6 Traditional Demand Theory

We have already discussed some examples of comparative statics in previous lectures and homework exercises. However, we have spent a big share of our energy discussing how to formulate and solve problems of rational choice in different versions. Now, we will switch the focus somewhat and try to be more systematic in our approach to comparative statics. In general, comparative statics is an exercise where we analyze how the behavior changes when different variables in the environment changes. That is, we try to predict what the behavioral response to different (to the consumer) exogenous changes should be. This sort of an exercise is what a large share of economics is about, and we will do it in many other applications in the rest of the class. For now, however, we will study what happens as prices and income changes for the consumer, which is the topic of traditional demand theory (one of the oldest and most well developed branches of economic theory).

The first fact to realize is trivial, but important: the optimal consumer choice depends on \( p_1, p_2 \) and \( m \). This should be obvious from a graph. In Figure 1 I’ve indicated an optimal solution given some initial prices and income \((x_1^*, x_2^*)\). In the left graph we see that when the price increases, then the old optimal bundle is no longer affordable, so the optimal solution

![Figure 1: A Change in Price or Income Changes the Solution](image-url)
with a higher price on good one must be some different bundle (somewhere along new budget line). To the right we see that when income increases there are now bundles better than the old optimal solution that are affordable, so the solution must change here as well.

Since we now will start to vary \( p_1, p_2 \) and \( m \) we want notation that makes this clear. We write

\[
x_1(p_1, p_2, m) \\
x_2(p_1, p_2, m)
\]

for the demand functions associated with some particular utility function. What this means in terms of our optimization problem is that \( x_1(p_1, p_2, m) \) and \( x_2(p_1, p_2, m) \) together constitute an optimal solution to

\[
\max_{x_1, x_2} u(x_1, x_2) \\
\text{s.t. } p_1 x_1 + p_2 x_2 \leq m
\]

**Example 1** Cobb-Douglas. We have actually already solved for the demand functions in class. \( x_1(p_1, p_2, m) \) and \( x_2(p_1, p_2, m) \) solves

\[
\max x_1^a x_2^{1-a} \\
\text{s.t. } p_1 x_1 + p_2 x_2 \leq m
\]

and we already know that

\[
x_1(p_1, p_2, m) = \frac{am}{p_1} \\
x_2(p_1, p_2, m) = \frac{(1-a)m}{p_2}
\]

Special property of Cobb-Douglas: demand for good 1 independent of price of good 2 and vice versa. This is not true in general. Also note that the parameter from the utility function \( a \) enters the formula for the demand generated by Cobb-Douglas preferences. Still, we don’t think of the demand function as a function of \( a \). The reason is that \( a \) is a deep parameter reflecting the preferences and comparisons of different as would be to compare different decision makers.
6.1 Changes in Income

Now, the demand function takes on a quantity for each triple \((p_1, p_2, m)\) so to graph these things we have to project down into lower dimensions. One of the obvious experiments to do is then to fix prices and see how the solution changes as income changes but prices are fix. This projection of the demand function is called an *Engel curve* and the graphical derivation of an Engel curve is shown in Figure 2. Observe that while the natural convention would be to plot \(x_1\) on the vertical axis and \(m\) on the horizontal, the opposite convention was established some time in the dark ages and economists got stuck with this.

Notice that whether the consumer increases or decreases the consumption of a particular good when the income increases depends on the preferences. That is, in the Cobb Douglas example we directly see that \(x_1(p_1, p_2, m) = \frac{am}{p_1}\) and \(x_2(p_1, p_2, m) = \frac{(1-a)m}{p_2}\) are both increasing in \(m\). In contrast, Figure 3, which is drawn with preferences that satisfy monotonicity and convexity, shows an example where the consumption if \(x_1\) is decreasing in \(m\) over an interval (observe, since consumption is zero when \(m = 0\) it must be that the consumption is initially increasing in \(m\) to make it possible for the curve to bend backwards).
The conclusion is thus that we cannot a priori say anything about whether the consumption is increasing or decreasing in income. For future reference we will simply create some terminology that assigns names to the different cases:

**Definition 2** Good 1(2) is said to be a normal good if \( x_1(p_1, p_2, m) \) \((x_2(p_1, p_2, m))\) is increasing in \(m\)

**Definition 3** Good 1(2) is said to be an inferior good if \( x_1(p_1, p_2, m) \) \((x_2(p_1, p_2, m))\) is decreasing in \(m\)

In concrete examples one can either just look at the demand function to see if it is increasing in \(m\) or not. If the demand function is more complicated, it is sometimes useful to observe that one can check this by looking at the partial derivative

\[
\frac{\partial x_1(p_1, p_2, m)}{\partial m}.
\]

In the Cobb-Douglas case \(\frac{\partial x_1(p_1, p_2, m)}{\partial m} = \frac{a}{p_1} > 0\). In general, normality corresponds to a positive partial derivative with respect to income, and inferiority corresponds with a negative partial derivative.
Typically normality is taken to mean that the quantity demanded is increasing in \( m \) for all \((p_1, p_2)\), but conventions differ and I will not be picky on things like that. Observe however that it is impossible for a good to be “always inferior”. If one wants to be careful, one also has to make ones mind up whether \( \frac{\partial x_1(p_1, p_2, m)}{\partial m} \geq 0 \) or \( \frac{\partial x_1(p_1, p_2, m)}{\partial m} > 0 \) is the criterion for normality, but you need not worry too much about that detail.

### 6.2 Changes in Prices

The next obvious experiment is to see how a demand function behaves as \( m \) and the price of the other good(s) are held constant. This projection of the demand function is called the demand curve. Figure 4 shows how this curve can in principle be derived from the usual indifference curve graph. Notice the convention (which has stuck for historical reasons) that (the “independent variable”) price is on the vertical axis.

Maybe more surprisingly (unless you’ve seen it in an earlier class) the effect on consumption from a change in the price is also ambiguous. If we again look at the Cobb-Douglas example we see that

\[
\frac{\partial x_1(p_1, p_2, m)}{\partial p_1} = -\frac{am}{p_1} < 0 \\
\frac{\partial x_2(p_1, p_2, m)}{\partial p_2} = -\frac{(1-a)m}{p_2} < 0,
\]

so in this case the effect from a price increase is what one would intuitively expect—a decrease.

However, it is possible that consumption of a good decreases when the price decreases, which graphically corresponds to an upwards sloping demand curve. Notice (we will come back to this) that the indifference curves in the graphical construction of an upward sloping demand in Figure 5 look remarkably similar to those of an inferior good.

The conventional terminology (which isn’t used as much as the normal/inferior distinction for reasons that will become apparent) is:

**Definition 4** Good 1(2) is said to be a ordinary good if \( x_1(p_1, p_2, m) \) \( x_2(p_1, p_2, m) \) is increasing in \( p_1 \) \( p_2 \)
Figure 4: Graphical Derivation of (Inverse) Demand Curve-Ordinary Good

**Definition 5** Good \((1, 2)\) is said to be a Giffen good if \(x_1(p_1, p_2, m)\) \((x_2(p_1, p_2, m))\) is decreasing in \(p_1\) \((p_2)\).

Again, sometimes the easiest way to check is by taking the partial derivative with respect to (own) price.
6.3 Examples

6.3.1 Cobb Douglas Preferences

We have that

\[ x_1(p_1, p_2, m) = \frac{am}{p_1} \]
and to draw an *Engel curve* we only need to set $a, p_1$ to some specific values and plot the relation between $m$ and $x_1$. Say for concreteness that $a = 1/2$ and $p_1 = 4$ ⇒

$$x_1(4, p_2, m) = \frac{1}{2} \frac{m}{4} = \frac{m}{8}$$

$$\Rightarrow \quad m = 8x_1,$$

so the Engel curve is a straight line starting at the origin with slope 8.

![Figure 6: The Engel Curve for $u(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{2}}$ given any $p_2$ and $p_1 = 4$](image)

In general we see that

$$x_1 = \frac{am}{p_1}$$

$$x_2 = \frac{(1 - a)m}{p_2}$$

Since the relationship between $x_1$ and $m$ is linear we see from this that the Engel curves are straight (upwards sloping) lines starting at the origin. Hence, good 1 is a normal good and everything is symmetric for good 2 so we conclude that both goods are normal for a consumer with Cobb Douglas preferences.

To sketch the *demand curve* for good 1 we fix $a$ and $m$ and vary $p_1$. If we again set the preference parameter $a = 1/2$ and set $m = 10$ we get

$$x_1 = \frac{1 \cdot 10}{2p_1} \Leftrightarrow p_1 = 5x_1,$$

so $x_1 = 1 \Rightarrow p_1 = 5$, $x_1 = 2 \Rightarrow p_1 = 2.5$, $x_1 = 5 \Rightarrow p_1 = 1...$
Figure 7: The Demand Curve for $u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$ given any $p_2$ and $m = 10$

Again we see that in general, the (inverse) demand curves are given by

\[ p_1 = \frac{am}{x_1}, \quad p_2 = \frac{(1 - a)m}{x_2}, \]

where you should think of $m$ as a constant when you plot these curves. Obviously this means that demand curves are downward sloping, so we conclude that both goods are ordinary

6.3.2 Perfect Substitutes

\[ u(x_1, x_2) = 5x_1 + x_2. \]

Recall from one of your homework exercises that this utility function generates interior solutions only in knife-edge cases. Indifference curves are straight lines, so the typical case is a corner solution (draw picture if you don’t see this!). The only tricky part is to determine when (that is for which prices & income) which corner is optimal and the easy way to this is to check what happens if the consumer spends everything on $x_1$, that is $(x_1, x_2) = \left( \frac{m}{p_1}, 0 \right)$. The corresponding value of the utility function is

\[ \frac{5m}{p_1}. \]
If on the other hand the consumer spends everything on $x_2$, then the utility is

$$\frac{m}{p_2}.$$  

Clearly, spending everything on $x_1$ ($x_2$) is better if $\frac{5m}{p_1} > \frac{m}{p_2} \iff \frac{p_1}{p_2} < 5$ ($\frac{5m}{p_1} < \frac{m}{p_2} \iff \frac{p_1}{p_2} > 5$), while the consumer is indifferent if $\frac{p_1}{p_2} = 5$. Thus the demand curve is

$$x_1(p_1, p_2, m) = \begin{cases} 
\frac{m}{p_1} & \text{if } \frac{p_1}{p_2} < 5 \\
\left[0, \frac{m}{p_1}\right] & \text{if } \frac{p_1}{p_2} = 5 \\
0 & \text{if } \frac{p_1}{p_2} > 5
\end{cases}.$$  

As an example, consider $p_1 = p_2 = 1 \Rightarrow$ slope of budget line: $-1$

$\Rightarrow$ Engel curve for good 1: $m = x_1$

$\Rightarrow$ Engel curve for good 2: $x_2 = 0$ for all $m$ (follows vertical axis)

Given that we only require the demand to be weakly increasing in income, this is consistent with normality.

Demand curve: Set $m = 10$, $p_2 = 1$

\begin{center}
\begin{tikzpicture}[scale=0.7]
\draw[->] (0,0) -- (10,0) node[right] {$x_1$};
\draw[->] (0,0) -- (0,10) node[above] {$p_1$};
\draw[dashed] (2,0) -- (2,10);
\draw[dashed] (5,0) -- (5,10);
\draw[thick] (0,5) .. controls (2,4) .. (5,0);
\node at (2,5) {5};
\node at (5,0) {5};
\end{tikzpicture}
\end{center}

Figure 8: Demand Curve for $u(x_1, x_2) = 5x_1 + x_2$ given $m = 10$ and $p_2 = 2$

$x_1 = 0$ for $p_1 > 5$

$[0, 2]$ for $p_1 = 5$

decreasing over $p_1$ in $[0, 5]$

If we allow ordinary goods to have ranges where the demand is constant, this is an ordinary good.
6.4 Substitutes and Complements

Final question—how is the demand of $x_1$ affected by a change in $p_2$. Know that for perfect complements the demand is decreasing since

$$ax_1(p_1, p_2, m) = bx_2(p_1, p_2, m)$$

The demand can then be found by solving

$$p_1x_1(p_1, p_2, m) + p_2\frac{a}{b}x_1(p_1, p_2, m) = m \Rightarrow x_1(p_1, p_2, m) = \frac{m}{p_1 + p_2\frac{a}{b}}$$

so $x_1 \downarrow$ when $p_2 \nearrow$. On the other hand, for perfect substitutes, either nothing happens or an increase in $p_2 \Rightarrow x_1 \nearrow$. Given these examples the following definitions seem natural:

6.4.1 (Gross) Substitutes

If $x_1(p_1, p_2, m)$ is increasing in $p_2$

6.4.2 (Gross) Complements

If $x_1(p_1, p_2, m)$ is deceasing in $p_2$

You can check that goods are neither substitutes or complements for Cobb-Douglas, substitutes for perfect substitutes and complements for perfect complements.

6.5 Income & Substitution Effects

This is covered in Varian Chapter 8 (8.1-8.5 and 8.7).

We found from straightforward indifference curve analysis that (in the case of a Giffen good) it is indeed possible that the demand is increasing in price. While we think that this is primarily a curiosity (that is, the conventional wisdom is that most goods are not like this) it is instructive to think about why this can happen.

Briefly put, the answer is that two things happen when the price of a good goes down. First of all the relative price changes making the good cheaper in terms of other good.
Moreover, the *purchasing power* of the consumer *increases* a fall in one price makes the consumer richer since consumption of all goods can now be increased.

We will now try to separate out these effects into *income* and *substitution* effects. You should be warned that there are two different ways to do this. Exactly as in Varian I will spend most time explaining the diagrammatically most straightforward decomposition (called the Slutsky decomposition), but check Varian 8.8 for another possible decomposition.

![Figure 9: Substitution and Income Effects](image)

The idea is that we can “control” for the fact that the consumer gets richer when the price falls by adjusting the income so that the *old optimal bundle is on the budget line with the new relative prices*. The optimal bundle for this fictitious budget set is *what the consumer would optimally choose under the new prices if income was taken away so as to make the old optimal bundle barely affordable*. Hence the difference between this and the old consumption can be thought of as the change in consumption that is attributed to the change in relative prices and this change is what is called the *SUBSTITUTION EFFECT*.

The *INCOME EFFECT* is then the change from this (hypothetical) bundle with new relative prices and adjusted income to the optimal bundle with the new prices and the (unchanged) income that the consumer actually has.
This is illustrated in Figure 9 where $x = (x_1, x_2)$ is the demand given ($p_1, p_2, m$) and $x'$ is the demand given ($p'_1, p_2, m$) (the only change is the price of good 1 that has decreased from $p_1$ to $p'_1$). Diagrammatically, the substitution effect is found by “pivoting” the budget line so that a new budget line with slope $-\frac{\bar{p}_1}{\bar{p}_2}$ that goes through the old optimal bundle is constructed. The substitution effect is then the difference between the demand given this new budget line and the original demand. The income effect is the difference between the demand given the price change (the “real thing” with unchanged income) and the (hypothetical) demand just constructed ($x''$ in the picture).

To explain this somewhat more carefully it is useful to use the notation for demand functions we’ve introduced.

1. The income that keeps $(x_1(p_1, p_2, m), x_2(p_1, p_2, m))$ exactly on the budget line when the price of good 1 changes from $p_1$ to $p'_1$ is

$$m' = p'_1 x_1 (p_1, p_2, m) + p_2 x_2 (p_1, p_2, m)$$

2. The TOTAL EFFECT on the demand of good 1 the price of good 1 changes from $p_1$ to $p'_1$ is

$$\Delta x_1 = x_1 (p'_1, p_2, m) - x_1 (p_1, p_2, m)$$

3. The SUBSTITUTION EFFECT is

$$\Delta x_1^S = x_1 (p'_1, p_2, m') - x_1 (p_1, p_2, m)$$

4. The INCOME EFFECT is

$$\Delta x_1^N = x_1 (p'_1, p_2, m) - x_1 (p'_1, p_2, m')$$

In words:

1. Total effect: effect on demand from a price change from $p_1$ to $p'_1$
2. Substitution effect: effect on demand from a price change from $p_1$ to $p'_1$ and a simultaneous income change from $m$ to $m'$ where $m'$ is calculated as to make the old demand exactly affordable at new prices.

3. Income effect: effect on demand from a change in income from $m'$ to $m$. (given new prices).

Observe finally that the decomposition is OK since

$$\Delta x_1^S + \Delta x_1^N = x_1 (p'_1, p_2, m') - x_1 (p_1, p_2, m) + x_1 (p'_1, p_2, m) - x_1 (p'_1, p_2, m') = x_1 (p'_1, p_2, m) - x_1 (p_1, p_2, m) = \Delta x_1$$

### 6.6 Example: Computing Substitution and Income Effects for Cobb-Douglas Preferences

Recall that if $u(x_1, x_2) = x_1^a x_2^{1-a}$, the demand functions are

$$x_1 (p_1, p_2, m) = \frac{am}{p_1}$$
$$x_2 (p_1, p_2, m) = \frac{(1-a)m}{p_2}.$$

For concreteness, set $a = \frac{1}{2}$ and let $(p_1, p_2, m) = (2, 2, 40)$. Consider a change in the price of good 1 from $p_1 = 2$ to $p'_1 = 1$ ⇒

$$x_1 (2, 2, 40) = \frac{140}{2} = 70$$
$$x_2 (2, 2, 40) = \frac{140}{2} = 70$$
$$x_1 (1, 2, 40) = \frac{140}{2} = 70$$

⇒ The total effect is given by

$$\Delta x_1 = x_1 (1, 2, 40) - x_1 (2, 2, 40) = 20 - 10 = 10$$
To compute the substitution effect we solve for

\[ m' = 1 \cdot x_1(2, 2, 40) + 2x_2 (2, 2, 40) = \\
= 1 \cdot 10 + 2 \cdot 10 = 30, \]

so the substitution effect is

\[ \Delta x_1^S = x_1 (1, 2, 30) - x_1 (2, 2, 40) = \frac{1}{2} \cdot 30 - 10 = 15 - 10 = 5. \]

The income effect is

\[ \Delta x_1^N = x_1 (1, 2, 40) - x_1 (1, 2, 30) = 20 - 15 = 5, \]

but really we already knew this since in general

\[ \Delta x_1^N = \Delta x_1 - \Delta x_1^S \]

and we had already computed \( \Delta x_1 = 10 \) and \( \Delta x_1^S = 5 \).

### 6.7 Sign of the Substitution Effect

Clearly, the income effect may be negative or positive depending on whether the good is normal or not, but the substitution effect can be signed, which is why the decomposition is of some use in economics.

**Claim** *The substitution effect is always negative.*

What this means is that when \( p_1 \) goes up (and income is adjusted), the demand goes down, while if \( p_1 \) goes down, then the demand goes down after the adjustment in income. To see this look at Figure 10, where \( x' = (x'_1, x'_2) \) is the optimal bundle given some prices and income (corresponding with the steeper budget line). Now, if the substitution effect tends to decrease the consumption when the price on good one goes down, the new optimal bundle must be to the left of \( x' \) on the pivoted budget line. Call this bundle \( x'' \). Now, the crucial thing to realize is that this bundle is *affordable given the old prices and income*, so
the consumer could have bought $x''$ before. Moreover, if preferences are monotonic, then everything to the northeast of $x''$ is strictly better than $x''$ which must be at least as good as $x'$ was affordable given the old prices and income. Hence, $x'$ could not have been optimal in the first place. The conclusion of this is that the optimal bundle after the pivot must be somewhere to the right of $x'$ on the pivoted budget line, so the demand increases when the price goes down. The exact same argument works for a price increase as well.

**Remark:** The argument uses the principle of *revealed preference*. We will not spend too much time on this in this class, but you should know that this principle makes it possible to “observe preferences”. Indeed, since we could get data that violates revealed preference, this makes our model of rational choice refutable, meaning that this is actually something that qualifies as a scientific theory. See chapter 7 in Varian for details on how revealed preference can be used to draw inference about preferences.

**WE CAN THEN CONCLUDE:**

1. $p'_1 < p_1 \Rightarrow x_1 (p'_1, p_2, m') \geq x_1 (p_1, p_2, m)$

2. $p'_1 > p_1 \Rightarrow x_1 (p'_1, p_2, m') \leq x_1 (p_1, p_2, m)$,
where
\[ m' = p'_1 x_1 (p_1, p_2, m) + p_2 x_2 (p_1, p_2, m) \]

### 6.8 Sign of the Income Effect

The income effect clearly depends on whether the good is normal or inferior and the only thing to watch out for is the direction of the shifts. First of all note that if \( x = (x_1, x_2) \) is the demanded bundle given \((p_1, p_2, m)\), then
\[
m' = p'_1 x_1 + p_2 x_2 \quad \text{(definition of pivot)}
\]
\[
m = p_1 x_1 + p_2 x_2 \quad \text{(since \( x \) is optimal \( \Rightarrow \) on budget line)}
\]

Combining these we have
\[
\Delta m = m' - m = x_1 (p'_1 - p_1) = x_1 \Delta p_1,
\]
so \( \Delta p_1 > 0 \Leftrightarrow \Delta m > 0 \). SAME SIGN, so

- Increased price \( \Rightarrow \) Increased Income
- Decreased price \( \Rightarrow \) Decreased Income,

so we say that

- The income effect is *negative* for normal goods (since \( p_1 \not\rightarrow m \not\rightarrow \) and the “effect” is shift from “hypothetical” to “actual” income)
• The income effect is positive for inferior goods.

6.9 A Giffen Good is “Very Inferior”

The easy way to think about these things is to imagine an increase in price. However, interpreting “−” as “opposite of the sign of the change in the price” and “+” as the same sign as the change in the price the expressions below are true no matter which way the price changes.

CASE 1: If good 1 is a normal good, everything is clear since

\[ \Delta x_1 = \Delta x_1^S + \Delta x_1^N \]

\[ \text{negative (always)} + \text{negative (def of normal good)} \]

⇒ Normal goods have downward sloping demand curves.

CASE 2: If good 1 is inferior, then

\[ \Delta x_1 = \Delta x_1^S + \Delta x_1^N \]

\[ ? + \text{Positive (def of inferior good)} \]

Hence, if the income effect is strong enough it may dominate the substitution effect ⇒ \( \Delta x_1 \) may be positive (meaning that it moves in the same direction as the change in the price), in which case good 1 is a Giffen good. The conclusion of this is that any Giffen good must be an inferior good (while the opposite implication doesn’t hold), so it is no coincidence that examples for inferior and Giffen good often coincide.

I am not an empirical economist, but, according to colleagues, there is no convincing empirical study that has been able to find a real-world Giffen good. I don’t think that is too surprising. The income effects need to be large for the income effects to dominate the substitution effects, and for that to be the case the good in question must be a fairly important good in the sense that a quite large share of the income is spent on it. Typical textbook examples of Giffen goods are potatoes on Ireland or rice in China (this is also the kind of goods empiricists have looked at in vain), but it seems doubtful that the consumption of the basic source of carbohydrates would decline as income rises. Rather, one would expect
people to eat as much potatoes (or more) as before and top it up with some meat, vegetables, and maybe a Guiness or two

7 Labor Supply

Varian pages 171-176 (but ignore his terminology about “endowment income effects”).

Before starting with equilibrium theory I will discuss one final application/interpretation of the standard consumer choice model that I have postponed because I wanted to talk about income and substitution effects before discussing it.

We now think about an agent who likes consumption, dislikes work (who doesn’t rather spend time watching “Who wants to...”) and is selling his/her time on the market. Let

- $C$ be consumption
- $p$ be the price of the consumption good
- $L$ be the amount of labor supplied
- $L$ be the endowment of time (i.e., 24 hours)
- $w$ be the wage (dollars per unit of time)
- $M$ be non-labor income

The sceptic may find it strange that although supply of labor has an obvious time dimension, consumption has not. This is a fair complaint, but as usual we are abstracting away from lots of realism in order to make the model as simple as possible. Still, this basic model has proven to be very useful and is still used in labor economics extensively.

7.1 The Budget Constraint

The obvious way to write down the budget constraint is to observe that Expenditures=total income, that is

\[ pC = M + wL. \]
This form is fine for setting up the relevant maximization problem, but not very useful for drawing indifference curve graphs. Instead, we observe that

\[ pC = M + wL. \]

\[ pC - wL = M \quad \Rightarrow \quad \text{add } w \bar{L} \text{ on both sides} \]

\[ pC + w (\bar{L} - L) = M + w \bar{L} \]

Finally let \( \bar{C} \) be the consumption the consumer would have if not working, that is

\[ \bar{C} = \frac{M}{p} \]

and we can write the budget constraint as

\[ pC + w (\bar{L} - L) = p\bar{C} + w \bar{L} \]

This form of the budget constraint should make clear that we can think of the labor supply problem in the same way as the standard model, where the consumer is purchasing consumption goods and leisure.

![Figure 12: The Budget Set for the Labor Supply Problem](image_url)

The way to think about it is that \textit{if the consumer doesn’t trade} then he/she consumes the endowment, which is \( \bar{C} \) units of the consumption good and \( \bar{L} \) units of leisure. However,
the consumer can trade leisure for consumption by supplying labor in the market. Note here that:

1. \( w \) is the price, or opportunity cost, of leisure. The point of this is that if you think that it is free to watch The Bachelor (or The Bachelorette) you should seriously rethink! What you pay for doing it is the income you would earn if you went out working the time spent on the couch (or, in a richer model where you can invest in future earnings opportunities by getting a good grade in Econ 301, your present value of the expected extra earning you would get by spending that extra time on your homework problem for Friday).

2. The slope of the budget line, \( w/p \) is usually referred to as the real wage.

### 7.2 Optimal Labor Supply Decisions

We now assume that the consumer has some preferences over consumption and leisure given by

\[
U (C, L - L)
\]

Substituting in the budget constraint gives the problem in the usual form

\[
\max_{0 \leq L \leq L} U (\bar{C} + \frac{w}{p} L, \bar{L} - L)
\]

and the first order condition will give a tangency condition of exactly the same form as before with the interpretation that the marginal rate of substitution between consumption and leisure has to be equalized with the slope of the budget constraint. The optimal labor supply decision is depicted in Figure 13.

You should all try to find the relevant first order conditions and think about them, I will restrict myself to graphical analysis.

**Question:** Suppose the wage goes up, what happens with labor supply?

Seems to be a relevant question. Often argued that reducing taxation on income will lead to more incentives to supply labor \( \Rightarrow \) more goods produced....
For simplicity let $M = 0$ (no non-labor income). To graph the budget set we then note that the intercept at the leisure axis in $\bar{L}$, while if the consumer has no leisure the (dollar) income is $w\bar{L}$, so that the intercept with the consumption axis is that $\frac{w\bar{L}}{p}$. Figure 14 shows the effect of increasing the wage from $w$ to $w'$ and we note that:

- An increase in the wage is exactly as a decrease in price of the consumption good.
- Hence, how consumption of leisure (supply of labor) is affected depends on whether leisure and the consumption good are substitutes or complements. You are encouraged to work out details!

An alternative way to look at it is to decompose the change in income and substitution effects:

**Substitution Effect** $w/p \downarrow$⇒leisure more expensive⇒work more/less leisure.

**Income Effect** $w/p \uparrow$⇒consumer richer and can increase consumption of both goods. If leisure is a NORMAL good⇒ consumer “buys” more leisure.
ThUS: INCOME AND SUBSTITUTION EFFECTS TEND TO GO IN OPPOSITE DIRECTIONS FOR NORMAL GOODS RATHER THAN FOR INFERIOR GOODS (AS IN STANDARD TWO GOODS MODEL).

Since there seem to be no a priori reason why leisure should be inferior we expect small responses in terms of labor supply and even backwards bending at high incomes. Eventually this means that the response to changes in the real wage is an empirical question and while the evidence is mixed small responses is the norm and backwards bending has been found in lots of studies.

7.3 Example: Cobb-Douglas Preferences

I assume no non-labor income. The problem is then to solve

\[
\max_{C,L} \alpha \ln C + (1 - \alpha) \ln (T - L)
\]

subj to \( pC = wL \)

or

\[
\max_{0 \leq L \leq T} \alpha \ln \left( \frac{wL}{p} \right) + (1 - \alpha) \ln (T - L)
\]
Figure 15: Substitution and Income Effects (Income Effect Dominating)

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\[
\frac{\alpha w}{pL} + \frac{(1 - \alpha)}{(L - L)} (-1) = 0 \Leftrightarrow \\
\alpha (L - L) = (1 - \alpha) L \\
L \left( \frac{w}{p} \right) = \alpha L
\]

- Labor supply constant function of real wage.
- Income and substitution effect cancel each other out completely.