17 Signaling Games

Consider the following formulation of the job market signalling model due to Spence;

- Types are given by $\alpha \in \{1, 2\}$
- μ_0 is the probability that $\alpha = 1$
- A Worker may choose (i.e. commit to) any education of length $t \geq 0$
- The utility is given by
$$u(w, t, \alpha) = w - \frac{t}{\alpha}$$
- Game: 1) Worker choose education. 2) Firms compete Bertrand for workers and get a profit
$$\pi = \alpha - w$$
if it manages to attract the worker.

Let $\mu(t)$ denote the probability that firms' asses that $\alpha = 1$. In (a perfect Bayesian) equilibrium, the firms must both optimize given beliefs, implying that

$$w(t) = 2 - \mu(t)$$

17.0.1 Separating Equilibria

- Given that we restrict attention to equilibria where firms behave optimally after any $t \geq 0$ is follows that $t_1 = 0$ in any separating (perfect Bayesian) equilibrium: the reason is that if $t_1 > 0$ and the low productivity worker deviates to $t = 0$, then
$$\begin{aligned} \mu(0) &\in [0, 1] \Rightarrow \\ w(0) - 0 &\geq 1 > 1 - t_1 = w(t_1) - t_1. \end{aligned}$$
We conclude that the deviation is profitable. In words the logic is simply that the low type reveals to be the worst possible worker. Clearly, it cannot be worth anything to the low productivity worker to do so, so the worker must take the least costly action.

- By consistency of beliefs

$$\mu(t_2) = \frac{0 \times \mu_0}{0 + \mu_0 \times 1} = 0$$

so $w(t_2) = 2 - \mu(t_2) = 2$. In order for type 1 to be better off at $t_1 = 0$ than with t_2 it must be that,

$$\begin{aligned} w(t_1) &\geq w(t_2) - t_2 \\ 1 &\geq 2 - t_2 \Leftrightarrow t_2 \geq 1 \end{aligned}$$

and for type 2 to be better of with t_2 than with $t_1 = 0$ it must be that

$$\begin{aligned} w(t_2) - \frac{t_2}{2} &\geq w(0) \\ 2 - \frac{t_2}{2} &\geq 1 \Leftrightarrow t(2) \leq 2 \end{aligned}$$

Hence, if

$$1 \leq t_2 \leq 2$$

neither type has an incentive to pretend to be the other, and by considering beliefs;

$$\mu(t) = \begin{cases} 0 & \text{if } t = t_2 \\ 1 & \text{if } t \neq t_2 \end{cases}$$

it is immediate that neither type has a profitable deviation. In fact, it is sufficient to use beliefs

$$\text{or } \widetilde{\mu}(t) = \begin{cases} 0 & \text{if } t \geq t_2 \\ 1 & \text{if } t < t(2) \end{cases},$$

which you can verify by drawing a graph. $\widetilde{\mu}$ is "nicer" than μ because beliefs are monotonic in education.

17.0.2 Pooling Equilibria

To support as large a set of education levels as pooling equilibria, suppose that $t_1 = t_2 = t^*$ is a pooling equilibrium and let beliefs be

$$\mu(t) = \begin{cases} \mu_0 & \text{if } t = t^* \\ 0 & \text{if } t \neq t^* \end{cases},$$

which obviously is consistent with Bayes rule where relevant. The best deviation for both types is then to $t = 0$ and this is not profitable for the low productivity/high cost of education type if

$$2 - \mu_0 - t^* \geq 1.$$

Clearly, the high productivity type has no incentive to deviate if the low productivity type has no incentive to deviate, so any

$$0 \leq t^* \leq 1 - \mu_0$$

can be supported as a pooling equilibrium.

17.1 The Role of Commitment

The insight with the job market signalling model is that it shows that the premium paid to workers with higher education doesn't necessarily mean that education increases productivity. It could, as is the case in the model, simply be that education allows more highly productive workers screen themselves out. However, if we think of education as something that happens in real time we may complain that once the low productivity worker has decided not to educate, then the firms know that it must be a high productivity worker if they observe strictly positive education. Hence, the firms should bid for the worker immediately after the low productivity worker has been screened out, which, it seems, would destroy any separating equilibrium.

There are models addressing this issue and, while there are some subtleties, the separation in the Spence model is rescued essentially by randomizations where there is a small probability ($\rightarrow 0$ as period length $\rightarrow 0$) that the low type pools with the high type.