

## Homework 2

1. Recall the problem from homework 1 about Tweedeldum and Tweedeldee, where Tweedeldum has a service that gives him 1000 weekday minutes, but no weekend minutes and Tweedeldee has a service that gives him 1000 weekend minutes, but no weekday minutes. In a carefully labeled graph, draw the preferences in such a way that, (i) both Tweedeldee and Tweedeldum prefers 500 weekday minutes and 500 weekend minutes to their initial endowment, but; (ii) 500 weekday minutes and 500 weekend minutes each is not a competitive equilibrium, (iii) Tweedeldee consuming 700 weekday minutes and 300 weekend minutes is an equilibrium.
2. Suppose that firms  $A$  and  $B$  compete in a Cournot duopoly in a market with inverse demand  $p(y) = 1 - y$  and constant marginal cost equal to zero. The profit for firm  $A$  as a function of  $y_A, y_B$  is thus

$$\Pi(p_A, p_B) = p(y_A + y_B) y_A = (1 - y_A - y_B) y_A$$

and  $y_A$  and  $y_B$  are chosen simultaneously by the two firms.

- (a) Set up the appropriate optimization problems and derive the best reply function for firm  $A$  and firm  $B$ , plot these in a graph with  $y_A$  and  $y_B$  on the axes and indicate where the Nash equilibrium is.
- (b) Calculate the equilibrium profit (call it  $\Pi^C$ ).
- (c) Solve the problem

$$\max_{y_A, y_B} p(y_A + y_B) (y_A + y_B) = \max_{y_A, y_B} (1 - y_A - y_B) (y_A + y_B),$$

and add all solutions to this problem in the graph with the best reply functions [*Hint: as a first step, let  $y = y_A + y_B$  and solve the monopoly problem*].

- (d) Redraw the graph and sketch the curve  $(1 - y_A - y_B) y_A = \Pi^C$ , that is, the set of points where firm  $A$  is indifferent with the Nash equilibrium. Also add the set of points that are as good for  $B$  as the Nash equilibrium.
  - (e) Compare with the example of voluntary provision of public goods from lecture. Are there any important differences?
3. Ivar and Sven were the only inhabitants in a miserable village in northern Sweden sometime before the enclosure reform. In between Ivar and Sven there was a big piece of common land where both of them could have their goats grazing. Assume that the common is the only available land for pasture and let  $g_I$  be the number of goats Ivar owns and  $g_S$  the number of goats that Sven owns. Moreover, since there is less grass for each goat the higher the number of goats on the field is, the value of having a goat on the field decreases in the total number of goats,  $G$ , that are grazing on the field. To be specific, assume that the value is  $V(G) = 9 - G$ . Finally, let the cost of buying and caring for a goat be  $c$  (independent of the number of goats owned by either Ivar or Sven).
    - (a) Suppose that during the spring, Ivar and Sven simultaneously decides how many goats to get (for simplicity, you may assume that the goats are perfectly divisible). Specify the payoff functions for Ivar and Sven as a function of the strategic variables (=decision variables).

- (b) Derive the best responses. For  $c = 1$ , calculate the Nash equilibrium of the model.
- (c) Suppose that Ivar and Sven would get together and decide on the number of goats collectively. What would they do?
- (d) Suppose instead that Sven and Ivar would build a fence and split the field into two equally sized separate fields, where the value of having a goat on Ivars' (part) of the field would be  $9 - 2g_I$  and symmetrically for Sven (why does this make sense). How would this affect the number of goats grazing compared with part b.? Would this make Ivar and Sven worse off or better off compared with part b.? For simplicity you may keep the assumption that  $c = 1$ . Explain intuitively the difference (a graph may be useful)
4. (Welfare measures) An important practical issue in public economics is how to measure the value of an intervention. For simplicity, suppose there is one market provided good  $x$  and another good  $y$ , which due to some unspecified market failure is unavailable in the absence of intervention. Consider a single agent, and assume that he/she is endowed with  $e$  units of the market good  $x$ . Finally, suppose the preferences are given by  $U(x, y)$ . Consider a policy where good  $y$  is made available at a price  $p$  (in units of good  $x$ ) per unit. The optimal consumption plan for the consumer is then given by the solution to

$$\begin{aligned} \max_{(x,y)} U(x, y) \\ \text{s.t. } x + py \leq e. \end{aligned}$$

Let  $(x^*, y^*)$  solve the utility maximization problem.

**WELFARE MEASURE # 1:** It seems reasonable to argue that the “dollar value ” of the intervention is given by the (under reasonable assumptions unique) value  $m^*$  satisfying

$$U(m^*, 0) = U(x^*, y^*).$$

The rationale is that the agent gets happiness  $U(x^*, y^*)$  government policy, so one could just give the agent  $m^*$  units of good  $x$  and forget about the intervention providing  $y$  altogether and the agent would be equally happy. We can say this compactly by saying that  $m^* - e$  is the dollar value of the intervention where,

$$\begin{aligned} U(m^*, 0) &= \max_{(x,y)} U(x, y) \\ \text{s.t. } x + py &\leq e. \end{aligned} \tag{1}$$

**WELFARE MEASURE # 2:** An alternative measure of the dollar value is to ask, how much money can we steal from the consumer after the intervention to make him/her indifferent with the situation before the price change. That is we seek some  $m^{**}$  such that

$$\begin{aligned} U(e, 0) &= \max_{(x,y)} U(x, y) \\ \text{s.t. } x + py &\leq m^{**}. \end{aligned} \tag{2}$$

and we would then take  $e - m^{**}$  as the dollar value of the intervention.

- (a) For  $U(x, y) = x + xy, p = 1$ , and  $e = 1$ , calculate the two welfare/willingness to pay measures.
- (b) For  $U(x, y) = x + \sqrt{y}, p = 1$ , and  $e = 1$ , calculate the two welfare/willingness to pay measures. What is the important qualitative difference with the answer above? What explains the difference?