

Econ 440 Midterm

1. **(25 pts)** Consider a world with three alternatives $A = \{x, y, z\}$ and three agents $\{1, 2, 3\}$.
 1. **(5 pts)** Construct a preference profile that generates a Voting cycle (Condorcet cycle).
 2. **(5 pts)** Consider the following sequential game. In stage 1 there is a vote between x and y . Then, in the second stage there is a vote between z and the winning alternative of the first stage vote. Assume that each agent vote as if she is pivotal (e.g. elimination of weakly dominant strategies). What is the winning policy?
 3. **(5 pts)** Do the same thing as in part b, but assume that the first stage vote is between x and z .
 4. **(5 pts)** Do the same thing as in parts b and c, but assume that the first stage vote is between y and z .
 5. **(5 pts)** Suppose that agent 1 can decide on which two alternatives to vote on in stage 1. What will happen? Why is this interesting from the point of view of understanding the role of legislative procedures?
2. **(25 pts)** Consider an economy with three agents, A, B, C , a private and a public good. Suppose preferences are given by $U^A(x, y)$, $U^B(x, y)$ and $U^C(x, y)$ and that each agents is endowed with one unit of the private good. The private good can be turned into the public good on a one-to-one basis,

1. **(10 pts)** Assuming that taxes are lump sum and equal for all agents and that

$$\begin{aligned}U^A(x, y) &= x + a \ln y \\U^B(x, y) &= x + b \ln y \\U^C(x, y) &= x + c \ln y.\end{aligned}$$

Are the reduced form preferences over y single-peaked or not?

2. **(5 pts)** Given the preferences above, does the median voter theorem apply or not? If it does, what is the median voter equilibrium if agents are labeled so that $a \leq b \leq c$
3. **(10 pts)** Define a voluntary provision equilibrium. Is it possible that all agents consume a level of the private good x_i satisfying $0 < x_i < 1$ under the assumption that $a < b < c$?
3. **(25 pts)** Consider the social choice problem considered in class, where A is a set of alternatives and F is a social welfare function.
 1. **(10 pts)** Give a concrete example of a social welfare function that is non-dictatorial and satisfies the Pareto principle, but violates the independence assumption.
 2. **(10 pts)** Give a concrete example of a social welfare function that is satisfies the Pareto principle and the independence assumption, but is dictatorial.
 3. **(5 pts)** a^* in A is said to be a Condorcet winner if a majority prefers a^* to any other a in A . Give an example of some restriction on preferences that makes sure that a Condorcet winner exists.
4. **(25 pts)** Consider the case with 3 agent and two possible policies, $\{x, y\}$. Suppose agents $i = 1, 2, 3$ gets utility $u_i(x) = 0$ if x is implemented and $u_i(y) = a_i$ if y is implemented, where a_i is a number (positive or negative)

In the true/false questions below you should provide as rigorous a proof you can if the statement is true and a counterexample (ideally a simple one) if the statement is false.

1. **(True/False, 5 pts)** Suppose that the agents cast votes simultaneously and the policy that receives the largest number of votes wins, then the only Nash equilibrium outcome is for the policy that is favored by a majority wins.

2. **(True/False, 5 pts)** Suppose that the agents cast votes simultaneously and the policy that receives the largest number of votes wins, then y wins if and only if $a_1 + a_2 + a_3 \geq 0$.
3. **(True/False, 5 pts)** Voting for the most preferred outcome is a weakly dominant strategy.
4. **(True/False, 5 pts)** If Voting is done sequentially with 1 voting first, 2 second and 3 last, where each player is able to observe all previous votes. Then all agents will always vote for the most preferred outcome in every backwards induction equilibrium.
5. **(True/False, 5 pts)** If Voting is done sequentially with 1 voting first, 2 second and 3 last, where each player is able to observe all previous votes. Then the winning policy in any backwards induction equilibrium is the policy that a majority prefers.