

Statistical Discrimination and Efficiency

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This paper asks whether statistical discrimination is a market failure. I consider the problem for a utilitarian social planner who operates in an environment that can generate statistical discrimination as an equilibrium phenomenon. It is found that there are potential efficiency gains from discrimination in terms of reduced “mismatch” between workers and jobs. Whether the solution to the planning problem involves discrimination depends on the trade-off between the informational gains of specialization and the losses in terms of increased investment costs.

1. INTRODUCTION

Models of statistical discrimination are often used to explain group inequality. The basic insight is that race or gender may be a useful signal of productivity, provided that productivity is imperfectly observable and correlated with group identity. Moreover, if workers are forward looking and improve productivity by human capital investments, then groups that are identical in everything except some observable “irrelevant” group identity may be treated differently in equilibrium (Arrow (1973), Coate and Loury (1993), Moro and Norman (2003*a*)). In the simplest models of statistical discrimination this is explained as a self-confirming prophesy: minority workers are unsuccessful because firms (correctly) believe that they likely have low skills and minority workers invest less in human capital since they (correctly) believe that they will be disadvantaged in the labour market.

Efficiency properties of statistical discrimination have never been extensively analysed, but it has been usually taken for granted that unequal treatment of identical groups must be a market failure. Indeed, statistical discrimination tends to generate higher average costs of investments than equilibria with equal treatment. To understand this, consider a worker from a statistically discriminated group who could acquire human capital at a cost below that of a worker from a favourably treated group who invests in human capital. As a result of the divergence in incentives, the worker may nevertheless decide not to invest. Statistical discrimination therefore creates an efficiency loss due to higher costs of human capital investments. What has not been noted in the existing literature is that statistical discrimination generates more precise information about individual workers than equal treatment. If society as a whole benefits from better information, this creates potential efficiency gains of statistical discrimination.

The main finding of this paper is that discrimination can be welfare improving. More specifically I show that a planner assigning equal welfare weights to all agents in a canonical model of statistical discrimination may find it optimal to discriminate. In fact, discrimination can even be a Pareto improvement for both groups compared with the best symmetric allocation.

The result is driven by the interaction between economics of specialization and a signal extraction problem. Agents who invest in human capital, called *qualified* workers, have a comparative advantage in one job (the complex task) while agents who do not invest have

a comparative advantage in another (the simple task). Investments in human capital are unobservable, but a noisy signal of the investment decision is available. This set-up generates a non-trivial matching problem, where workers are sorted to jobs on the basis of the signal and, possibly, group identity.

To understand why the group characteristic is useful, compare a situation where the fraction of qualified workers is the same in both groups with one where, say, group a has a higher fraction of qualified workers than the other group, b , but where the total mass of qualified workers is unchanged. If job assignments are changed so that the a group is more likely and the b group is less likely to be assigned to the complex task, fewer workers are misassigned. This is because when discrimination is introduced, qualified a workers and unqualified b workers are less likely to be misassigned. At the same time, an unqualified a worker or a qualified b worker is more likely to be misassigned, but this loss is dominated because more a workers than b workers are qualified. Discrimination thus mitigates the matching problem.

The logic above implies that the factor frontier unambiguously expands when group differences in investments increase for a constant total number of qualified workers. Exactly how this affects the solution to the planning problem depends on the technology, and to clarify the role of the production technology I consider two cases: one with a linear production function and one where the two labour inputs are complementary in the usual neoclassical sense.

The planning problem may be formulated as a problem where the planner selects investment behaviour in each group subject to incentive, feasibility, and participation constraints. In the case with a linear technology, this problem separates into two identical problems if the constraints are ignored. A necessary condition for discrimination to be efficient is therefore that the constraints bind at the optimum. This occurs whenever the (colour-blind) unconstrained optimum cannot be implemented, in which case there is under-investment in human capital in any feasible symmetric allocation. By a transfer of resources between groups, investments in one group can be increased at the expense of investments in the other. Because of the improved matching of workers to jobs this may increase surplus relative the best incentive feasible symmetric allocation.

In the linear case, efficient discrimination always corresponds with a resource transfer from the group that specializes as unqualified to the other group. Discrimination can therefore never be Pareto improving in this case. This changes if tasks are complementary. There is then a technological reason for workers to specialize. Examples can therefore be constructed where the informational gains of specialization are exploited and, at the same time, the group that is designated as unqualified labour is fully compensated. Discrimination based on “stereotypes” may thus result in a Pareto improvement relative to the best symmetric allocation.

2. THE MODEL

The underlying economic environment is a simplified version of the one considered in Moro and Norman (2003a,b), which in turn is closely related to models considered by Arrow (1973) and Coate and Loury (1993). The main difference is that wages and job assignments in this paper are determined by a central planner instead of a decentralized labour market.

2.1. *Human capital investments and the screening technology*

There are two groups of workers, each consisting of a continuum. A worker is identified by her cost of investment, c , and group identity, a or b . Population size is normalized to unity, with fractions λ^a and $\lambda^b = 1 - \lambda^a$ in each group. Workers decide whether to invest in human capital. Those who invest, called *qualified* workers, incur cost c , but acquire skills that are essential in complex jobs (see Section 2.2). Investment costs are independent of group identity and

distributed according to density g with support $[\underline{c}, \bar{c}]$. The corresponding cumulative distribution is denoted by G .

Workers are risk neutral with separable utility in money income w and the cost c : the utility for a worker who becomes qualified is $w - c$ and the utility for an unqualified worker is w .

Investments are unobservable, but after the investment decision each worker draws a noisy signal $\theta \in \{\theta_L, \theta_H\}$ with conditional probability distributions given by

$$\begin{array}{rcc}
 & \theta_H & \theta_L \\
 \text{worker qualified} & p & 1 - p \\
 \text{worker unqualified} & 1 - p & p.
 \end{array} \tag{1}$$

The symmetry, as well as the restriction to binary signals, is only for expositional simplicity.

2.2. *The production technology and the set of feasible production plans*

There are two jobs, the *complex task* and the *simple task*. The planner assigns workers to tasks based on the observable signal and group identity. I let $t = (t^a, t^b)$ denote a generic *task assignment rule*, where $t^j : \{\theta_L, \theta_H\} \rightarrow [0, 1]$ and $t^j(\theta)$ is the proportion of workers from group j with signal θ assigned to the complex task. I denote by $\pi = (\pi^a, \pi^b)$ the fractions of qualified workers in each group and, with some abuse of notation, the effective input of labour in the complex and simple task as a function of t and π as $C(\pi, t)$ and $S(\pi, t)$ respectively. These are given by

$$\begin{aligned}
 C(t, \pi) &= \sum_j \lambda^j (t^j(\theta_H) p \pi^j + t^j(\theta_L) (1 - p) \pi^j). \\
 S(t, \pi) &= \sum_j \lambda^j [(1 - t^j(\theta_H))(p \pi^j + (1 - p)(1 - \pi^j)) \\
 &\quad + (1 - t^j(\theta_L))((1 - p) \pi^j + p(1 - \pi^j))].
 \end{aligned} \tag{2}$$

The interpretation of (2) is that only qualified workers contribute to the effective input of labour in the complex task, whereas all workers are equally productive in the simple task.¹ This can be relaxed considerably. The qualitatively important assumption is that human capital enhances productivity more in one job than in the other.

Output is produced from the two types of labour according to a differentiable and concave constant returns production function $y : R_+^2 \rightarrow R_+$, where $y(C, S)$ denotes the output given labour inputs (C, S) . To economize on notation I define $Y(\pi)$ as the maximized output for any given π ,

$$Y(\pi) \equiv \max_{t \in [0, 1]^4} y(C(t, \pi), S(t, \pi)). \tag{3}$$

2.3. *The planning problem*

The planner seeks to maximize social surplus, the difference between aggregate earnings and investment costs. Since investments are unobservable, the maximization is subject to incentive compatibility constraints. The planner must also respect *ex-post* individual rationality constraints and has no exogenous resources available, so the total spending can be no more than the output produced.

Formally, the planner selects a task assignment rule t , fractions of qualified workers $\pi = (\pi^a, \pi^b)$ in each group j , a *wage scheme* $w = (w^a, w^b)$, where $w^j : \{\theta_H, \theta_L\} \rightarrow R_+$

1. $p\pi^j$ is the mass of qualified j workers with signal θ_H and $(1 - p)(1 - \pi^j)$ is the mass of unqualified signal θ_H workers. The latter group enters in $S(t, \pi)$, but not in $C(t, \pi)$. Workers with signal θ_L are treated in the same way.

describes the wage as a function of the noisy signal for each j , to solve

$$\max_{\{w,t,\pi\}} y(C(t, \pi), S(t, \pi)) - \sum_{j=a,b} \lambda^j \int_{\underline{c}}^{G^{-1}(\pi^j)} cg(c)dc \quad (4)$$

$$\text{subject to } \pi^j = G((2p-1)(w^j(\theta_H) - w^j(\theta_L))) \quad \text{for } j \in \{a, b\} \quad (\text{IC})$$

$$w^j(\theta) \geq 0 \quad \text{for each } \theta \in \{\theta_H, \theta_L\}, j \in \{a, b\} \quad (\text{IR})$$

where the constraint that aggregate consumption cannot exceed production is already built into the objective function in (4). Observing that any excess output is pure waste, the objective function and the constraints in (IR) are self-explanatory. The constraints in (IC) are somewhat less obvious. Incentive compatibility means that all qualified (unqualified) workers are weakly better off (worse off) if investing in human capital. We note that $(2p-1)(w^j(\theta_H) - w^j(\theta_L))$ is the difference in expected earnings between a qualified and an unqualified worker from group j . By incentive compatibility, a worker from group j must invest (not invest) if this difference is strictly larger (smaller) than c . The R.H.S. of (IC) is thus the unique fraction of investors in group j consistent with incentive compatibility given wage scheme w^j .

It is convenient to transform (4) into a more tractable form. Since t only enters in the objective function, task assignments must maximize output given π , so t may be eliminated from the problem and $y(C(t, \pi), S(t, \pi))$ replaced by the maximized output $Y(\pi)$. Moreover, by observing that the way to maximize incentives for a given amount of resources is to only reward workers with the high signal one can combine (IC) and (IR) to eliminate wages from the problem. After these simplifications we obtain a reduced form problem given by

$$\max_{\pi \in [0,1]^2, \alpha \in \Delta} Y(\pi) - \sum_{j=a,b} \lambda^j \int_{\underline{c}}^{G^{-1}(\pi^j)} cg(c)dc$$

$$\text{subject to } \pi^j \leq G\left(\frac{(2p-1)\alpha^j Y(\pi)}{\lambda^j(p\pi^j + (1-p)(1-\pi^j))}\right) \quad \text{for } j \in \{a, b\}, \quad (5)$$

where $\Delta = \{\alpha \in R_+^2 \mid \alpha^a + \alpha^b = 1\}$ and α^j may be interpreted as the share of resources spent on group j . Proposition 1 below summarizes the exact relationship between the programs and how wage schemes implementing the solution to (5) can be constructed.

Proposition 1. *If (π^*, w^*) solves (4), then there is some $\alpha^* \in \Delta$ such that (π^*, α^*) solves (5). Conversely, if (π^*, α^*) solves (5), then there exists some $w^* \in R_+^4$ such that (π^*, w^*) solves (4). Moreover, if (π^*, α^*) solves (5), one optimal choice of w^* is given by*

$$w^{j*}(\theta_H) = \frac{s^{j*}\alpha^{j*}Y(\pi^*)}{p\pi^{j*} + (1-p)(1-\pi^{j*})} \quad \text{and} \quad w^{j*}(\theta_L) = \frac{(1-s^{j*})\alpha^{j*}Y(\pi^*)}{p(1-\pi^{j*}) + (1-p)\pi^{j*}}, \quad (6)$$

for $j = a, b$, where $0 \leq s^{j*} \leq 1$ is the unique solution to

$$\pi^{j*} = G\left((2p-1)\left(s^{j*}\frac{\alpha^{j*}Y(\pi^*)}{\lambda^j(p\pi^j + (1-p)(1-\pi^j))} - (1-s^{j*})\frac{\alpha^{j*}Y(\pi^*)}{\lambda^j(p(1-\pi^{j*}) + (1-p)\pi^{j*})}\right)\right). \quad (7)$$

The idea is simple, and the proof is left to the reader (who may consult Norman, 1999). In a solution to (4), some share α^j of output $Y(\pi)$ is spent on wages to group j . Directly by

inspection of constraint (IC) in (4) it can be seen that *incentives to invest* are maximized if the $\lambda^j(p\pi^j + (1-p)(1-\pi^j))$ workers who draw signal θ_H split $\alpha^j Y(\pi)$ equally and workers who draw θ_L are kept at the reservation utility, generating a fraction of qualified workers given by the R.H.S. of the constraint in (5). If the planner does not want to maximize incentives the constraints in (5) do not bind. It is then optimal to pay also low signal workers above the reservation wage and (6) and (7) provide a constructive procedure to construct *one* optimal wage scheme from a solution to (5).

3. SPECIALIZATION AND THE FACTOR FRONTIER

The purpose of this section is to clarify the intuition discussed in the introduction for how discrimination improves matching. I show how the factor frontier expands when groups are treated asymmetrically given a constant total measure of qualified workers, and interpret this in terms of fewer misassigned workers.

I let the scalar x denote some fraction of qualified workers, common to both groups, and σ and ρ the fraction of agents with signal θ_H and θ_L respectively, that are assigned to the complex task. The input of labour in each task may then be written as

$$\begin{aligned} C(\sigma, \rho, x) &= \sigma px + \rho(1-p)x \\ S(\sigma, \rho, x) &= (1-\sigma)[px + (1-p)(1-x)] + (1-\rho)[(1-p)x + p(1-x)]. \end{aligned} \tag{8}$$

The kinked line in the middle of the left graph in Figure 1 depicts the *factor frontier*. Factor inputs are $(C, S) = (0, 1)$ if all workers are in the simple task. Initially, the best way to increase the input of complex labour is by use of workers with high signals. Eventually, the planner runs out of workers with high signals, which accounts for the kink at point A , where workers are assigned to tasks in “accordance to signals” and labour inputs are $C = px$ and $S = (1-p)x + p(1-x)$. The slope to the right of A is steeper because workers with low signals are less likely to be qualified, so each unit of complex labour comes at a larger cost in terms of simple labour than to the left of A .

Two more frontiers are depicted in the left graph in Figure 1. These are derived as the first, but with fractions of qualified workers given by αx and $(2-\alpha)x$ respectively, where $\alpha < 1$ and $(2-\alpha)x < 1$. If the fraction of qualified workers is αx in one group, $(2-\alpha)x$ in the other, and groups are of equal size, the total measure of qualified workers still equals x . The segmented line through points D, A and E in the graph to the right in Figure 1 depicts the factor frontier for this economy.² The slope of the line segment between the vertical intercept and point D is the same as the slope of the line between the vertical intercept and C in the left graph, and the slopes of the other line segments can also be directly translated from the graph to the left.

The expansion of the frontier in Figure 1 may be interpreted as a reduction in the number of misassigned workers. For example, compare point D , where $\frac{1}{2}(1-p)(1-(2-\alpha)x)$ unqualified workers are assigned to the complex task, with point F . At F , $\sigma(1-p)(1-x)$ unqualified workers are put in the complex task, where, to keep the input of simple labour constant, $\sigma > \frac{1}{2}$ (since the fraction of high signal workers increases in the proportion of qualified workers). The measure of workers *employed* in each task is the same in D and F , but the difference in the measure of unqualified workers in the complex task is

$$\sigma(1-p)(1-x) - \frac{1}{2}(1-p)(1-(2-\alpha)x)$$

2. The graph is drawn for α large enough for high signal workers from the group investing at rate αx to be more likely to be qualified than low signal workers from the other group. If α is low enough, low signal workers from the high investment group are more likely to be qualified than high signal workers from the low investment group. The reader may verify that the frontier expands also in this case (the frontiers then only coincide at the intercepts).

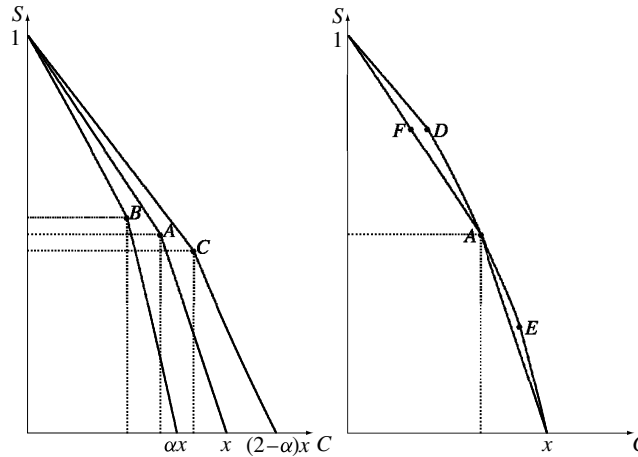


FIGURE 1
Discrimination expands the set of feasible effective factor inputs

$$\begin{aligned}
 &> \frac{1}{2}((1 - p)(1 - x) - (1 - p)(1 - (2 - \alpha)x)) \\
 &= \frac{1}{2}(1 - p)(1 - \alpha)x > 0,
 \end{aligned}
 \tag{9}$$

since $\alpha < 1$. The matching is thus better at D than at F , which reflects that the creation of a group with a higher prior probability of being qualified is useful, since the posterior probability is increasing in the prior. The only places where discrimination is useless are where the planner does not act on the additional information provided by group identity (point A and the corners).

4. THE LINEAR CASE

Consider a linear production function $y(C, S) = \alpha C + \beta S$, where $\alpha > \beta > 0$. The maximized output, $Y(\pi)$ defined in (3), can then be found by maximizing the *per capita* output for each group separately and be expressed explicitly as

$$Y(\pi) = \sum_{j=a,b} \lambda^j Y^*(\pi^j),
 \tag{10}$$

where $Y^*(\pi^j)$, the *per capita* output in group j , is depicted in Figure 2 and given by

$$Y^*(\pi^j) \equiv \max\{\alpha\pi^j, \alpha p\pi^j + \beta((1 - p)\pi^j + p(1 - \pi^j)), \beta\}.
 \tag{11}$$

For small π^j , the expected output of a high signal worker in the complex task is below β , so, for π^j lower than a threshold, all workers are assigned to the simple task and output is β . Similarly, for π^j sufficiently large, all workers are assigned to the complex task, while in a middle range only workers with the high signal are assigned to the complex task (values for the thresholds in Figure 2 are found by equalizing expected productivity in the two tasks for each signal). The important qualitative feature of Y^* is the convexity, which reflects that the rate at which output increases in π^j is higher when more agents are in the complex task.

4.1. Example where the optimal solution involves discrimination

I now construct an example where there is discrimination in the optimal solution to the planner's problem. Rather than solving the full problem directly I first solve for the best symmetric allocation.

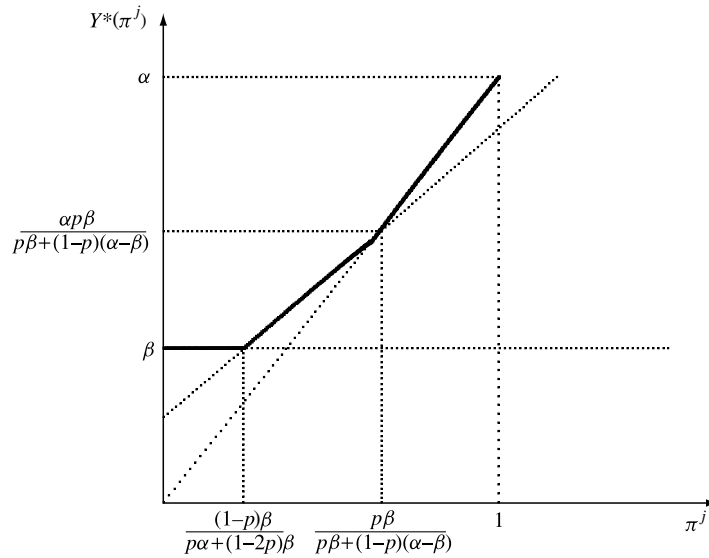


FIGURE 2

The maximized *per capita* output in group *j* is convex

It is then showed that the planner can do better by treating groups asymmetrically, implying that also the optimal solution to (5) must be discriminatory.

Set $\alpha = 2$, $\beta = 1$, and let G be uniform over $[0, 1]$. The maximized *per capita* output in (11) then simplifies to $Y^*(x) = \max\{2x, x + p, 1\}$, investment costs as a function x are $\int_0^{G^{-1}(x)} cg(c)dc = \frac{x^2}{2}$, and problem (5) with the extra constraint $\pi^a = \pi^b = x$ can be written

$$\begin{aligned} & \max_{0 \leq x \leq 1} \left(\max\{2x, x + p, 1\} - \frac{x^2}{2} \right) \\ & \text{subject to } x \leq \min \left\{ \frac{(2p - 1) \max\{2x, x + p, 1\}}{px + (1 - p)(1 - x)}, 1 \right\}. \end{aligned} \tag{12}$$

Figure 3 depicts the objective function to (12). The three solid curves plot surplus as a function of x for each of the potentially optimal pure task assignment rules (all in simple task, high signal only in complex task, all in complex task). The upper envelope, the thicker line, is thus the relevant maximand.³ The maximand is strictly decreasing on $[0, 1 - p]$ and strictly increasing on $[1 - p, 1]$, so the solution to (12) is either to set $x = 0$ or equal to the highest value satisfying the constraint.

It is easy to check that $x = \frac{1}{2}$ satisfies the constraint with equality for $p = \sqrt{\frac{3}{8}}$. For $x \in [\frac{1}{2}, p]$ we have that $\max\{2x, x + p, 1\} = x + p$, so

$$\frac{d}{dx} \left[\frac{(2p - 1)(x + p)}{px + (1 - p)(1 - x)} \right] = \frac{(2p - 1)(1 - 2p^2)}{(px + (1 - p)(1 - x))^2} \leq 4(2p - 1)(1 - 2p^2) < 1, \tag{13}$$

3. An increase in p shifts the middle line upwards, but leaves the others unchanged. Hence, the larger is p , the bigger is the range of x where workers are assigned in “accordance to signals”. The dotted lines in Figure 3 are the extreme cases, which should be intuitive. If $p = \frac{1}{2}$, the signal is uninformative, so only the prior matters for task assignments. If $p = 1$, the signal is perfect, and workers are always assigned to task in accordance to signals.

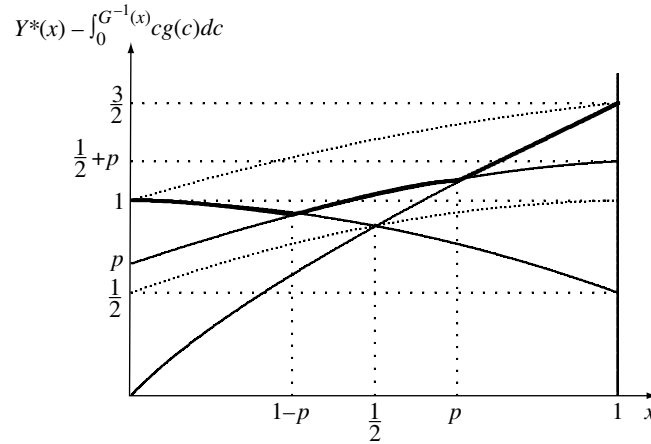


FIGURE 3
Social surplus as a function of π

where the first inequality follows since $px + (1 - p)(1 - x) \geq \frac{1}{2}$ whenever $x \geq \frac{1}{2}$ and the second from evaluating at $p = \sqrt{\frac{3}{8}}$. Hence, the L.H.S. in the constraint to (12) increases slower than x over $[\frac{1}{2}, p]$, implying that no $\frac{1}{2} < x < p$ satisfies the constraint. Straightforward algebra also shows that no $x > p$ satisfies the constraint, so $x = \frac{1}{2}$ is the largest fraction of qualified workers satisfying the constraint in (12). The associated value of the objective function is

$$\frac{1}{2} + \sqrt{\frac{3}{8}} - \frac{1}{8} < \frac{1}{2} + \frac{5}{8} - \frac{1}{8} = 1. \tag{14}$$

We conclude that $x = 0$ solves (12), since this generates social surplus 1.

Next, consider the asymmetric allocation with $(\pi^a, \pi^b) = (1, 0)$, which generates social surplus $\lambda^a \frac{3}{2} + (1 - \lambda^a) > 1$. The solution to (12) can thus be improved upon if this is incentive feasible. Wages $w^b(\theta_H) = w^b(\theta_L) = 0$ suffice to support $\pi^b = 0$, so all resources may be devoted to provide incentives for group a . From the constraint in (5) (for $\alpha^a = 1$) we find that $(\pi^a, \pi^b) = (1, 0)$ is incentive feasible if

$$1 \leq \frac{(2p - 1)(2\lambda^a + (1 - \lambda^a))}{\lambda^a p} = \frac{(2p - 1)(1 + \lambda^a)}{p \lambda^a}. \tag{15}$$

For $\lambda^a \leq \frac{1}{2}$ (15) holds for all $p \geq \frac{3}{5}$, and since $\sqrt{\frac{3}{8}} > \frac{3}{5}$ and group labels are arbitrary we conclude that discrimination is constrained optimal for all λ^a . In fact, it is almost immediate to extend the example to all $p \in [\frac{3}{5}, \sqrt{\frac{3}{8}}]$ by observing that a decrease in p tightens the constraint in (12).

In the example, and any other discriminatory solution to (5) with a linear technology, resources are transferred from the group with a low fraction of qualified workers to the other group. These distributive consequences *cannot* be undone by transfers: discrimination helps the planner because a lump sum transfer between groups helps the planner exploit a non-concavity in the maximized production function.

4.2. *Generalizing the example*

To conclude the treatment of the linear case, I show that the example is robust in the sense that if other parameters are held fixed, there is always a non-negligible set of values of λ^a and p for which any solution to the planning problem is discriminatory. Formally

Proposition 2. *Suppose $y(C, S) = \alpha C + \beta S$, $\underline{c} \geq 0$ and that there exists $\pi \in [0, 1]$ such that $\alpha\pi - \int_{\underline{c}}^{G^{-1}(\pi)} cg(c)dc > \beta$. Then, there is a $\bar{\lambda} > 0$ such that for any $\lambda \leq \bar{\lambda}$ there is an open interval $P \subset [\frac{1}{2}, 1]$ such that if $p \in P$ and $\lambda^j = \lambda$ for some group j , then $\pi^a \neq \pi^b$ in any solution to (5).*

The assumption that $\underline{c} \geq 0$ ensures that free-riding is a problem if signals are sufficiently uninformative (if $\underline{c} < 0$ some agents like to invest). The assumption that $\alpha\pi - \int_{\underline{c}}^{G^{-1}(\pi)} cg(c)dc > \beta$ for some π is necessary since otherwise the unconstrained optimum is for all workers to remain unqualified and be all assigned to the simple task. The proof is in the Appendix.

5. EXAMPLE WITH COMPLEMENTARITIES IN PRODUCTION

I now consider an example where tasks are complementary. Unlike the linear case, marginal productivities now depend on the factor ratio, and, since workers in both tasks are needed for efficiency in production, the planning problem may now have a discriminatory solution even if it coincides with the unconstrained maximum of the objective. This in turn makes it possible to generate examples where the “discriminated” group is fully compensated.

I let the production function be given by $y(C, S) = \sqrt{CS}$ and G be of the form

$$G(c) = \begin{cases} 0 & \text{for } c < 0 \\ \frac{1}{2} & \text{for } 0 \leq c \leq c^* \\ 1 & \text{for } c \geq c^*. \end{cases} \tag{16}$$

As in Section 4.1 I first solve the planning problem (5) with the additional constraint that $\pi^a = \pi^b = x$. Using the definitions in (8), the maximized output as a function of x can be expressed as

$$Y(x) = \max_{\sigma, \rho, \epsilon \in [0, 1]^2} \sqrt{C(\sigma, \rho, x) \cdot S(\sigma, \rho, x)}. \tag{17}$$

The advantage of the particular form of the cost function (16) is that $x = \frac{1}{2}$ and $x = 1$ are the only potential solutions to the planning problem, so (17) needs only to be solved for these two cases. For $x = \frac{1}{2}$ one can use first-order conditions to check that the unique solution to (17) is to set $(\sigma, \rho) = (1, 0)$.⁴ For $x = 1$ the signal is irrelevant, but any rule that assigns half the agents to each task is optimal. The maximized values of output at the two potential solutions are thus

$$Y(\frac{1}{2}) = \frac{1}{2}\sqrt{p} \quad \text{and} \quad Y(1) = \frac{1}{2}. \tag{18}$$

The associated values of the social surplus are $\frac{1}{2}\sqrt{p}$ and $\frac{1}{2} - \frac{1}{2}c^*$ respectively.

I now assume that groups are of equal size and allow the planner to discriminate. There are many cases to consider, depending on how c^* relates to p . Rather than to provide an exhaustive list of all possibilities I only consider two cases that highlight the most interesting possibilities.

Case 1. $c^* < 1 - \sqrt{p}$ and $c^* \leq \frac{2p-1}{2p}$. In this case the best symmetric allocation is when all workers are qualified and this is incentive feasible, so the maximal social surplus with equal

4. Detailed derivations are available from the author. While calculations are non-trivial the solution $(\sigma, \rho) = (1, 0)$ is not particularly surprising given the kink in the factor frontier (see Figure 1).

treatment is $\frac{1}{2} - \frac{1}{2}c^*$. If either group is designated as qualified workers, output is still $\frac{1}{2}$ and investment costs are cut in half, so social surplus is increased to $\frac{1}{2} - \frac{1}{4}c^*$. One wage scheme that implements this outcome is $w^a(\theta_H) = \frac{1}{2p}$, $w^a(\theta_L) = 0$ and $w^b(\theta_H) = w^b(\theta_L) = \frac{1}{2}$. A simple calculation verifies that budget balance holds with equality and one can check that the suggested solution achieves the unconstrained maximum of the planners' objective function.

Note that the average income in each group is $\frac{1}{2}$, so the "discriminated" group b is fully compensated for designating themselves as unqualified workers. In fact, all b workers are strictly better off than in the best symmetric solution while a workers are indifferent. Of course, wage schemes making all agents strictly better off can also be constructed by giving slightly more to group a .

Case 2. $1 - \sqrt{p} < c^* < 2(1 - \sqrt{p})$ and $c^* \leq \frac{2p-1}{2p}$.⁵ In this case the additional output from making all agents qualified is not worth the cost in terms of additional investments in the planning problem with equal treatment, so $x = \frac{1}{2}$ is the optimal investment rate and the associated surplus is $\frac{1}{2}\sqrt{p}$. However, by designating one group as qualified workers, surplus is increased since $\frac{1}{2} - \frac{1}{4}c^* > \frac{1}{2}\sqrt{p}$ under the assumption on c^* . Working out the details one can show that it is possible to increase both the expected income and the expected utility for all agents also in this case.

The example is highly stylized, but it shows that asymmetric treatment of the groups can, possibly, be a Pareto improvement from the best symmetric allocation: the "discriminated" group may be fully compensated with lump-sum transfers. Hence, even if society cares about group equality, as Loury (1987) argues is reasonable, there may be gains from discrimination on the basis of "stereotypes" that potentially can be enjoyed by all agents in the economy.

6. DISCUSSION

6.1. *Introducing direct revelation mechanisms*

The planning problem studied in this paper does not allow workers to report whether they invested or not. If we are interested in weak implementation, this is a restriction. As long as wages are independent of reports, workers are indifferent between truth-telling and lying. Hence, a mechanism where wages depend on only the signal and task assignments depend on only the report can implement first best task assignments as an equilibrium.⁶ Of course, any report is consistent with equilibrium in such a mechanism and it is easy to show that the planning problem (4) studied in this paper characterizes the best allocation implementable as a unique equilibrium.

However, the strongest argument against use of direct mechanisms and in favour of the approach taken in this paper is that a mechanism relying heavily on cooperation of indifferent workers, which any direct mechanism must do in order to improve on the solution to (4), is not robust to natural perturbations of the model. If workers have direct preferences over jobs following a non-degenerate distribution, it is impossible to implement first best matching. This generates a multidimensional screening problem (where the type of a worker is the cost of investment and a job preference parameter). I cannot solve this problem in general, but it can be shown that the solution to the multidimensional problem approaches the solution to (4) as the variation in (continuously distributed) direct job preferences becomes negligible.

5. It is easy to check that the inequalities are not inconsistent. One example is $p = 2/3$ and $c^* = 1/5$.

6. With participation and resource constraints, inefficiencies in investments are still possible.

6.2. *Related equilibrium analysis*

The novel feature of this paper is that discrimination on the basis of payoff-irrelevant characteristics arises in the solution to a *planning problem*. Other papers have shown that discrimination of identical groups may arise as an *equilibrium phenomenon*.

The most closely related equilibrium model of discrimination is in Moro and Norman (2003a,b), who consider roughly the same economic environment as in this paper, but where the labour market is competitive.⁷ In the case of a linear technology, Moro and Norman (2003b) show that the equilibrium conditions for the different groups are separable. Discrimination is then a pure coordination failure, and the Pareto dominant equilibrium is symmetric.

While the results for planning problem and the equilibrium model are different, they are not difficult to reconcile. In the competitive model, agents are paid their expected productivities, so each group is paid their contribution to output in the aggregate. As observed in Section 4, discrimination in the optimum requires a net transfer between groups. The crucial difference is thus that the planning problem has a single resource constraint for the whole economy, which, if relevant for the solution, breaks the separability between groups.

Results for the equilibrium model with complementarities in production are more in line with the planning model. For reasons similar to those in Section 5, the best equilibrium may then be discriminatory.

It is natural to ask whether there is more or less discrimination in a decentralized equilibrium or in the social optimum, but the model is silent on this. On the one hand, there are always symmetric equilibria in the equilibrium model. If the benchmark is a symmetric equilibrium, the planner therefore trivially chooses a higher level of discrimination than the market when the command optimum is discriminatory. On the other hand, the market may coordinate on an asymmetric equilibrium even though the social optimum is an allocation with equal treatment of groups.

6.3. *Previous literature on discrimination and efficiency*

An early discussion on discrimination and efficiency is in Cain (1985), who concludes that neither taste based models (see Becker, 1957) nor early models of statistical discrimination (following Phelps, 1972) provide an efficiency rationale for intervention. However, the models discussed by Cain assumed an exogenous skill distribution and a single job, so wage setting is purely a matter of distribution.

Lundberg and Startz (1983) added a human capital investment to an otherwise standard model of statistical discrimination. The testing technology is more noisy for one group than the other, implying that equilibrium wages generate weaker incentives to invest for the group with more noisy signals. "Free-riding" in human capital investments is therefore more severe for this group, and investments as well as average wages are lower in equilibrium. It is shown that a policy that forbids employers to condition wages on group identity may increase social surplus. The idea is that the free-riding problem is partially alleviated (worsened) in the disadvantaged (dominant) group when wages are averaging the marginal products across groups, shifting human capital investments from high to low cost units. Similar exercises can also be found in Schwab (1986) and Haagsma (1993).

The most important difference between this paper and the previous literature is that the planning problem allows me to better separate gains or losses from discrimination from inefficiencies due to free-riding type problems that always occur in positive models of statistical discrimination.

7. This model in turn is closely related to models in Arrow (1973), Coate and Loury (1993) and Fang (2001).

7. CONCLUDING REMARKS

This paper studies the planning problem for an environment where discrimination between identical groups can be supported as an equilibrium phenomenon. The advantage of discrimination is that it creates additional information that helps the planner avoid mistakes in job assignments. At the same time, workers from the discriminated group who could acquire human capital at a lower cost than the marginal worker from the other group remain unqualified, so discrimination leads to higher costs of human capital investments. This latter effect is well understood (see for example Lundberg and Startz, 1983), so while the net effect of the trade-off can go either way, the paper is focused on the possibility that discrimination may increase social surplus and, with complementarities, even result in a Pareto improvement.

APPENDIX. PROOF OF PROPOSITION 2

To emphasize that the maximized value of *per capita* output depends on the parameter p I write

$$Y^*(\pi, p) = \max\{\alpha\pi, \alpha p\pi + \beta((1-p)\pi + p(1-\pi)), \beta\} \quad (\text{A.1})$$

for the maximized *per capita* output in (11). If the solution to the planning problem involves equal treatment of groups the associated maximized social surplus is

$$V(p) = \max_{0 \leq \pi \leq 1} Y^*(\pi, p) - \int_{\underline{c}}^{G^{-1}(\pi)} cg(c)dc$$

subject to $\pi \leq G\left(\frac{(2p-1)Y^*(\pi, p)}{p\pi + (1-p)(1-\pi)}\right)$, (A.2)

which is the problem (5) given the technology $y(C, S) = \alpha C + \beta S$ with the additional constraint that $\pi^a = \pi^b = \pi$. I first show:

Lemma A1. *There exists some $\underline{p} > \frac{1}{2}$ such that $V(p) = \beta$ for all $p \leq \underline{p}$.*

Proof. Let $\underline{\pi}(p) = \frac{(1-p)\beta}{p\alpha + (1-2p)\beta}$. This (see Figure 2) is the critical value of π when it becomes worth the while to assign the workers with high signals to the complex task, so $Y^*(\pi, p) = \beta$ for all $\pi \leq \underline{\pi}(p)$. Straightforward differentiation shows that $\underline{\pi}(p)$ is decreasing in p . Next consider the equation

$$\pi = G\left(\frac{(2p-1)\beta}{p\pi + (1-p)(1-\pi)}\right), \quad (\text{A.3})$$

where we adopt the convention that $G(1) = 1$ if $\frac{(2p-1)\beta}{p\pi + (1-p)(1-\pi)} \geq \bar{c}$. Since the argument is positive for all $p > \frac{1}{2}$ and $\pi \in [0, 1]$ and the R.H.S. is strictly decreasing in π there is a unique solution $0 < \bar{\pi}(p) \leq 1$ for each p . The R.H.S. is decreasing in p (strictly so if $\frac{(2p-1)\beta}{p\pi + (1-p)(1-\pi)} < \bar{c}$) and $\lim_{p \rightarrow \frac{1}{2}} \frac{(2p-1)\beta}{p\pi + (1-p)(1-\pi)} = 0$ for any π , so given that $\underline{c} \geq 0$ it follows that $\bar{\pi}(p) \rightarrow 0$ as $p \rightarrow \frac{1}{2}$. Since $\underline{\pi}(p) \rightarrow \frac{\beta}{\alpha} > 0$ as $p \rightarrow \frac{1}{2}$ it follows that there is some $\underline{p} > \frac{1}{2}$ so that $\underline{\pi}(p) > \bar{\pi}(p)$ for all $\frac{1}{2} < p \leq \underline{p}$. What this means is that for all $p \leq \underline{p}$ the maximized *per capita* output is $Y^*(\pi, p) = \beta$ for all $\pi \leq \bar{\pi}(p)$, *i.e.* under the assumption that β is the maximized *per capita* output (this assumption built into equation (A.3) by replacing $Y^*(\pi, p)$ with β), the maximized *per capita* output is β for all fractions of investors that are incentive feasible. Hence the social surplus is $\beta - \int_{\underline{c}}^{G^{-1}(\pi)} cg(c)dc$ for all incentive feasible π for $p \leq \underline{p}$, which achieves its maximum value $V(p) = \beta$ at $\pi = 0$. \parallel

Proof of Proposition. Let $\hat{\pi}$ satisfy $\alpha\hat{\pi} - \int_{\underline{c}}^{G^{-1}(\hat{\pi})} cg(c)dc > \beta$ (such $\hat{\pi}$ exist by hypothesis). Suppose without loss that $\lambda^a \leq \lambda^b$ and let $\lambda = \lambda^a$. Also, let $p^* > \frac{1}{2}$ be some value of p such that $V(p^*) = \beta$, which exists by Lemma A1 and observe that $(\pi^a, \pi^b) = (\hat{\pi}, 0)$ is incentive feasible if

$$\hat{\pi} \leq G\left(\frac{(2p-1)(\lambda Y^*(\hat{\pi}, p^*) + (1-\lambda)\beta)}{\lambda(p^*\pi + (1-p^*)(1-\pi))}\right) < G\left(\frac{(2p^*-1)\beta}{\lambda(p^*\pi + (1-p^*)(1-\pi))}\right).$$

Since the argument of the expression on the R.H.S. is monotonically decreasing in λ and goes to infinity as $\lambda \rightarrow 0$ there exists some $\bar{\lambda} > 0$ such that $(\hat{\pi}, 0)$ is incentive feasible for any $\lambda \leq \bar{\lambda}$. Finally, $V(p)$ is continuous and $V(p^*) = \beta$, while the maximized solution with discrimination allowed gives a surplus strictly greater than β . The conclusion that there is a neighbourhood P of p^* such that discrimination is optimal then follows trivially since the value function to the full problem (that allows discrimination) also is continuous in p .

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