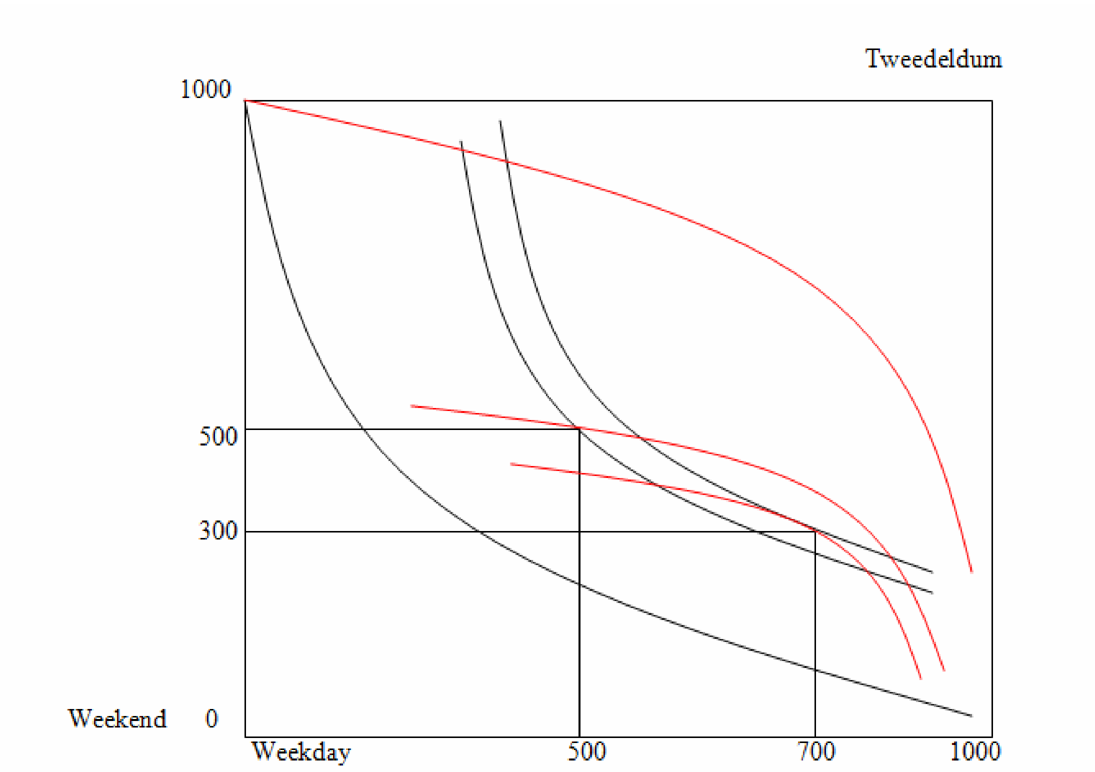
Econ 440
September 2007
Peter Norman

Homework 2
Answer Key

1.



2.

a.

$$\max_{y_A} \quad \Pi_A = (1 - y_A - y_B)^* y_A$$

FOC: $1 - 2y_A - y_B = 0$

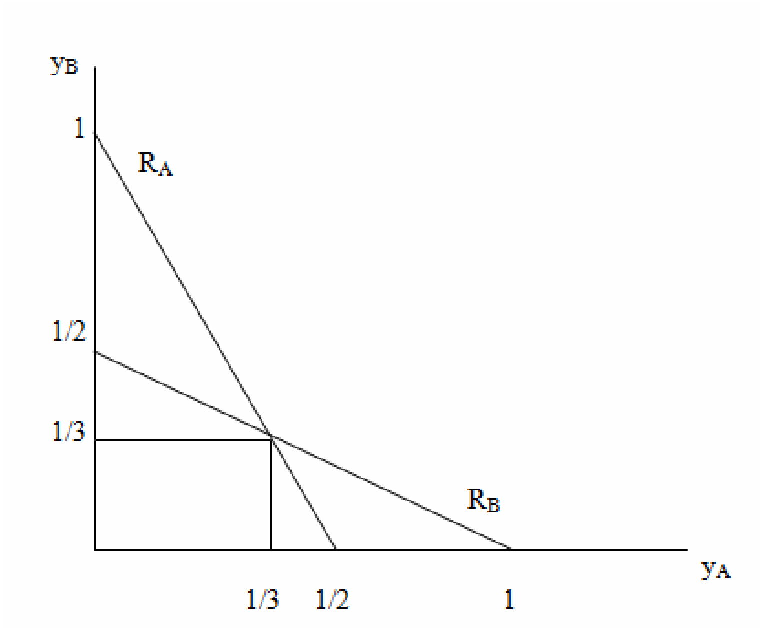
$$R_A(y_B) = (1 - y_B) / 2$$

By the same way,

$$R_B(y_A) = (1 - y_A) / 2$$

By solving reaction functions simultaneously,

Nash Equilibrium: $y_A = 1/3$ $y_B = 1/3$



b.

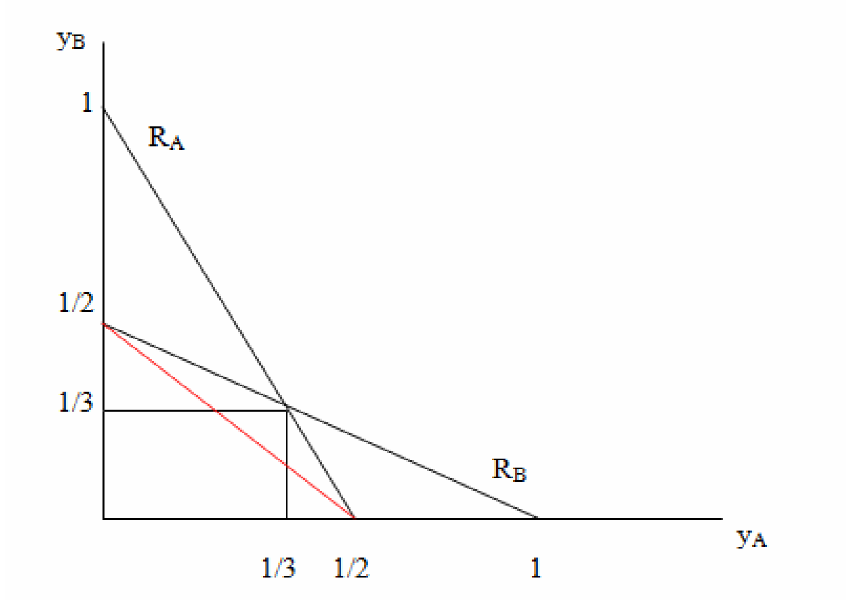
$\prod_C = (1 - 2/3)*(1/3) = 1/9$ (same for both firms)

c.

$\max_y (1 - y)*y$

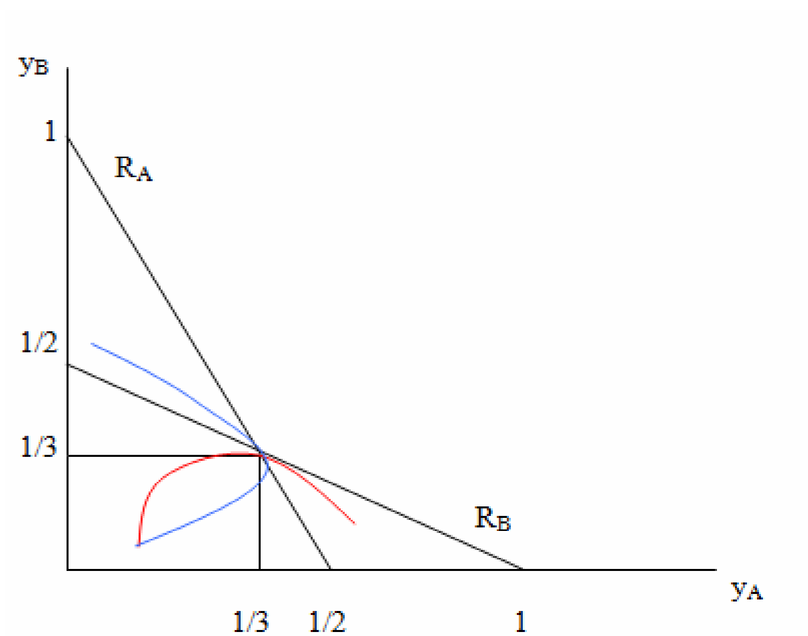
FOC: $1 - 2y = 0$ $y = 1/2$ $(y = y_A + y_B)$

$y_A + y_B = 1/2$



Solution set to the problem is (y_A^* y_B^*) such that $y_A^* + y_B^* = 1/2$ (red line above)

d.



e. In voluntary provision of public good example, there is "underprovision". Here, there is "overproduction". There exists a region on the southwest of the equilibrium where both agents are strictly better off.

3.

a.

Payoff functions are:

$\prod_I = (9 - g_I - g_S - c)*g_I$

$\prod_S = (9 - g_I - g_S - c)*g_S$

b.

$$\max_{g_I} \quad \prod_I = (9 - g_I - g_S - c)*g_I$$

FOC: $9 - 2g_I - g_S - c = 0$

$g_I = (9 - g_s - c)/2$

By the same way,

$g_S = (9 - g_I - c)/2$

$c = 1$

By solving reaction functions simultaneously,

$g_I = 8/3$

$g_S = 8/3$

c.

$$\text{Max}_{G} \quad (9 - G - c)*G$$

FOC: $9 - 2G - c = 0$

$G = (9 - c) / 2$

If $c = 1$

$G = 4$

$G = g_S + g_I = 4$

They choose g_S and g_I in such a way that there will be 4 goats in total.

$g_S = 2$

$g_I = 2$

is a possible solution.

d.

$\max_{g_I} \prod_I = (9 - 2g_I - c)*g_I$

FOC: $9 - 4g_I - c = 0$

$g_I = (9 - c)/4$

$c = 1$

$g_I = 2$

By the same way,

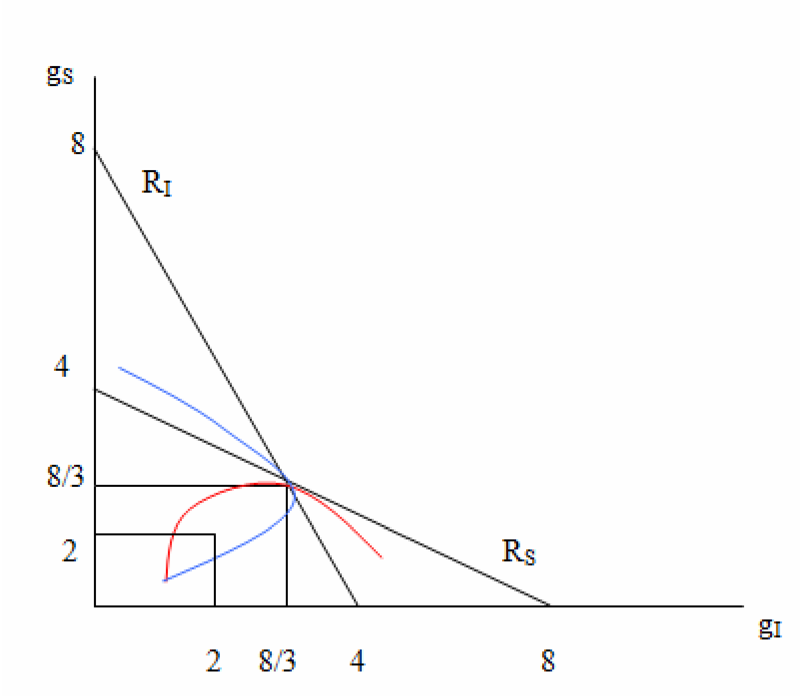
$g_S = 2$

Number of goats grazing decreases compared with part b.

$\prod_I^* = (9 - 2*2 - 1)*2 = 8$

In part b, it was: $\prod_I = (9 - 8/3 - 8/3 - 1)*8/3 = 64/9$

Thus, both of them are better off compared with part b.



In part b, there is overproduction. There exists a region on the southwest of the equilibrium where both agents are strictly better off. (2,2) is in this region.

4.

$$\max_{x,y} U(x,y)$$

s.t $\quad x + py \leq e$

$$\max L = U(x,y) - \lambda\,(x + py - e)$$

FOC: $U_1(x,y) - \lambda = 0$

$U_2(x,y) - \lambda p = 0$

$U_1(x,y) = \lambda$

$U_2(x,y) = \lambda p$

Solving FOC's simultaneously, we have;

$U_2(x,y) / U_1(x,y) = p$

a.

$U(x,y) = x + xy$

$U_1(x,y) = 1 + y$

$U_2(x,y) = x$

$p = 1$

$x / (1 + y) = 1$

$x = 1 + y$

$x + y = 1$ (by using constraint of the maximization problem and $e = 1$)

Solving both gives;

$y = 0 \quad x = 1$

welfare measure 1: $m^* = U(m^*, 0) = U(1,0) = 1$

value of intervention = $m^* - e = 1 – 1 = 0$

welfare measure 2: $x = 1 + y$ (from FOC)

$U(e,0) = e = 1 = x + xy$

$x(1+y) = 1$ substitute $x = 1+y$,

$x^2 = 1 \quad x = 1 \quad y = 0$

Since $x+y = m^{**}$

$m^{**} = 1$

value of intervention = $e - m^{**} = 1 – 1 = 0$

b.

$U(x,y) = x + y^{1/2}$

$U_1(x,y) = 1$

$U_2(x,y) = (1/2)y^{-1/2}$

$p = 1$

From FOC;

$(1/2)*y^{-1/2} = 1$

$y = 1/4$ and $x = 3/4$

welfare measure 1: $m^* = U(m^*, 0) = U(3/4,1/4) = 5/4$

$m^* = 5/4$

value of intervention : $m^* - e = 5/4 – 1 = 1/4$

welfare measure 2: $(1/2)*y^{-1/2} = 1$ (from FOC)

$y = 1/4$

$x = m^{**} - 1/4$

$1 = U(1,0) = x – y^{1/2}$

$1 = (m^{**} - 1/4) – ½$

$m^{**}= 3/4$

value of intervention : $e - m^{**} = 1 – 3/4 = 1/4$

In part a, value of intervention is zero. The reason for this is that even if there is no intervention (good y does not exist), the agent has the same utility level. In fact, he does not want to consume good y even if it is available. That is why the intervention has no value for the agent.

In part b, once the good y is made available; the agent can increase his utility by changing his consumption plan.