

Midterm 1

1. Consider an economy with two goods, x and y and two agents, A and B . Suppose that the agents are endowed with $e^A = (e_x^A, e_y^A)$ and $e^B = (e_x^B, e_y^B)$ units of the two goods. Also suppose that preferences over different consumption bundles are represented by utility functions $u^A(x, y)$ and $u^B(x, y)$ respectively.
 - (a) Define a competitive equilibrium.
 - (b) Define a Pareto optimal allocation.
 - (c) True/False: All competitive equilibria are Pareto Optimal. Explain using a wisely chosen graph. No credit unless explanation satisfactory.
 - (d) True/False: All Pareto optimal allocations are competitive equilibria. Explain using a wisely chosen graph. No credit unless explanation satisfactory.
 - (e) Suppose instead that preferences are given by $u^A(x^A, x^B, y)$ and $u^B(x^A, x^B, y)$. Would this affect the answers above? Why? Why not? No credit unless answer contains a reasonable explanation.
2. Consider a variant of the voluntary contribution model for public goods that we considered in class. Let x be a private good and y be a public good. Assume agents A and B are endowed with one unit of the private good each and that the private good can be transformed to the public good on a one-to-one basis. Assume that the agents decide simultaneously how much to provide and let y^A be the contribution by A and y^B be the contribution by B . Suppose that the preferences are

$$\begin{aligned}U^A(x, y) &= ax + (1 - a)y \\U^B(x, y) &= bx + (1 - b)y,\end{aligned}$$

where $a > 0$ and $b > 0$.

- (a) Define a Nash equilibrium for this voluntary contribution model.
 - (b) Define a Pareto optimal allocation for this environment.
 - (c) Is it possible to find values of a and b such that the Nash equilibrium of the voluntary contribution model is inefficient? (If yes, give a concrete example. If no, explain/prove).
 - (d) Is it possible to find examples of a and b such that the Nash equilibrium of the voluntary contribution model is efficient? (If yes, give a concrete example. If no, explain/prove).
3. Suppose that Bart and Lisa cares about their private consumption and a public good. The public good is “binary”, meaning that it is a project that can either be implemented or not. Write y for the public good. Given that it is a binary decision, we may then without loss of generality write $y = 0$ when the project is not implemented and $y = 1$ when it is implemented. Barts preferences are given by

$$U_B(x, y) = \begin{cases} x & \text{if } y = 0 \\ x + b & \text{if } y = 1 \end{cases}$$

and Lisa's preferences are

$$U_L(x, y) = \begin{cases} x & \text{if } y = 0 \\ x + l & \text{if } y = 1 \end{cases}$$

Assume that $b > 0$ and $l > 0$ and that Bart and Lisa have endowments $e_B = e_L = 1$ in units of the private good and that the public good costs 1 unit of the private good to implement.

- (a) Under what conditions on b and l is the efficient decision to build the public good?
- (b) Consider the following game. Bart and Lisa simultaneously decide how much to contribute. Hence, a strategy for Bart is to pick a contribution y_B between 0 and 1 (there is no point in contributing more). For Lisa, a strategy is some y_L between 0 and 1. Suppose that if the contributions are sufficient, that is if $y_B + y_L \geq 1$, then the project is implemented and that the difference between 1 and $y_B + y_L$ is redistributed to the agents (you may for simplicity assume that Bart and Lisa gets 50% each). If the contributions are not sufficient, then they get returned to the players (i.e., Bart gets y_B back and Lisa gets y_L back).

Write down the relevant payoff function in terms of the strategic variables (y_B and y_L) for Bart and Lisa. Assuming that provision is efficient, is there always an equilibrium in the game where the public good is provided? Be careful to explain the argument carefully.

- (c) Again assuming that provision is efficient and assume that $b < 1$ and $l < 1$, is there an equilibrium when the public good is not provided?