

## Consumer Theory: Review Problems

Most public economics (and any other applied economic theory) is based on the standard consumer theory. While you have seen this both in principles and intermediate courses some of you may benefit from reviewing this material. I suggest that you pick up your notes and/or textbook from intermediate micro and solve the questions below to see how much you remember, and then try to fill in the gaps as soon as you can.

You are not required to hand in answers. As a service, I will have a review session where I solve some of the problems on the board, but I will not post any answer key.

1. Consider the standard textbook world with two goods and an exogenous income wealth. That is, let  $x_1$  and  $x_2$  denote quantities of good 1 and 2 respectively and let the prices be denoted by  $p_1$  and  $p_2$ . Finally, let  $m$  denote the “income” or “wealth” for some consumer. Recall that the **budget set** (set of affordable consumption bundles) consists of all  $(x_1, x_2)$  such that  $p_1x_1 + p_2x_2 \leq m$  and  $x_1 \geq 0$  and  $x_2 \geq 0$ .
  1. For  $p_1 = 3, p_2 = 6$  and  $m = 12$ , draw the budget set exactly. Be explicit about intercepts with the  $x_1$  and  $x_2$ -axis.
  2. Demonstrate how the budget set is affected if  $p_1$  changes from 3 to 4 and everything else is unchanged.
  3. Demonstrate how the budget set is affected if  $m$  changes from 12 to 16, but prices are kept at  $(p_1, p_2) = (3, 6)$
  4. Next, compare the budget set when  $(p_1, p_2) = (3, 6)$  and  $m = 12$  with the budget set when  $(p_1, p_2) = (6, 12)$  and  $m = 24$ . Explain.
2. True/False: In general, the budget set given  $(p_1, p_2, m)$  is the same as given  $(p'_1, p'_2, m') = (tp_1, tp_2, tm)$  if  $t$  is strictly positive. If you argue that the claim is true, provide an explanation/proof. If you argue that the claim is false, provide a counter example.
3. A professor is allocated a research budget of \$2000, which can be used only for copying and postage. However, the chair of the department doesn't think that it is worth the effort to charge for small copying expenses and has therefore decided that the first 4000 pages are free, but that if copying exceeds 4000 pages then 10 cent/copy is charged (also for the first 4000 copies). For the postage, assume that each letter is 33 cents and that a charge will be imposed from the very first letter. Draw the budget set. **Hints:** ignore the impossibility of sending 1/2 of a letter and you may also want to change the scales to eliminate a bunch of zeros.
4. Now suppose that the prices on the market are  $(p_1, p_2) = (2, 1)$  and the income is  $m = 20$ . However, the government thinks that good  $x_1$  is very very important and gives away 4 units for free.
  1. Draw the budget set.
  2. The (per capita) cost for the policy to give away 4 units for free is 8 dollars. Hence, for the same cost the government could simply increase the disposable income of the consumer to 28. Which is better for the consumer? **Hint:** use a graph with some appropriately drawn indifference curves and a budget set.

3. Suppose instead that the government pays a subsidy  $s = 1$  for each unit of good  $x_1$  the consumer buys (total cost will thus be the number of units the consumer buys). Draw the new budget set for the consumer. Sketch some indifference curves such that the best consumption bundle for the consumer is  $(10, 10)$  with this subsidy. What is the total cost for the subsidy paid by the government? Suppose the government would give this to the consumer instead of subsidizing consumption. Would the consumer be better or worse off? Interpret!
5. Carefully depict some of the indifference curves for the following utility functions.
1.  $u(x_1, x_2) = x_1 + \sqrt{x_2}$
  2.  $u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$
  3.  $u(x_1, x_2) = 55 + \left(x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}\right)^2$
6. Lisa consumes  $x_1$  and  $x_2$  and has 15 units of good 1 and 10 units of good 2 before going to the marketplace. At the market place one unit of good 1 can be exchanged for 2 units of good 2 (so that 1 unit of good 2 can also be traded for  $\frac{1}{2}$  unit of good 1).
1. Graph Lisas budget set carefully.
  2. Suppose that there is a “transaction tax” so that for each unit of good 1 sold or bought,  $\frac{1}{4}$  unit of good 1 must be given to the Sheriff of Nottingham. Carefully graph the budget set.
7. Again consider the standard textbook consumer choice model with two goods and an exogenous income  $m$ . Suppose that the preferences are represented by utility function

$$u(x_1, x_2) = x_1 x_2.$$

For those who are not used to this notation, the utility is simply the product of the quantity consumed of the two goods. Suppose that  $p_1 = 1$  and  $p_2 = 2$  and  $m = 10$ .

1. Sketch at least 2 indifference curves with at least 5 points on each curve being exactly calculated.
2. The optimal consumption bundle is the solution to a maximization problem which may be written as

$$\begin{aligned} & \max_{x_1, x_2} x_1 x_2 \\ & \text{subject to } x_1 + 2x_2 \leq 10. \end{aligned}$$

In words, this notation says that the consumer seeks to get as high utility as he/she possibly can, while staying in the budget set. Solve this problem (exactly) and illustrate the solution in a graph.