first note that a movement in the direction of full insurance is strictly better for both types, so the only possible equilibrium pooling contract involves full insurance)

$$\begin{aligned} \pi_H u(x'_B) + (1 - \pi_H)u(x'_G) &< \pi_H u(x_B^*) + (1 - \pi_H)u(x_G^*) \\ \pi_L u(x'_B) + (1 - \pi_L)u(x'_G) &> \pi_L u(x_B^*) + (1 - \pi_L)u(x_G^*) \\ \|x' - x^*\| &\leq \varepsilon \end{aligned}$$

In words (see Figure 15), for every  $\varepsilon$  there is a contract within  $\varepsilon$  distance from  $x^*$  such that

1. The low risk type has a strict incentive to take it
2. The high risk type has a strict incentive to go with the original contract.

Moreover,

$$\begin{aligned} \Pi(x'_B, x'_G, \pi_L) &= \pi_L(m_B - x'_B) + (1 - \pi_L)(m_B - x'_B) \\ &> \pi_L(m_B - x_B^*) + (1 - \pi_L)(m_B - x_B^*) - \varepsilon \\ &> \widehat{\pi}(m_B - x_B^*) + (1 - \widehat{\pi})(m_B - x_B^*) = 0 \end{aligned}$$

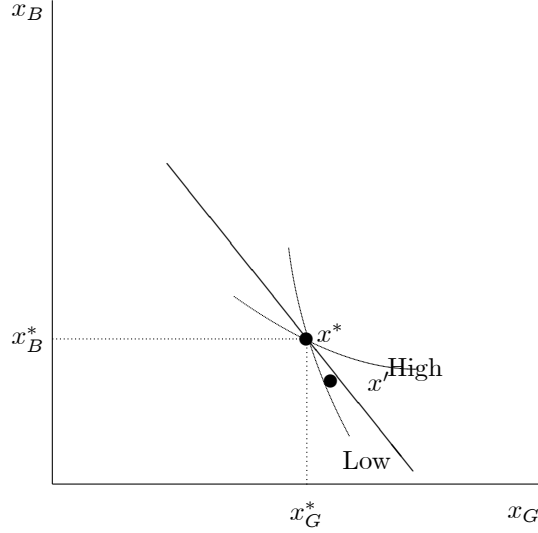


Figure 15: Breaking a Pooling Contract

if  $\varepsilon$  is small enough. We conclude that the suggested deviation breaks any possible pooling contract. ■

**Lemma 2** *No hybrid equilibrium can exist.*

Same argument essentially.

We thus conclude that the only possibility is a separating equilibrium, that is, one contract designed for each type. Again, each contract in the equilibrium set must make a zero profit.

**Proposition 4** *The only possible equilibrium is one where;*

1. *The high risk type purchases full insurance at actuarially fair rates  $x_B^H = x_G^H = \pi_H m_B + (1 - \pi_H) m_G$ .*
2. *The low risk type get a partial insurance contract  $(x_B^L, x_G^L)$  where  $(x_B^L, x_G^L)$  is the unique solution to*

$$\begin{aligned} \pi_H u(x_B^L) + (1 - \pi_H) u(x_G^L) &= u(\pi_H m_B + (1 - \pi_H) m_G) \\ \pi_L x_B^L + (1 - \pi_H) x_G^L &= \pi_L m_B + (1 - \pi_L) m_G \end{aligned}$$

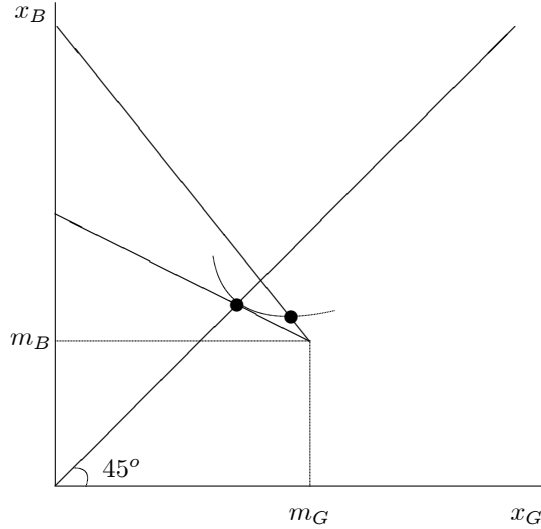


Figure 16: The Only Equilibrium Candidate

To see this, look at the figure. The two lines are the zero profit lines for the high and the low risk customers (the flatter corresponding to type  $H$ ). It is rather clear that type  $L$  must be offered full insurance, since moving along the zero profit line towards full insurance makes the low risk type worse off. Then, the low risk type is happier the closer to full insurance he/she gets along the zero profit line, but the contract must be picked so that type  $H$  doesn't have an incentive to get the contract designed for  $L$ . The best (for type  $L$ ) such contract is where the indifference curve for type  $H$  intersects the zero profit line (for  $L$ ).

**Corollary 1** *There may be no equilibrium in the model (something that occurs if there are relatively few high risks).*

The deviation to consider is a “pooling contract” offering full insurance at a rate which is slightly worse than a fair rate. It is obvious that  $H$  will prefer such a contract, and if

$$u(\hat{\pi}m_B + (1 - \hat{\pi})m_G) > \pi_L u(x_B^L) + (1 - \pi_L) u(x_G^L)$$

which will hold true if the proportion of type  $H$  is sufficiently small, then we conclude that there exists no equilibrium.

**Remark 2** *In parts, analysis agrees with monopoly case; high risk type gets full insurance and (if there is an equilibrium) low risk type gets partial insurance.*

**Remark 3** *One difference is that in competitive case, if the high risks would just admit being high risks, this would make all consumers better off. In monopoly case, it would make all consumers worse off.*

**Remark 4** *The possibility of nonexistence of equilibria has led to many “fixes” being proposed. Many of them “work”, but, in the end, the timing in the original Rothschild and Stiglitz paper is very natural.*

**Remark 5** *Fixes are either explicitly game theoretic or more loose, but the key issue is what would happen “after the deviator proposes something that shouldn’t happen”. The analysis here has been taking for granted that the firms offering the candidate equilibrium contracts are stuck with their commitments. Many alternatives assumes that firms can withdraw contract offers that lose money, which changes the analysis quite a bit.*