

Homework 4

1. Consider a consumer with preferences over an uncertain income stream given by

$$U(x_1, x_2) = \pi u(x_1) + (1 - \pi) u(x_2),$$

where π is the probability of state 1. For each of the following functional forms for $u(x)$, sketch the indifference curves and classify the preferences as risk averse, risk neutral or risk loving.

- (a) $u(x) = \sqrt{x}$
 - (b) $u(x) = x^2$ (you need only to consider (x_1, x_2) with $x_1 \geq 0$ and $x_2 \geq 0$ as negative consumptions don't make much sense)
 - (c) $u(x) = ax + b$ where $a > 0$.
 - (d) $u(x) = \ln x$
2. Lisa has decided to invest all her wealth in stocks. She is only considering 2 stocks suggested by her financial advisor: Mron and Nron. Stocks in both Mron and Nron cost 1 dollar each and Lisa has 40 dollars to invest. Each stock in Mron will be worth 10 cents with probability $\frac{1}{2}$ and 10 dollars with probability $\frac{1}{2}$. Each stock in Nron will be worth 10 cents with probability $\frac{1}{2}$ and 10 dollars with probability $\frac{1}{2}$. Moreover, the value of the Mron stock and the value of the Nron stock are independent outcomes, so to sum up we have that:

probability	dollar value of 1 Mron stock	dollar value of 1 Nron stock
$\frac{1}{4}$	0.1	0.1
$\frac{1}{4}$	10	0.1
$\frac{1}{4}$	0.1	10
$\frac{1}{4}$	10	10

Label consumption when both stocks are worth 10 cents c_{LL} . and consumption when both stocks are worth 10 dollars c_{HH} . Next, let the consumption when Mron is worth 10 dollars and Nron is worth 10 cents be c_{HL} , and the consumption when Mron is worth 10 cents and Nron is worth 10 dollars be c_{LH} . Finally, assume that Lisas' expected utility function is

$$U(c_{HH}, c_{HL}, c_{LH}, c_{LL}) = \frac{1}{4}\sqrt{c_{HH}} + \frac{1}{4}\sqrt{c_{HL}} + \frac{1}{4}\sqrt{c_{LH}} + \frac{1}{4}\sqrt{c_{LL}}$$

- a. Suppose Lisa puts all her wealth in Mron. Calculate $c_{HH}, c_{HL}, c_{LH}, c_{LL}$ and Lisas expected utility.
- b. Suppose Lisa puts all her wealth in Nron. Calculate $c_{HH}, c_{HL}, c_{LH}, c_{LL}$ and Lisas expected utility.
- c. Suppose Lisa puts 20 dollars each in Mron and Nron respectively. Calculate $c_{HH}, c_{HL}, c_{LH}, c_{LL}$ and Lisas expected utility. **Demonstrate** that it cannot be optimal for Lisa to invest all her wealth in a single asset. This can be done by constructing an alternative portfolio that Lisa think is better. Explain as well as you can what is going on.

3. Gunnar is a farmer who owns 5 pigs. His neighbor Leif is willing to buy them all for 1000 Kronas each. If he doesn't sell, the pigs will multiply and he will have 50 pigs to sell next year. However, pigs are costly to raise, so he'll have to pay 800 Kronas per pig for food, sty-rental etc. Moreover, the pork price is uncertain, so with probability $\frac{1}{2}$ he will get 800 Kronas per pig when selling in the next period, whereas with probability $\frac{1}{2}$ he can sell them for 1000 Kronas each also in the next period.

- (a) Suppose that Gunnars' ex post utility function is $u(c) = \sqrt{c}$. Will Gunnar accept Leifs' offer?
 (b) Find some other function $u(c)$ that gives a different answer in terms of the acceptance decision.

4. Consider the adverse selection in insurance problem discussed in class. Assume that there are two types, $J = L, H$, with preferences given by

$$\pi_J u(x_B) + (1 - \pi_J) u(x_G)$$

where $\pi_L < \pi_H$ is interpreted as the probability of a loss, x_B is the consumption in case of a loss, and x_G is the consumption in case of no loss. The consumption in the absence of insurance is m_B if the loss occurs and m_G when there is no loss (for both type consumers). Assume that the consumer is risk averse unless anything else is stated.

- (a) In a carefully labeled graph, draw an indifference curve for type L and an indifference curve for type H that intersects some arbitrary point $x = (x_B, x_G)$.
 (b) Define an "isoprofit" as a set of contracts where the expected profit is constant, that is (x_B, x_G) such that

$$\pi_J x_B + (1 - \pi_J) x_G = k$$

In one or two carefully labeled graphs, draw the isoprofit for types L and H and explain in words why they are different.

- (c) In a carefully labeled graph, depict the optimal contract that a monopolist would sell to type $J = L, H$ if the monopolist would know that the consumer is of type J . Explain (using indifference and isoprofit curves) why this contract is the best for the monopolist.
 (d) In a carefully labeled graph, depict the equilibrium contract for type $J = L, H$ if there is a competitive insurance industry that can observe type J . What is the difference with the monopoly solution?
 (e) Using yet another and even more carefully labeled graph, explain (with care) what would happen if the monopolist would try to sell the optimal contract for type L (call it x^L) from above to type L and the optimal contract (x^H) for type H to type H in a world where the monopolist cannot see who is of which type.
 (f) In a new and even prettier graph, show why it must be the case that type L will get full insurance in the optimal contract for the monopolist (still assuming that type is unobservable).
 (g) Depict the optimal contract for the monopolist (still assuming that type is unobservable). In particular, it should be clear how the utility of each type compares to the utility of no insurance. Explain this property intuitively.
5. Consider a world with a single student/worker and two firms that compete in Bertrand fashion for the worker. The student/worker can be of ability $a \in \{1, 2\}$, where "ability" is interpreted as the productivity the worker has when hired by one of the firms. Let $\Pr[a = 2] = \pi > 0$. Before going on the job market,

the student may invest in (completely useless and unproductive) education. Assume that the utility of a productivity 1 student is

$$w - e,$$

where w is the wage and $e \geq 0$ is the length of education, whereas the utility of a productivity 2 student is

$$w - \frac{1}{2}e.$$

- (a) Let e^1 denote the education level chosen by type 1 and e^2 the education level chosen by type 2. If $e^1 = e^2 = e^*$, what inference can the firms make from observing e^* ? If $e^1 \neq e^2$, what inference can the firms make from observing e^1 and e^2 . What inference can the firms make if observing $e \neq e^1, e^2$?
- (b) If $e^1 = e^2 = e^*$, what wage will be paid by the firms if the student worker has e^* units of education?
- (c) If $e^1 \neq e^2$, what wage will the firms pay if observing e^1 and what wage will they pay if observing e^2 ?
- (d) In a graph, construct an equilibrium where the two types are chosen different levels of education.
- (e) In the equilibrium above, how does “private returns to education” compare with “social returns to education”? Assuming this model would correctly describe the educational process, would it make sense to subsidize education?