16 Oligopoly

Oligopoly refers to a situation with a few firms on the market and the central assumption that differs from the competitive model is that the firms understand that their actions affect the market price. There is an industry of different models, but most of these are in some way or another variations on the two really central models of oligopoly. These are:

1. The *Cournot model,* where the strategic variable is a quantity choice.

2. The *Bertrand model,* where the strategic variable is the price charged.

Besides being important in themselves, these models also illustrates well how the tools from game theory is used in economics. We will try to keep things as simple as possible, so we will typically only consider *duopoly* versions of the models (two firms).

16.1 The Cournot Model

We assume that:

- There are two firms, 1 and 2 (will consider n firms later)
- Constant marginal cost equal to c
- The firms produce a *homogenous good* with inverse demand

$$p(y) = a - by$$

Now, we want to cast the model as a *game,* so we need to describe:

1. Who the *players* are. This is no problem-we've already said that the firms are firm 1 and firm 2.

2. What the available *strategies a*re for each player. With quantity competition that is just

A quantity $y_1 \geq 0$ for firm 1

A quantity $y_2 \leq 0$ for firm 2

Note that we are assuming that quantities are chosen *simultaneously* since otherwise we would specify the strategy of one firm to be a *contingent plan (*a quantity for each quantity chosen by the other firm)

3. Finally we need to complete the model and describe the *payoff functions,* that is how the payoffs of the players depend on the strategies chosen by the players. Here we have some work remaining.

We will assume that firms care about their profits exactly as in the competitive model and the monopoly model, that is firm 1 cares about

$$\pi_1 = py_1 - cy_1$$

However, with only two firms we want to think about firms who understand how the price is affected by quantities on the market. Now, with a homogenous good the equilibrium price depends on the **sum** of quantities chosen by the firms, that is the price given quantities y_1 and y_2 is

$$p(y_1 + y_2) = a - b(y_1 + y_2),$$

So, we can write firm 1's profit as a function of the *strategies (*that is quantities) as

$$\begin{aligned} \pi_1(y_1, y_2) &= p(y_1 + y_2) - cy_1 = \\ &= (a - b(y_1 + y_2) - c)\, y_1 \end{aligned}$$

Note that the profit is now described fully in terms of the strategies (and parameters), so we have described a game between firm 1 and firm 2. Since there is an infinite number of possible quantity combinations we can not write it as a payoff matrix, but we have described a more elaborate version of a game exactly like the prisoners' dilemma or the battle of the sexes.

16.2 Nash Equilibrium in the Cournot Model

Recall that a Nash equilibrium is a situation where each player does the best he/she can given what the other player(s) are doing. Hence a pair of quantities (y_1^*, y_2^*) is a Nash equilibrium in the Cournot model if

$$\begin{aligned} \pi_1(y_1^*, y_2^*) &\geq \pi_1(y_1, y_2^*) \text{ for all } y_1 \geq 0 \text{ and} \\ \pi_2(y_1^*, y_2^*) &\geq \pi_2(y_1^*, y_2) \text{ for all } y_2 \geq 0 \ . \end{aligned}$$

But, that is just saying that

$$\begin{aligned} y_1^* &\text{ solves } \max_{y_1 \geq 0} \pi_1(y_1, y_2^*) \text{ and} \\ y_2^* &\text{ solves } \max_{y_2 \geq 0} \pi_2(y_1^*, y_2) \end{aligned}$$

What this means is that we can just take first order conditions as usual (for a fixed quantity by the other firm) and then solve the 2 first order conditions out for the Nash equilibrium. Firm 1 solves the problem

$$\max_{y_1 \geq 0} \pi_1(y_1, y_2) = \max_{y_1 \geq 0} (a - b(y_1 + y_2) - c)\, y_1$$

and the first order condition is

$$a - b(y_1 + y_2) - c - by_1 = 0$$

Solving for y_1 we get the *best response,* the optimal choice of y_1 given any y_2

$$y_1(y_2) = \frac{a - c - by_2}{2b}$$

Symmetrically, we get a best response for firm 2

$$y_2(y_1) = \frac{a - c - by_1}{2b}$$

A Nash equilibrium is a situation where each firm is doing the best they can given what the other firms is doing, so $y_1^* = y_1(y_2^*)$ and $y_2^* = y_2(y_1^*)$, so a Nash equilibrium is a solution to the system

$$\begin{aligned} y_1 &= \frac{a - c - by_2}{2b} \\ y_2 &= \frac{a - c - by_1}{2b} \end{aligned}$$

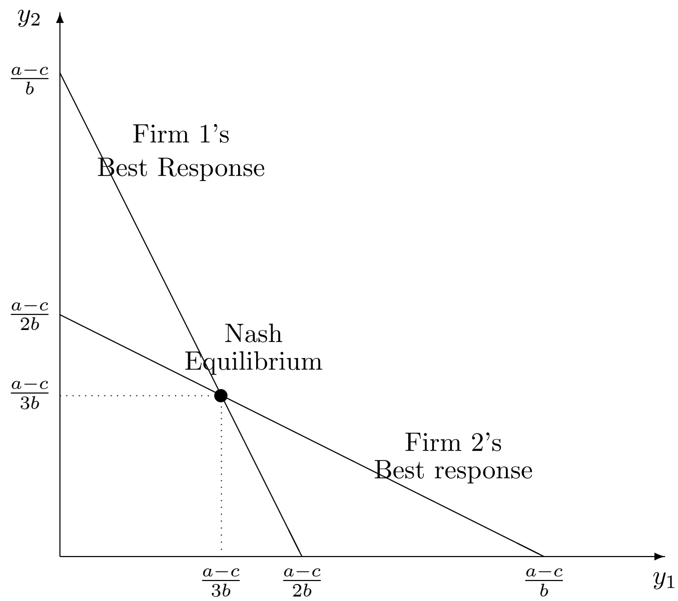

Figure 1: Best Responses and Nash Equilibrium in Cournot Model

The best responses are drawn in Figure 1 where you should observe a few things:

1. Evaluating the best response for a firm when the other firm picks an output=0 we have $y_1(0) = y_2(0) = \frac{a-c}{2b}$. If you recall the monopoly analysis *this is exactly the same as the solution to the monopoly problem* (verify), which should make perfect sense since a firm who believes that the other firm will produce nothing should behave as a monopolist.

2. Moreover, if, say, firm 2 would produce enough output so that $p(y_2) = c$,which would be the competitive (break-even price) then it is rather intuitive that additional output from firm 1 would mean that the price would get below the marginal cost⇒loss for the firm. But $q_1 = \frac{a-c}{b}$ is exactly that price since

$$p(\frac{a-c}{b}) = a - b\left(\frac{a-c}{b}\right) = c,$$

so the intercepts where the best response is zero has exactly the interpretation as the points where the competitor produces an output that yields the competitive price.

Solving the system for a Nash equilibrium we can actually "cheat" a little bit since the equations are symmetric and therefore ought to have a symmetric solution $y_1^* = y_2^*$. Then we have only a single equation to solve, namely

$$
\begin{aligned}
y_1^* &= \frac{a-c-by_1^*}{2b} = \frac{a-c}{2b} - \frac{y_1^*}{2} \Leftrightarrow \\
\frac{3}{2}y_1^* &= \frac{a-c}{2b} \Leftrightarrow y_1^* = y_2^* = \frac{a-c}{3b}
\end{aligned}
$$

You may also find it obvious from the picture that $\frac{a-c}{3b}$ by observing that the intercepts of the best responses are at $\frac{a-c}{b}$ and $\frac{1}{2}\frac{a-c}{b}$ respectively on both axis.

16.3 Comparison with Monopoly Model

Suppose that the firms instead of acting independently would get together and think about if they could improve on the situation in the Cournot equilibrium. They would then simply set $y = y_1 + y_2$ to maximize the monopoly profit, that is solve

$$\max_y \, (a - by - c)\, y$$

We've already solved this problem and the solution is (check!)

$$y^m = \frac{a-c}{2b}.$$

Plugging this into the profits and comparing with the profits under oligopoly you can check that the monopoly profit is more than twice the per firm profit in the Cournot equilibrium so it is possible for both firms to gain by forming a cartel. To see this more clearly in the graph we note that if we plot "isoprofits" (combinations of y_1 and y_2 such that profits are constant) for firm 1 then anyone of these solves

$$\pi_1 = (a - b(y_1 + y_2) - c)\, y_1$$

for some π_1. Now, **since the best response is the profit maximizing choice given the particular** y_2 these must have a zero slope when intersecting the best response. Moreover, profits are increasing the less the other firm produces so we can depict these isoprofits for firm 1 as in Figure 2. Doing the same thing for firm 2 and combining with the line consisting of points where $y_1 + y_2$ equals the monopoly output we get a very instructive picture, which

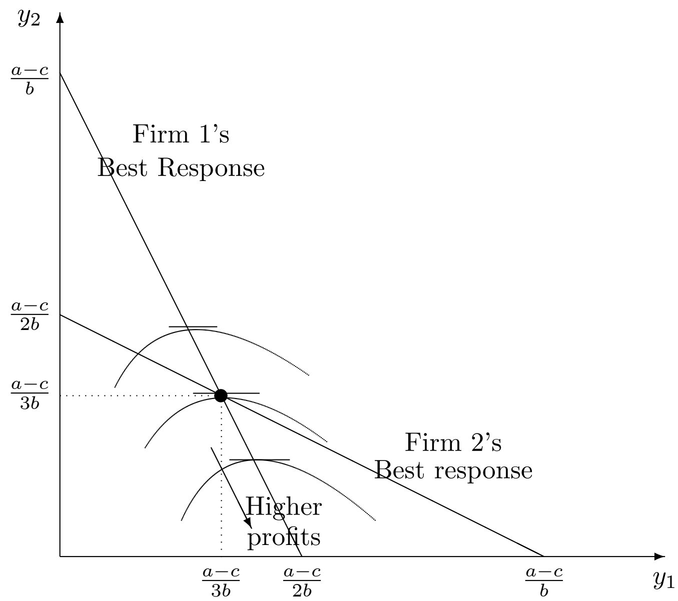

Figure 2: Isoprofits in Cournot Model

is the one in Figure 3. In the picture the line between $\left(\frac{a-c}{2b}, 0\right)$ and $\left(0, \frac{a-c}{2b}\right)$ are the points where industry output equals the monopoly output (firms acting as a cartel). In the figure we have also drawn in the isoprofits going through the Nash equilibrium of the Cournot model and everything in the shaded area consists of points where both firms are better off than in the Cournot equilibrium. Finally, the point on the intersection between the "cartel line" and the 45^0 line is the point where the firms agree to split the monopoly output in two.

This picture illustrates nicely both the temptation and the problem with a cartel agreement. The reason the firms want to cooperate is that by reducing output relative the equilibrium they both can gain. The problem is that if they try to do this, there is always a temptation to defect since the firms are not playing best responses.

16.4 The Stackelberg Model

A natural variation on the Cournot model is to ask what would happen if one firm would make its decision before the other firm. Then the players are still (in the duopoly version) firms 1 and 2 and their profit functions are the same, but:

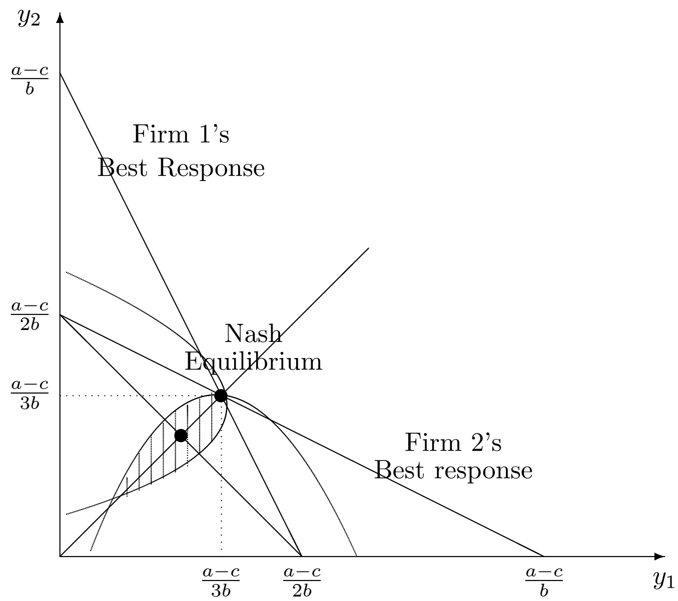

Figure 3: Cartel would be Better for Firms than Cournot Competition

- A strategy for firm 1 is some $y_1 \geq 0$
- A strategy for firm 2 is a *contingent plan that specifies the output as a function of* y_1. We write $r(y_1)$, where r is for "response".

Exactly as in the example with the "Battle of the sexes with first mover advantage" there will be lots of Nash equilibria in this game. However, the most interesting equilibrium is the equilibrium *where firm 2 behaves optimally after any "history of play",* that is the equilibrium where firm 2 would chose an optimal quantity no matter which quantity firm 1 picks. This is the "credible" Nash equilibrium which we refer to as the "backwards induction equilibrium".

16.4.1 The "Followers" Problem

In the backwards induction equilibrium the follower (firm 2) should maximize profits given any choice of y_1, that is firm 2 solves

$$\max_{y_2} \left(a - b(y_1 + y_2) - c\right) y_2$$

But this is just like the Cournot problem that determines the best responses which we already solved when analyzing the Cournot model. Without any further calculation we then know that the (dynamic) response of firm 2 must be

$$r(y_1) = \frac{a - c - by_1}{2b}$$

in a backwards induction equilibrium.

16.4.2 The "Leaders" Problem

Now, exactly as Bruce would foresee that if he moved first and went to the game rather than the opera, the leader firm in the Stackelberg model can figure out what the follower will do given any quantity choice by the leader. The leaders problem thus *takes into account that* $y_2 = r(y_1)$ so the leader solves

$$\max_{y_1} (a - b (y_1 + r (y_1)) - c) y_1$$

or, after substituting the response of firm 2

$$\begin{aligned} & \max_{y_1} \left(a - b \left(y_1 + \frac{a - c - by_1}{2b} \right) - c \right) y_1 \\ = & \max_{y_1} \left(a - by_1 - \left(\frac{a - c - by_1}{2} \right) - c \right) y_1 \\ = & \max_{y_1} \frac{1}{2} (a - c - by_1) y_1 \end{aligned}$$

It turns out that this is the monopoly problem, so the solution is to set (check if necessary!)

$$y_1^* = \frac{a - c}{2b}$$

This coincidence with the monopoly output is not a general feature of the Stackelberg Model but has to do with the geometry on the linear demand and cost functions, but the obvious consequence is that when the leader takes the optimal responses by the follower into consideration, it increases its output relative to the Cournot model. The equilibrium is

$$\begin{aligned} y_1^* &= \frac{a - c}{2b} \\ y_2^* &= r(y_1^*) = \frac{a - c - by_1^*}{2b} = \frac{a - c}{2b} - \frac{1}{2}\frac{a - c}{2b} = \frac{a - c}{4b} \end{aligned}$$

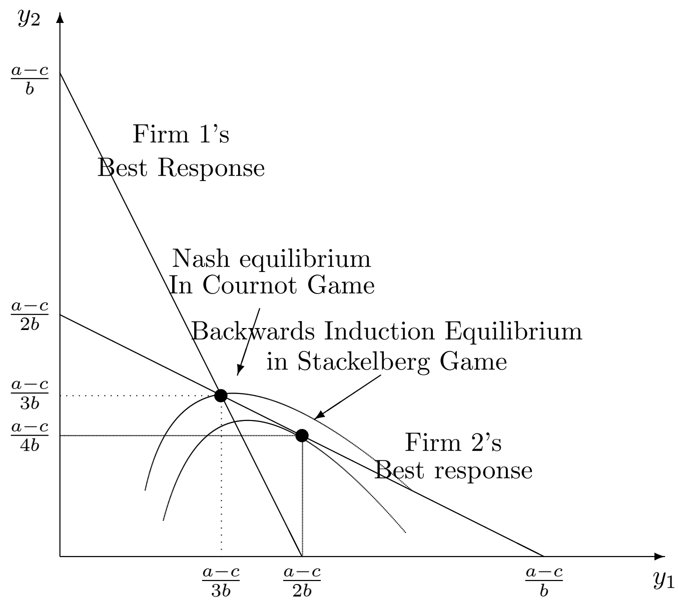

Figure 4: Equilibrium in Stackelberg Model

The most useful thing to understand this is from Figure 4. The key insight is that when the leader takes the response *function* rather than a given (conjectured) output into consideration, then the firm will pick the best point on the followers best response. But the best point is just a tangency between the isoprofit and the best response and since the slope of the isoprofit going through the Nash equilibrium in the Cournot game is zero it should be clear from the picture that this means that the leader-firm will increase the output relative the simultaneous model, which in turn implies that the follower will reduce its output. Since the leader ends up on a higher profit level (isoprofit closer to the monopoly output) there is a *first mover advantage* in the model.

16.5 Bertrand Competition

In the *Bertrand model* everything is as in the Cournot model except that firms choose prices instead of quantities. Note that

- If $p_1 < p_2$ then all consumers go to firm 1 that will then sell $q(p_1)$ units while firm 2 sells nothing.

- If $p_1 > p_2$ then all consumers go to firm 2 that will then sell $q(p_2)$ units while firm 1 sells nothing.

- If $p_1 = p_2$ then the firms split the consumers, getting half of them each.

Let

$$q(p) = A - Bp$$

be the (direct) demand. Now note that:

1. Neither firm could set a price below c in equilibrium since the lowest price firm would then would make a loss.

If firm 2 would set a price $p_2 > c$, then the profit of firm one would be

$$\begin{array}{rcl} 0 \text{ if } p_1 & > & p_2 \\ (p_2 - c)\dfrac{(A - Bp_2)}{2} > 0 \text{ if } p_1 & = & p_2 \\ (p_1 - c)(A - Bp_1) \text{ if } p_1 & < & p_2 \end{array}$$

Hence no firm could charge a price above c in equilibrium either since by undercutting the other firm by a small amount (a penny) the firm with the price a penny lower would get the whole market rather than just half of it. Hence the equilibrium is for both firms to charge a price equal to marginal cost, so with price competition two firms are sufficient to generate the competitive outcome.

While it may seem counter-intuitive that the firms don't have any market power the situation is very much like an auction where two agents value an object equally high. Thinking of it that way it is not that surprising that the equilibrium bids have to be the value of the object since if the agents would bid below their values, one of them could win for sure and still get some surplus out of the outcome while the other would get nothing.