1. \[ \max_{y_A} \prod_A = (1 - y_A - y_B) \cdot y_A \]

FOC: \( 1 - 2y_A - y_B = 0 \)

\( R_A (y_B) = (1 - y_B) / 2 \)

By the same way,

\( R_B (y_A) = (1 - y_A) / 2 \)
By solving reaction functions simultaneously,

Nash Equilibrium: \( y_A = 1/3 \quad y_B = 1/3 \)

\[
\prod_C = (1 - 2/3) \times (1/3) = 1/9 \quad \text{(same for both firms)}
\]

c. 

\[
\text{Max} \ (1 - y) \times y
\]

FOC: \( 1 - 2y = 0 \quad y = 1/2 \quad \text{(} y = y_A + y_B \text{)}
\]

\( y_A + y_B = 1/2 \)
Solution set to the problem is \(( y_A^* \ y_B^* )\) such that \( y_A^* + y_B^* = 1/2 \) (red line above)

d.

e. In voluntary provision of public good example, there is “underprovision”. Here, there is “overproduction”. There exists a region on the southwest of the equilibrium where both agents are strictly better off.
3.

a. Payoff functions are:

\[ \Pi_I = (9 - g_t - g_s - c)g_t \]
\[ \Pi_S = (9 - g_t - g_s - c)g_s \]

b. \[
\max_{g_I} \Pi_I = (9 - g_t - g_s - c)g_t
\]

FOC: \[ 9 - 2g_t - g_s - c = 0 \]
\[ g_t = (9 - g_s - c)/2 \]

By the same way,
\[ g_s = (9 - g_t - c)/2 \]
\[ c = 1 \]

By solving reaction functions simultaneously,
\[ g_t = 8/3 \]
\[ g_s = 8/3 \]

c. \[
\max_G (9 - G - c)G
\]

FOC: \[ 9 - 2G - c = 0 \]
\[ G = (9 - c) / 2 \]

If \( c = 1 \)
\[ G = 4 \]
\[ G = g_S + g_I = 4 \]

They choose \( g_S \) and \( g_I \) in such a way that there will be 4 goats in total.

\[ g_S = 2 \]
\[ g_I = 2 \]

is a possible solution.

d.

\[
\max_{g_I} \prod I = (9 - 2g_I - c)g_I 
\]

FOC: \[ 9 - 4g_I - c = 0 \]

\[ g_I = (9 - c)/4 \]

\[ c = 1 \]
\[ g_I = 2 \]

By the same way,

\[ g_S = 2 \]

Number of goats grazing decreases compared with part b.

\[ \prod I^* = (9 - 2*2 - 1)*2 = 8 \]

In part b, it was: \[ \prod I = (9 - 8/3 - 8/3 - 1)*8/3 = 64/9 \]

Thus, both of them are better off compared with part b.
In part b, there is overproduction. There exists a region on the southwest of the equilibrium where both agents are strictly better off. (2,2) is in this region.

4.

\[
\begin{align*}
\max_{x,y} & \quad U(x,y) \\
\text{s.t} & \quad x + py \leq e \\
\max L &= U(x,y) - \lambda (x + py - e) \\
\text{FOC:} & \quad U_1(x,y) - \lambda = 0 \\
& \quad U_2(x,y) - \lambda p = 0 \\
& \quad U_1(x,y) = \lambda \\
& \quad U_2(x,y) = \lambda p \\
\text{Solving FOC's simultaneously, we have;} \\
& \quad U_2(x,y) / U_1(x,y) = p
\end{align*}
\]
a.

\[ U(x, y) = x + xy \]
\[ U_1(x, y) = 1 + y \]
\[ U_2(x, y) = x \]
\[ p = 1 \]
\[ x / (1 + y) = 1 \]
\[ x = 1 + y \]
\[ x + y = 1 \text{ (by using constraint of the maximization problem and } e = 1) \]

Solving both gives;
\[ y = 0 \quad x = 1 \]

\textit{welfare measure 1:} \quad m^* = U(m^*, 0) = U(1,0) = 1

value of intervention = \[ m^* - e = 1 - 1 = 0 \]

\textit{welfare measure 2:} \quad x = 1 + y \quad \text{(from FOC)}
\[ U(e,0) = e = 1 = x + xy \]
\[ x(1+y) = 1 \quad \text{substitute } x = 1+y, \]
\[ x^2 = 1 \quad x = 1 \quad y = 0 \]

Since \( x+y = m^{**} \)

\[ m^{**} = 1 \]

value of intervention = \[ e - m^{**} = 1 - 1 = 0 \]

b.

\[ U(x, y) = x + y^{1/2} \]
\[ U_1(x, y) = 1 \]
\[ U_2(x, y) = (1/2)y^{-1/2} \]
\( p = 1 \)

From FOC;
\[(1/2)^*y^{-1/2} = 1\]
\[y = 1/4 \text{ and } x = 3/4\]

\textit{welfare measure 1: } \( m^* = U(m^*, 0) = U(3/4, 1/4) = 5/4 \)
\[m^* = 5/4 \]

value of intervention : \( m^* - e = 5/4 - 1 = 1/4 \)

\textit{welfare measure 2: } \((1/2)^*y^{-1/2} = 1\) (from FOC)
\[y = 1/4\]
\[x = m^{**} - 1/4\]
\[1 = U(1,0) = x - y^{1/2}\]
\[1 = (m^{**} - 1/4) - 1/2\]
\[m^{**} = 3/4\]

value of intervention : \( e - m^{**} = 1 - 3/4 = 1/4 \)

In part a, value of intervention is zero. The reason for this is that even if there is no intervention (good y does not exist), the agent has the same utility level. In fact, he does not want to consume good y even if it is available. That is why the intervention has no value for the agent.

In part b, once the good y is made available; the agent can increase his utility by changing his consumption plan.