Homework 3

1. Consider a world with three alternatives \{x, y, z\} and three agents \{1, 2, 3\}.

(a) Construct a preference profile that generates a Voting cycle (Condorcet cycle).

(b) Consider the following sequential game. In stage 1 there is a vote between x and y. Then, in the second stage there is a vote between z and the winning alternative of the first stage vote. Assume that each agent vote as if she is pivotal (e.g., elimination of weakly dominant strategies). What is the winning policy?

(c) Do the same thing as in part b, but assume that the first stage vote is between x and z.

(d) Do the same thing as in parts b and c, but assume that the first stage vote is between y and z.

(e) Suppose that agent 1 can decide on which two alternatives to vote on in stage 1. What will happen? Why is this interesting from the point of view of understanding the role of legislative procedures?

2. Consider an economy with three agents, A, B, C, a private and a public good. Suppose preferences are given by \(U^A(x, y), U^B(x, y)\) and \(U^C(x, y)\) and that each agents is endowed with one unit f the private good. The private good can be turned into the public good on a one-to-one basis (just like we’ve always assumed in class).

(a) Assuming that taxes are lump sum and equal for all agents and that
\[
U^A(x, y) = a \ln x + (1 - a) \ln y
\]
\[
U^B(x, y) = b \ln x + (1 - b) \ln y
\]
\[
U^C(x, y) = c \ln x + (1 - c) \ln y.
\]
Are the reduced form preferences over y single-peaked or not?

(b) Given the preferences above, does the median voter theorem apply or not? If it does, what is the equilibrium if agents are labeled so that \(a \leq b \leq c\).

(c) Define a Lindahl equilibrium for this economy. For the preferences above, derive the conditions from which you can solve for the Lindahl equilibrium (it is somewhat algebra-intensive to solve for the Lindahl equilibrium in closed form for general \(a, b, c\), so you don’t need to do that).

(d) Define a voluntary provision equilibrium. For the preferences above, derive the conditions from which you can solve for the voluntary provision equilibrium (it is somewhat algebra-intensive to solve for the Lindahl equilibrium in closed form for general \(a, b, c\), so you don’t need to do that).

(e) Give an example of a case when the Lindahl equilibrium coincides with the median voter equilibrium.

3. Suppose that Bart and Lisa cares about their private consumption and a public good. The public good is “binary”, meaning that it is a project that can either be implemented or not. Write \(y\) for the public good. Given that it is a binary decision, we may then without loss of generality write \(y = 0\) when the project is not implemented and \(y = 1\) when it is implemented. Bart’s preferences are given by
\[
U_B(x, y) = \begin{cases} 
  x & \text{if } y = 0 \\
  x + b & \text{if } y = 1 
\end{cases}
\]
and Lisas preferences are

\[ U_L(x, y) = \begin{cases} 
    x & \text{if } y = 0 \\
    x + l & \text{if } y = 1 
\end{cases} \]

Assume that \( b > 0 \) and \( l > 0 \) and that Bart and Lisa have endowments \( e_B = e_L = 1 \) in units of the private good and that the public good costs 1 unit of the private good to implement.

(a) Under what conditions on \( b \) and \( l \) is the efficient decision to build the public good?

(b) Consider the following game. Bart and Lisa simultaneously decides how much to contribute. Hence, a strategy for Bart is to pick a contribution \( y_B \) between 0 and 1 (there is no point in contributing more).

For Lisa, a strategy is some \( y_L \) between 0 and 1. Suppose that if the contributions are sufficient, that is if \( y_B + y_L \geq 1 \), then the project is implemented and that the difference between 1 and \( y_B + y_L \) is redistributed to the agents (you may for simplicity assume that Bart and Lisa gets 50% each). If the contributions are not sufficient, then they get returned to the players (i.e., Bart gets \( y_B \) back and Lisa gets \( y_L \) back).

Write down the relevant payoff function in terms of the strategic variables \((y_B \text{ and } y_L)\) for Bart and Lisa. Assuming that provision is efficient, is there always an equilibrium in the game where the public good is provided? Be careful to explain the argument carefully.

(c) Again assuming that provision is efficient and assume that \( b < 1 \) and \( l < 1 \), is there an equilibrium when the public good is not provided?

(d) Now let’s consider an alternative game where Bart and Lisa moves sequentially. Suppose that Bart begins by making a contribution \( y_B \). Then, Lisa observes the contribution made by Bart and makes a contribution \( y_L \). Suppose that the public good is provided if \( y_B + y_L \geq 1 \), but that the corrupt person who runs the mechanism i) takes the contributions without providing the public good if \( y_B + y_L < 1 \), and; ii) takes the “excess contributions” (the difference \( y_B + y_L - 1 \)) if \( y_B + y_L > 1 \).

Solve for the backwards induction equilibrium.