

MIDTERM
ANSWER KEY

1.

$$C(y) = 1 + y + 0.5y^2$$

$$P(y) = 2 - 0.5y$$

$$\text{Profit of the monopolist} = \Pi = P(y) \cdot y - C(y)$$

$$\Pi = (2 - 0.5y) \cdot y - (1 + y + 0.5y^2)$$

$$\text{Max}_y \Pi = (2 - 0.5y) \cdot y - (1 + y + 0.5y^2)$$

FOC :

$$2 - y - 1 - y = 0$$

$$y^* = 1/2$$

Plugging into inverse demand function;

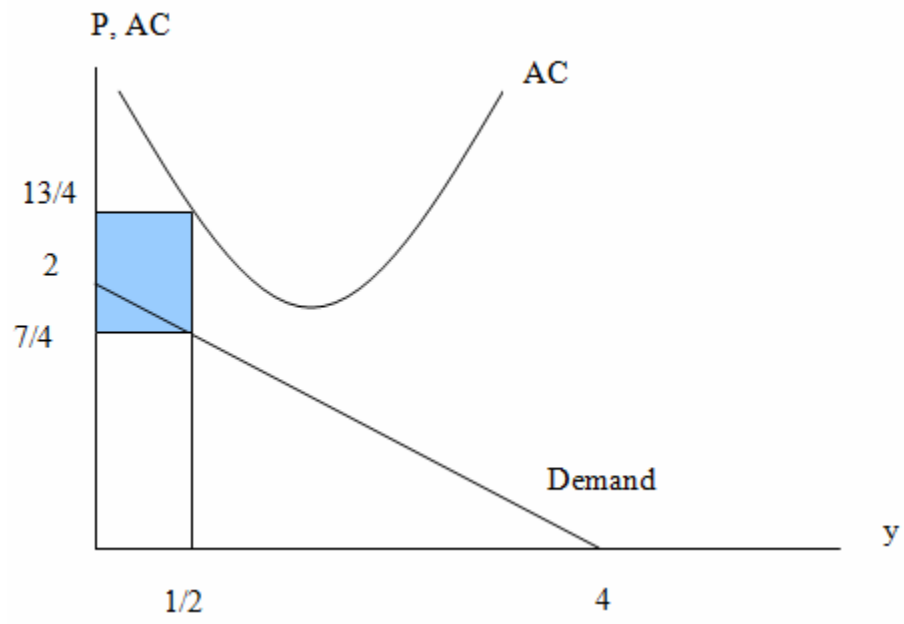
$$p^* = 7/4$$

Let's calculate the profit;

$$\Pi = (1/2) \cdot (7/4) - (13/8) = -6/8$$

Profit is negative even if the monopolist maximizes its profit. Therefore, it is not profitable for the monopolist to produce.

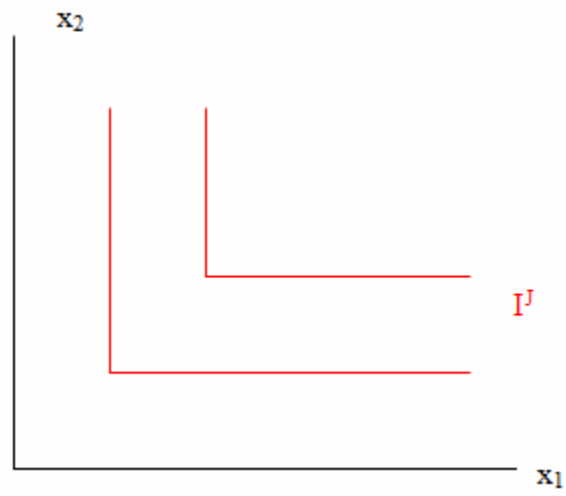
The loss of the monopolist is illustrated with the blue shaded area below.



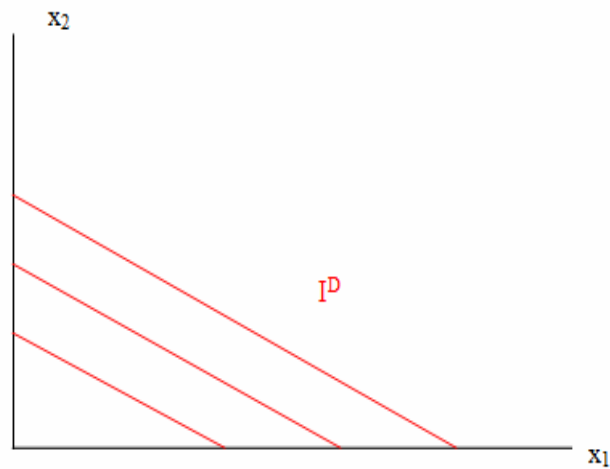
2.

a.

Jim:



Dwight:



b.

Because of the nature of Jim's preference: $x_1^J = x_2^J$ and

$$p_1 x_1^J + p_2 x_2^J \leq 2p_1 + 8p_2$$

Dwight maximizes his utility function subject to his budget constraint:

$$\text{Max } x_1^D + 2x_2^D + 3$$

Subject to;

$$p_1 x_1^D + p_2 x_2^D \leq 6p_1 + 2p_2$$

$$L = x_1^D + 2x_2^D + 3 - \lambda(p_1 x_1^D + p_2 x_2^D - 6p_1 - 2p_2)$$

FOC:

$$1 - \lambda p_1 = 0$$

$$2 - \lambda p_2 = 0$$

$$p_1/p_2 = 1/2$$

$$p_2 = 2p_1$$

Plugging into Jim's budget constraint;

$$p_1 x_1^J + 2p_1 x_1^J = 2p_1 + 16p_1$$

$$x_1^J = 6$$

Since $x_1^J = x_2^J$

$$x_2^J = 6$$

By market clearing constraints;

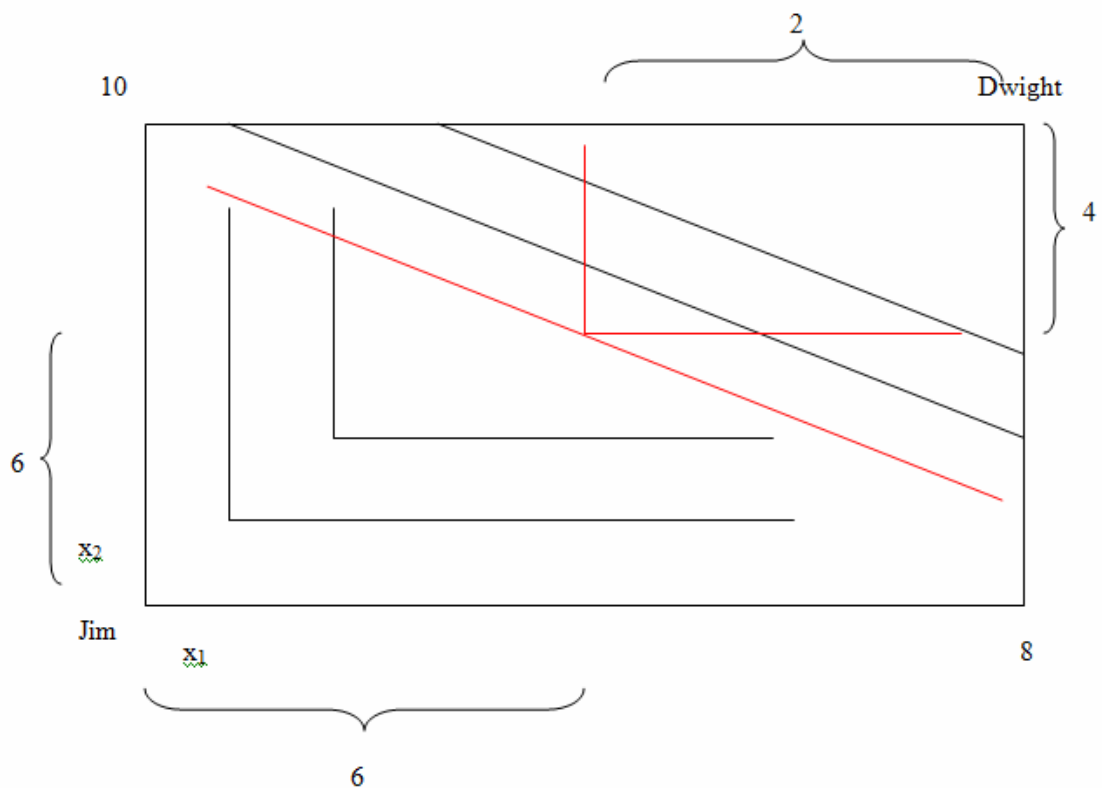
$$x_1^J + x_1^D = 8$$

$$x_2^J + x_2^D = 10$$

Then,

$$x_1^D = 2$$

$$x_2^D = 4$$



3.

a.

(y^{1*}, y^{2*}) is a Nash equilibrium if;

$$y^{1*} \text{ solves } \max_{y^1} U^1(1 - y^1, y^1 + y^{2*}) = (1 - y^1) * (y^1 + y^{2*})$$

and

$$y^{2*} \text{ solves } \max_{y^2} U^2(1 - y^2, y^{1*} + y^2) = (1 - y^2) * (y^{1*} + y^2)$$

b.

$$\max_{y^1} U^1(1 - y^1, y^1 + y^{2*}) = (1 - y^1) * (y^1 + y^{2*})$$

$$\text{FOC: } 1 - 2y^{1*} - y^{2*} =$$

$$y^{1*} = (1 - y^{2*}) / 2$$

$$\max_{y^2} U^2(1 - y^2, y^{1*} + y^2) = (1 - y^2) * (y^{1*} + y^2)$$

$$\text{FOC: } 1 - 2y^{2*} - y^{1*} =$$

$$y^{2*} = (1 - y^{1*}) / 2$$

c.

Solving FOC's simultaneously;

$$y^{2*} = 1/3$$

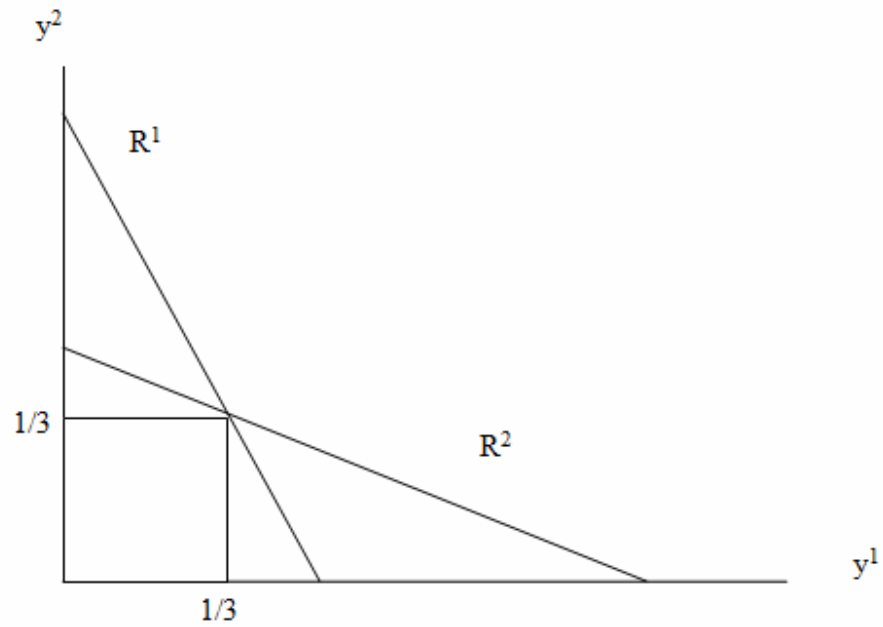
$$y^{1*} = 1/3$$

since

$$x^{1*} = 1 - y^{1*} \quad \text{and} \quad x^{2*} = 1 - y^{2*}$$

$$x^{1*} = 2/3$$

$$x^{2*} = 2/3$$



d.

$$\max x^1 y$$

s.t.

$$x^2 y \geq u$$

$$x^1 + x^2 + y \leq 2$$

$$\max L = x^1 y - \lambda(x^2 y - u) - \gamma(x^1 + x^2 + y - 2)$$

FOC:

$$y - \gamma = 0$$

$$-\lambda y - \gamma = 0$$

$$x^1 - \lambda x^2 - \gamma = 0$$

Solving simultaneously gives;

$$y = 1$$

and since the equilibrium is symmetric,

$$x^1 = 1/2$$

$$x^2 = 1/2$$

e.

A Lindahl mechanism can be implemented to obtain a symmetric Pareto optimal allocation.

A Lindahl Equilibrium is a vector $p^* = (p^{1*}; p^{2*})$ and an allocation $(x^{1*}; x^{2*}; y^*)$ such that

- $p^{1*} + p^{2*} = 1$
- Each consumer maximizes utility, that is $(x^J; y^*)$ solves

$$\begin{aligned} \text{Max}_{x,y} \quad & U^J(x; y) = xy \\ \text{s.t.} \quad & 1 - x - p^{J*}y \geq 0 \end{aligned}$$

- Market clears

$$x^{1*} + x^{2*} + y^* = 2$$

$$\max_{x,y} L = xy + \lambda(1 - x - p^{J*}y)$$

FOC:

$$y^* - \lambda = 0$$

$$x - \lambda p^{J*} = 0 \quad \text{for} \quad J = 1, 2$$

Then,

$$x^{1*} = yp^{1*} \quad \text{and} \quad x^{2*} = yp^{2*}$$

Adding side by side;

$$x^{1*} + x^{2*} = y(p^{1*} + p^{2*})$$

Since $p^{1*} + p^{2*} = 1$,

$$x^{1*} + x^{2*} = y$$

And by using market clearing condition,

$$y^* = 1$$

$$x^{1*} + x^{2*} = 1$$

Since the equilibrium is symmetric,

$$x^{2*} = 1/2$$

$$x^{1*} = 1/2$$

Lindahl Equilibrium coincides with the Symmetric Pareto Optimum.

4.

a.

$$U^A(x^A, y^A, p) = x^A y^A - (y^A + y^B)^2$$

Replacing $x^A = 2 - y^A$

$$U^A = (2 - y^A)y^A - (y^A + y^B)^2$$

b.

By the same way,

$$U^A = (2 - y^B)y^B - (y^A + y^B)^2$$

(y^{A*}, y^{B*}) is a Nash equilibrium if;

$$y^{A*} \text{ solves } \max_{y^A} U^A = (2 - y^A)y^A - (y^A + y^B)^2$$

and

$$y^{B*} \text{ solves } \max_{y^B} U^A = (2 - y^B)y^B - (y^A + y^B)^2$$

c.

$$\text{Max}_{y^A} U^A = (2 - y^A)y^A - (y^A + y^B)^2$$

FOC:

$$2 - 2y^A - 2(y^A + y^B) = 0$$

$$y^A = (1 - y^B) / 2$$

$$\text{Max}_{y^B} U^B = (2 - y^B)y^B - (y^A + y^B)^2$$

FOC:

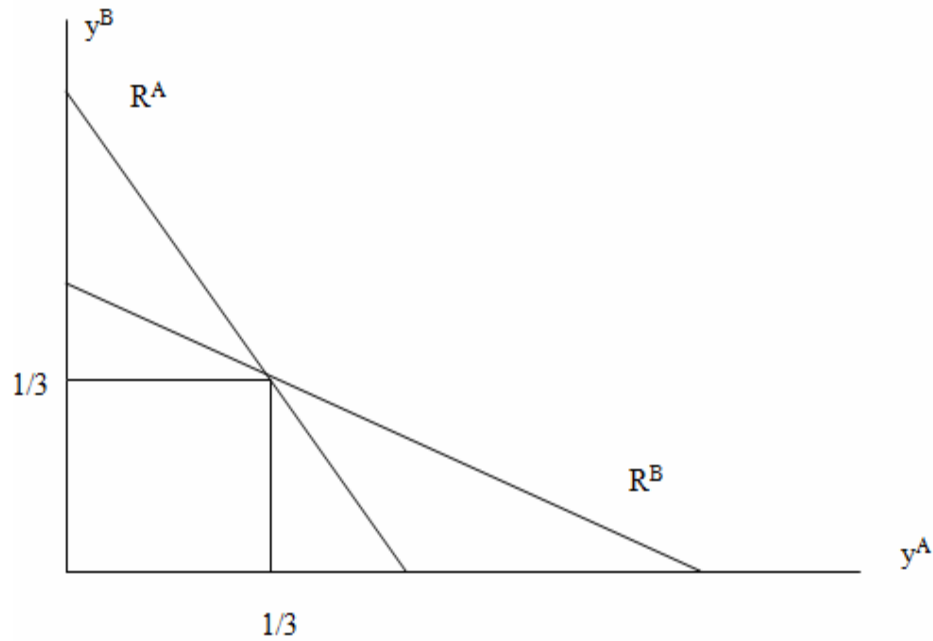
$$2 - 2y^B - 2(y^A + y^B) = 0$$

$$y^B = (1 - y^A) / 2$$

Solving FOC's simultaneously gives,

$$y^A = 1/3$$

$$y^B = 1/3$$



d.

The symmetric pareto optimal equilibrium is (y^A, y^B) if it solves the following problem;

$$\text{Max}_{y^A, y^B} y^A(2 - y^A) - (y^A + y^B)^2$$

subject to;

$$y^B(2 - y^B) - (y^A + y^B)^2 \geq u$$

$$\text{max}_{y^A, y^B} L = y^A(2 - y^A) - (y^A + y^B)^2 - \lambda(y^B(2 - y^B) - (y^A + y^B)^2 - u)$$

FOC:

$$y^A : 2 - 2y^A - 2(y^A + y^B) + 2\lambda(y^A + y^B) = 0$$

$$y^B : -2(y^A + y^B) - \lambda[2 - 2y^B - 2(y^A + y^B)] = 0$$

Since the problem is symmetric, we know that $y^A = y^B$,

By replacing in FOC's and solving simultaneously, we have

$$y^{A*} = y^{B*} = 1/5$$

$$x^{A*} = x^{B*} = 9/5$$

e. Nash Equilibrium is not Pareto efficient. We can easily check:

$$\text{(same for both players) Nash equilibrium utility} = (1/3)*(5/3) - (4/9) = 1/9$$

$$\text{(same for both players) Pareto optimal level of utility} = (1/5)*(9/5) - (4/25) = 5/25$$

Pareto optimal level of utility is greater than the Nash equilibrium utility level for both players. It is possible to make everybody better off. Thus, Nash equilibrium cannot be Pareto optimal.