14 Cost Minimization

Optional Reading: Varian, Chapters 20, 21.1-21.3 & 22.1-22.7.

In principle, everything we want to know about competitive firms can be derived from profit maximization problem. One can derive:

- Factor demands
- Firm supply

Adding over all firms in the economy we get market factor demands and supply of goods and combining with the demand for goods and supply of factors, which come from either exogenous resource constraints (land) or adding over demands and supplies from standard consumer problems (or if a "factor" is an intermediate good from other profit maximization problems) we get a full blown equilibrium model.

However, as we saw in the example, the profit maximization problem is rather complex and sometimes ill-defined and it turns out that it is useful to separate the problem to maximize profits into two steps:

Step 1 For a given level of output y, find the cheapest way to produce any given level of output.

Step 2 Select the best level of output.

The advantages of this approach are that

1. Tractability: It is easier to think about the relevant trade-offs when we break up problem in parts.

2. The first step, the *cost minimization problem,* is the same regardless of whether the market (for the output good) is competitive, the firm is a monopolist or if there is some intermediate situation with "imperfect" competition. This facilitates comparisons of different market forms.

14.1 The Cost Minimization Problem

We ask, which is the cheapest way to produce a given level of output for a firm that takes factor prices as given and has access to some technology summarized by the production function $f(x_1, x_2)$. That is

$$\begin{aligned} \min_{x_1,x_2} w_1x_1 + w_2x_2 \\ \text{s.t. } f(x_1,x_2) &= y. \end{aligned}$$

Now, you may think that this is something different from what we have done before due to the "min" operator replacing the "max". However:

- If (x_1^*, x_2^*) is a solution to the minimization problem that means that $w_1x_1^* + w_2x_2^* \leq w_1x_1 + w_2x_2$ for all (x_1, x_2) that satisfies $f(x_1, x_2) = y$.

- But $w_1x_1^* + w_2x_2^* \leq w_1x_1 + w_2x_2 \Leftrightarrow \Leftrightarrow(w_1x_1^* + w_2x_2^*) \geq \Leftrightarrow(w_1x_1 + w_2x_2)$

- $\Rightarrow \Leftrightarrow(w_1x_1^* + w_2x_2^*) \geq \Leftrightarrow(w_1x_1 + w_2x_2)$ for all (x_1, x_2) that satisfies $f(x_1, x_2) = y$. Which means that (x_1^*, x_2^*) solves

$$\begin{aligned} \max_{x_1,x_2} \Leftrightarrow w_1x_1 \Leftrightarrow w_2x_2 \\ \text{s.t. } f(x_1,x_2) = y. \end{aligned}$$

Now, in principle the constraint can be solved for x_2 as a function of x_1 and you can then plug this into the objective function and dervve a solution by taking first order conditions in the exact same way as when we solved utility maximization problems. Alternatively, Lagrangian methods can be applied (if you know how you are welcome to use Lagrangians when solving constrained optimization problems. For this particular problem it is actually more convenient due to the potentially non-linear constraint). Since I want to keep the math at a very basic level I will make sure that all problems you will see can be solved without Lagrangians (that is, $f(x_1, x_2)$ will have simple enough form so that you can solve it out).

We will return later to the (less important) problem *how to calculate solutions.* Here, the more important thing is to see conceptually how *cost minimization relates to profit* *maximization.* Write

$$x_1(w_1, w_2, y)$$

$$x_2(w_1, w_2, y)$$

For the solution to the cost minimization problem (which you can derive in exactly the same way as the solution to the utility maximization problem in consumer theory). In analogue with the utility maximization problem *the solution will depend on the parameters (the exogenous variables) of the problem.* That is, exactly as price and income changes will change the best consumer bundle in consumer theory, factor price changes and how much you are supposed to produce will change the factor inputs that produces the target output in the cheapest possible way. Now, we can define

$$\begin{aligned} C(w_1, w_2, y) &= w_1 x_1(w_1, w_2, y) + w_2 x_2(w_1, w_2, y) = \\ &= \begin{array}{c} \min_{x_1, x_2} w_1 x_1 + w_2 x_2 \\ \text{s.t } f(x_1, x_2) = y \end{array} \end{aligned}$$

This function $C(w_1, w_2, y)$ is called the *minimal cost function* or simply the *cost function.*

Now the problem to maximize profits is

$$\begin{aligned} &\max_{x_1, x_2, y} py \Leftrightarrow w_1 x_1 \Leftrightarrow w_2 x_2 \\ \text{subj to } y &\leq f(x_1, x_2) \end{aligned}$$

Let y^* be a profit maximizing output level (together with some factor inputs of course). We note that the optimal factor inputs must solve

$$\begin{aligned} &\max_{x_1, x_2} py^* \Leftrightarrow w_1 x_1 \Leftrightarrow w_2 x_2 \\ \text{subj to } y^* &\leq f(x_1, x_2), \end{aligned}$$

where py^* is just a constant. This problem is equivalent (see discussion on max versus min above) with the cost minimization problem.

The important consequence of this is that this means that if we have a given cost function we can look for the profit maximizing output level by solving the problem

$$\max_y py \Leftrightarrow C(w_1, w_2, y).$$

This is a simple enough univariate calculus problem and sometimes referred to as *the firm supply problem (*i.e., in Varian chapter 22)

14.2 Solving the Cost Minimization Problem

We will again proceed both graphically and by actually solving the problem using calculus. Here these approaches complements each other to a larger extent than previously. The picture gives a clear intuition, but the relationship with profit maximization is not seen as easily. The calculus approach is less intuitive, but here the way cost minimization is used as an intermediate step to solve the profit maximization problem is easier to see.

14.2.1 Graphical Treatment

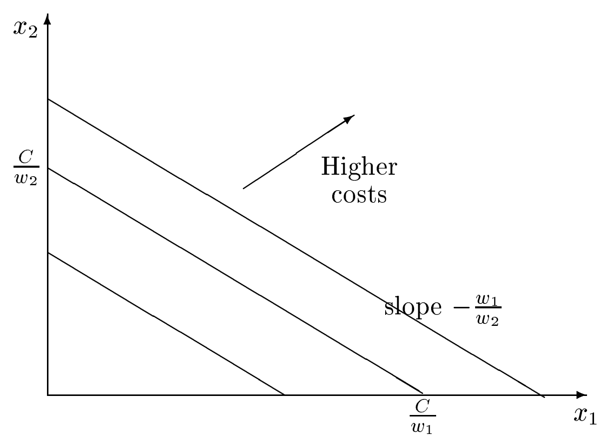

Figure 1: Lines with Constant Costs

The idea is to proceed as we did when "solving" the profit maximization problem with a picture in Section **??**. Consider all combinations of factor inputs that corresponds to some given cost level C, that is

$$\begin{aligned} C &= w_1 x_1 + w_2 x_2 \Leftrightarrow \\ x_2 &= \frac{C}{w_2} \Leftrightarrow \frac{w_1}{w_2} x_1, \end{aligned}$$

which defines a family of straight lines called "isocosts" in Varian. The cost minimization problem is to produce a given output at the lowest possible cost, so in terms of a graph it is then clear that if we put in the level curve of $f(x_1, x_2)$ that shows the combinations of inputs that gives this level of output y (i.e., the isoquant corresponding to y) the solution to the cost minimization problem occurs *at the line that touches the given isoquant which is closest to the origin of the graph.* Once again it is then apparent that the solution must occur where there is a tangency between the isoquant and the isocost since otherwise it is possible to move towards lower cost levels and still produce the same output.

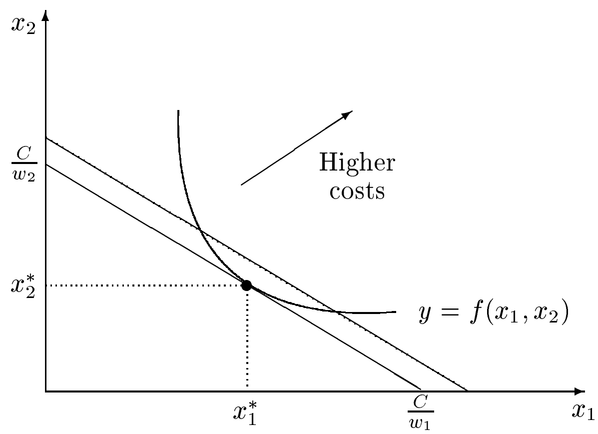

Figure 2: The Cost Minimization Problem

In our discussion of technology we showed that the slope of the level curve is given by

$$\Leftrightarrow \frac{\frac{\partial f(x_1,x_2)}{\partial x_1}}{\frac{\partial f(x_1,x_2)}{\partial x_2}} \equiv TRS(x_1, x_2),$$

so the optimum condition is

$$\frac{\frac{\partial f\left(x_1^*,x_2^*\right)}{\partial x_1}}{\frac{\partial f\left(x_1^*,x_2^*\right)}{\partial x_2}} = \frac{w_1}{w_2}$$

This makes perfect sense:

- The technical rate of substitution tells you how much extra factor 2 needed if output is to be kept constant and factor 1 reduced by a small unit.

  "The rate at which firms can substitute factors"

- The relative factor price $\frac{w_1}{w_2}$ gives the "rate at which factors can be exchanged in the market".

14.3 Some Examples

14.3.1 Fixed Proportions

$$\begin{aligned} C(w_1, w_2, y) &= \min_{x_1, x_2} w_1 x_1 + w_2 x_2 \\ \text{s.t } y &= \min\{x_1, x_2\} \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} x_1(w_1, w_2, y) &= y \\ x_2(w_1, w_2, y) &= y \end{aligned}$$

$\Rightarrow$

$$C(w_1, w_2, y) = (w_1 + w_2)\, y$$

14.3.2 Cobb Douglas

$$\begin{aligned} C(w_1, w_2, y) &= \min_{x_1, x_2} w_1 x_1 + w_2 x_2 \\ \text{s.t } y &= x_1^a x_2^b \end{aligned}$$

Solving the constraint we get

$$x_2 = y^{\frac{1}{b}} x_1^{-\frac{a}{b}}$$

Plugging into the objective we get

$$C(w_1, w_2, y) = \min_{x_1} w_1 x_1 + w_2 y^{\frac{1}{b}} x_1^{-\frac{a}{b}}$$

Or, you may write this as

$$\max_{x_1} \Leftrightarrow w_1 x_1 \Leftrightarrow w_2 y^{\frac{1}{b}} x_1^{-\frac{a}{b}}$$

The first order condition is

$$\Leftrightarrow w_1 \Leftrightarrow w_2 y^{\frac{1}{b}} \left(\Leftrightarrow \frac{a}{b} \right) x_1^{-\frac{a}{b}-1} = 0$$

or

$$\begin{aligned} w_1 &= w_2 y^{\frac{1}{b}} \frac{a}{b} x_1^{-\left(\frac{a+b}{b}\right)} \Leftrightarrow \text{multiply with } x_1^{\frac{a+b}{b}} \\ w_1 x_1^{\frac{a+b}{b}} &= w_2 y^{\frac{1}{b}} \frac{a}{b} x_1^{-\left(\frac{a+b}{b}\right)} x_1^{\frac{a+b}{b}} = w_2 y^{\frac{1}{b}} \frac{a}{b} \end{aligned}$$

or

$$\begin{aligned} x_1^{\frac{a+b}{b}} &= \frac{w_2}{w_1} \frac{a}{b} y^{\frac{1}{b}} \Leftrightarrow \\ x_1 (w_1, w_2, y) &= \left(\frac{w_2}{w_1} \frac{a}{b} \right)^{\frac{b}{a+b}} y^{\frac{1}{a+b}} \end{aligned}$$

Symmetrically we get (either by observing the symmetry or by plugging back in constraint) that

$$x_2 (w_1, w_2, y) = \left(\frac{w_1}{w_2} \frac{b}{a} \right)^{\frac{a}{a+b}} y^{\frac{1}{a+b}}$$

and plugging this into the objective we get the cost function

$$C (w_1, w_2, y) = w_1 \left(\frac{a w_1}{b w_2} \right)^{\frac{b}{a+b}} y^{\frac{1}{a+b}} + w_2 \left(\frac{b w_2}{a w_1} \right)^{\frac{a}{a+b}} y^{\frac{1}{a+b}},$$

which looks really ugly. However, competitive analysis assumes that prices and factor prices are exogenous for the firm. Hence, from the perspective of analyzing the supply problem for the firm *the really important property of the cost function is to say how costs change with output.* This exercise is for fixed factor prices and we note that we then may write the resulting cost function for a Cobb Douglas technology as

$$C (y) = K y^{\frac{1}{a+b}},$$

where

$$K = w_1 \left(\frac{a w_1}{b w_2} \right)^{\frac{b}{a+b}} + w_2 \left(\frac{b w_2}{a w_1} \right)^{\frac{a}{a+b}}$$

We then see that

$$C' (y) = K \frac{1}{a+b} y^{\frac{1}{a+b}-1}$$

is:

1. Increasing in y if $a + b < 1$. That is, the *marginal cost* is increasing when there is *decreasing returns to scale.*

2. Decreasing in y if $a + b > 1$. That is, the marginal cost is decreasing with increasing returns to scale.

3. Constant in y if $a + b = y$. That is the marginal cost is constant with constant returns to scale.

14.3.3 A Remark about Notation

Once again, note that:

- w_1, w_2, y are *parameters* of the cost minimization problem

- x_1, x_2 are the *choice variables.*

The solution to the cost minimization problem will then in general give the choice variables as functions of the parameters. We write these as

$$x_1 (w_1, w_2, y)$$
$$x_2 (w_1, w_2, y)$$

and call them conditional factor demands. Now, plugging in these in the objective we get the *cost function*

$$C (w_1, w_2, y) = w_1 x_1 (w_1, w_2, y) + w_2 x_2 (w_1, w_2, y) .$$

One of the more confusing aspects of economics is that sometimes we write something as a function of a long list of parameters and sometimes we write the same thing as a function of a shorter list, maybe just a single parameter. This practice simply reflects that for some purposes we want to keep a bunch of parameters constant and for other purposes we want to see what happens when we change these parameters. For that reason, the list of parameters that is explicitly introduced in the notation depends on the question we want to ask.

The cost function is a perfect example. If we want to study factor substitution we keep w_1, w_2 in the notation and study how factor shares are affected when the relative factor price changes. However, often times we will not experiment with changes in factor prices and then we write

$$x_1(y)$$

$$x_2(y)$$

for the conditional factor demands and

$$C(y) = w_1 x_1(y) + w_2 x_2(y)$$

For the cost function. This is purely a matter of convenience and for any particular technology there is a particular cost function and the formula for this function typically involves w_1 and w_2. However, this is now implicitly incorporated in "the functional relation".

15 Average, Marginal and Total Costs

We will now take the cost function derived in the section on cost minimization as given and introduce some terminology that is useful in order to think about firm supply decisions. Since we are not going to chine factor prices we are lazy and write $C(y)$ rather than $C(w_1, w_2, y)$, but it is conceptually important that you keep in mind we are still studying exactly the same creature. Now:

- $\frac{C(y)}{y}$ is called the average cost
- $\frac{dC(y)}{dy}$ is called the marginal cost.

Often we think about cost functions that have a fixed cost component (costs for setting up a plant or R&D etc.) and write

$$C(y) = C_v(y) + F,$$

where $C_v(y)$ is the variable cost function and F is the fixed cost. We than call

- $\frac{C_v(y)}{y}$ the average variable cost

- $\frac{F}{y}$ the average fixed cost

There are lots of relations between these curves and you can read about this in Varian. However, a few facts are important for understanding of graphs:

15.0.4 Fact 1: The Area below the Marginal Cost Curve=Total Variable Costs

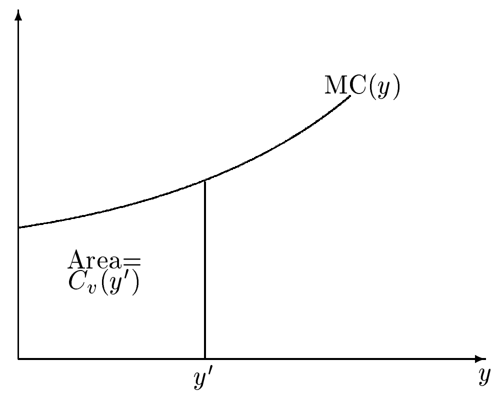

Figure 3: The Area Below MC-Curve=Total Variable Costs

Idea: MC-cost of last unit

Total variable costs-sum of all MCs from 1st to last unit

Mathematically this is essentially just saying that integration is the opposite of differentiation, so for those of you who knows what an integral is, this should be obvious:

$$\begin{aligned} C(y) &= C_v(y) + F \Rightarrow \\ \frac{dC(y)}{dy} &= \frac{dC_v(y)}{dy} \\ \int_0^{y'} \frac{dC(y)}{dy} dy &= \int_0^{y'} \frac{dC_v(y)}{dy} dy = [C_v(y)]_0^{y'} = C_v(y') \end{aligned}$$

If you don't know what an integral is, ignore this and think about this in terms of small discrete units.

15.0.5 Fact 2: MC and AVC curve starts at same place

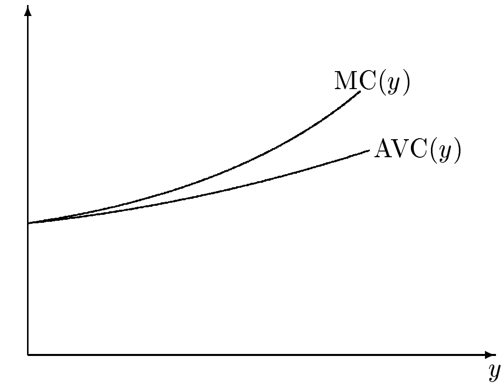

Figure 4: MC and AVC Curves Starts at same Place

Idea: Average variable cost of first small unit and marginal cost for the first small unit is the same thing. I.e., Marginal cost at zero is

$$\frac{dC(0)}{dy} = \frac{dC_v(0)}{dy} = \lim_{y \to 0} \frac{C_v(y) \Leftrightarrow C_v(0)}{y} = \lim_{y \to 0} \frac{C_v(y)}{y}$$

and $\frac{C_v(y)}{y}$ is just the average cost.

15.0.6 Fact 3: AVC decreasing whenever MC curve is below AVC curve and AVC increasing whenever MC curve is above AVC curve.

Idea: The way to decrease an average is to add numbers that are below the average

Math:

$$\begin{aligned} \frac{d}{dy}\frac{C_v(y)}{y} &= \frac{\frac{dC_v(y)}{dy} y \Leftrightarrow C_v(y)}{y^2} = \\ &= \frac{\frac{dC_v(y)}{dy} \Leftrightarrow \frac{C_v(y)}{y}}{y} = \frac{MC(y) \Leftrightarrow AVC(y)}{y} \end{aligned}$$

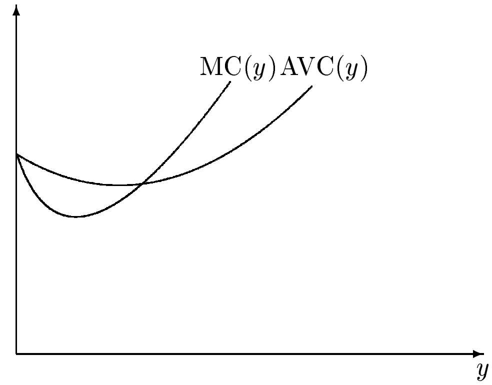

Figure 5: MC Decreasing iff AVC¿MC

16 Supply of a Competitive Firm

Optional Reading: Varian, Chapter 22.

A competitive firm take the price of the product(s) it sells as given, so the profit maximizing level of output is the solution to

$$\max_y py \Leftrightarrow C(y)$$

Observe here the analytical advantage of using the cost function rather than to write down the "complete" problem

$$\max_{x_1, x_2} pf(x_1, x_2) \Leftrightarrow w_1 x_1 \Leftrightarrow w_2 x_2.$$

The cost function $C(y)$ includes all relevant information about the production function $f(x_1, x_2)$ and factor prices and as you will see this means that we can graphically depict the firm supply function in pictures that should be familiar from Econ 101. Now, the first order condition to the "firm supply" problem is

$$p \Leftrightarrow C'(y) = 0 \Leftrightarrow p = C'(y).$$

Which simply says that firms should produce output up to the point where the marginal cost of production is equal to the price. This should be highly intuitive since if the last produced unit would cost more than the price to produce, then the firm would be better off reducing output. If on the other hand, the last produced unit would cost less to produce than the price the firm gets on the market, then profits would increase if output is increased.

16.1 How to Handle Multiple Solutions to $p = MC$

It is important to understand what the condition $p = MC$ is. It is a *necessary* condition for an *interior solution.* Hence there are two things to worry about. First of all it may be that there are several output levels consistent with the condition. Second of all, the solution may be a *boundary solution* (that is producing nothing).

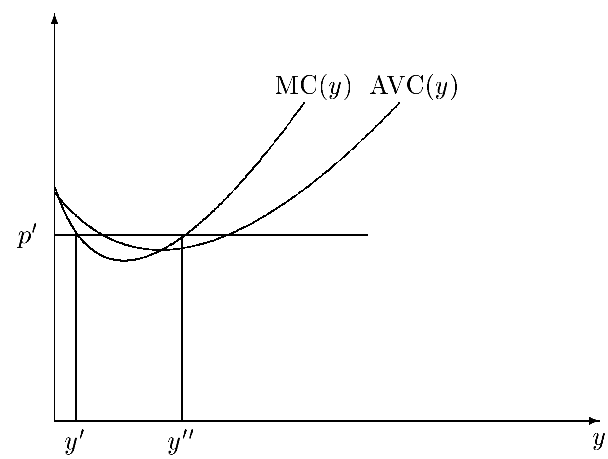

Figure 6: Solution can't be where MC curve slope downwards

In many circumstances it is reasonable to think that the marginal cost curve is U-shaped and this case is also the favorite case for undergraduate economics textbooks. Then, it may very well be that there are two levels of output y that satisfies $p = C'(y)$ as in Figure 6.

Here the basic insight is that **whatever the solution to the profit maximization problem is, it must be at the upwards sloping part of the marginal cost curve.** Hence, in the case depicted in Figure 6 we see that y' and y'' both are output levels such that price equals marginal costs. However, y'' generates a higher profit than y', so if either y' or y'' indeed is the solution, then it must be y''.

That y'' generates a higher profit can be understood from the picture directly. We established above that the total variable costs equals the area below the marginal cost curve. Hence the difference in variable costs between y'' and y' is the area **below the MC curve** in between y' and y''. The difference in revenues is the rectangle with base given by the line between y' and y'' and height p. Hence, the extra profit from increasing production from y' to y'' is the area in between the horizontal line at p' and the marginal cost curve, meaning that producing at y'' gives a higher profit than producing at y'.

Another way to understand the same thing is that the profits can be written as

$$\begin{aligned} \pi(y) &= py \Leftrightarrow C(y) = py \Leftrightarrow C_v(y) \Leftrightarrow F \\ &= y\left(p \Leftrightarrow AVC(y)\right) \Leftrightarrow F \end{aligned}$$

Hence, conditional on $p < AVC(y'')$ (see discussion below on this) it follows that:

- If $y'' > y'$ and
- $AVC(y'') < AVC(y')$, then
- $\pi(y'') > \pi(y')$

Now, if you look at the picture you should see that the marginal cost curve is below p for all y in between y' and y''. Hence the marginal cost for each unit in between y' and y'' is **lower than the marginal cost for the units in between** 0 and y'. Hence, the average variable cost must be lower at y'' than the cost at y'.

Note from this discussion that if the marginal cost curve is always decreasing, then there CAN NOT be an interior solution to the supply problem. You should draw a graph and convince yourself that this is so and why!

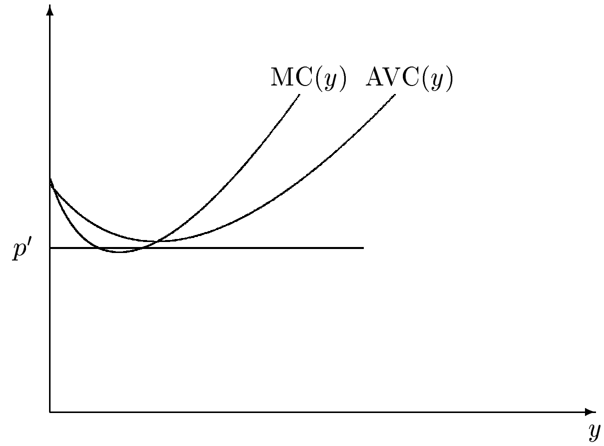

Figure 7: $y = 0$ Solves Problem if Price too Low

16.2 The Possibility of a Boundary Solution

In general we know that the first order condition is *necessary* only for *interior solutions.* However, if

$$p < AVC(y)$$

for all choices of y, then the best thing the firm can do is to produce nothing. Such a situation is depicted in Figure 7. It should be clear from the picture that

$$\pi(y) = y\,(p \Leftrightarrow AVC(y)) \Leftrightarrow F < \Leftrightarrow F = \pi\,(0)$$

for any $y > 0$. The conclusion is that:

- If the candidate solution on the upwards sloping part of the marginal cost curve occurs where the marginal cost curve is below the average variable cost curve, then this is indeed the solution to the profit maximization problem.

- However, if not, then the solution is to set $y = 0$.

16.3 The (Inverse) Supply Curve

Combining the "shutdown condition" with the fact that if the shutdown condition is satisfied we can depict the *supply curve* of the firm as in Figure 8. Again, you should note that conceptually we think of supply as a function of the price, i.e., the problem

$$\max_{y \geq 0} py \Leftrightarrow C(y)$$

has price as an exogenous parameter. Hence, it gives an optimal solution for each p (if problem well-defined as we will assume). However, for the graphs it is more convenient to put p on the vertical axis since that corresponds to the natural way to draw the cost curves.

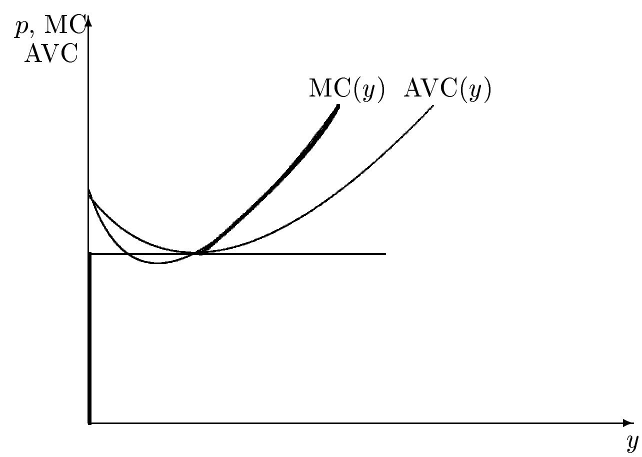

Figure 8: The Firm Supply Curve

16.4 Profits and Producer Surplus

In the case when there is a fixed cost, we do have to include that in the calculation of the profit for the firm. In terms of the graph this means that we have to use the average cost curve rather than the average variable cost curve. Note that the larger is output, the closer is the average cost curve to the average variable cost curve, which simply reflects that $\frac{F}{y}$ is decreasing in y with $\lim_{y \to \infty} \frac{F}{y} = 0$.

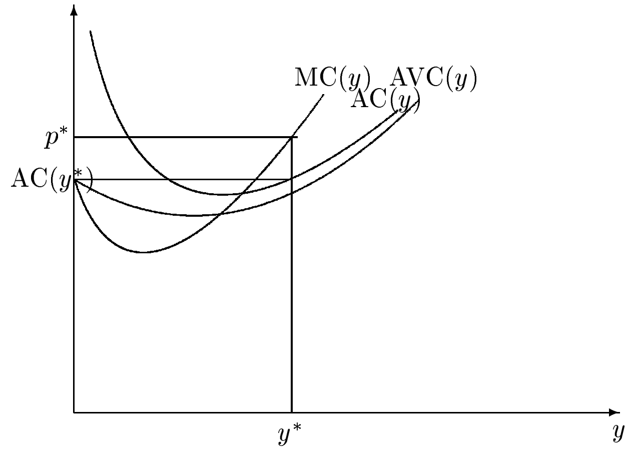

Figure 9: Firm Profit Given price p^*

In Varian and other textbooks it is common to call the profit net of the fixed cost F the *producer surplus.* The reason for this name (I think) is that diagrammatically it is the analogue of the *consumer surplus* in utility theory (don't panic-I haven't even mentioned that in the class, but you may recall this name from Econ 101).

Now, the distinction between profits and producer surplus is kind of trivial and unimportant when solving problems using calculus. Whether one maximizes

$$py \Leftrightarrow C(y) = py \Leftrightarrow C_v(y) \Leftrightarrow F$$

or

$$py \Leftrightarrow C_v(y)$$

doesn't matter at all since the problems only differ by a constant. However, for drawing pictures it is actually good not to have the fixed cost included. The reason is that we can measure total variable costs as either the rectangle with base y^* and height $AVC(y^*)$ or as the area between the marginal cost curve and the horizontal axis (Fact 1 in our discussion of cost curves). Hence, we can measure the producer surplus in any of the two ways depicted in Figure 10.

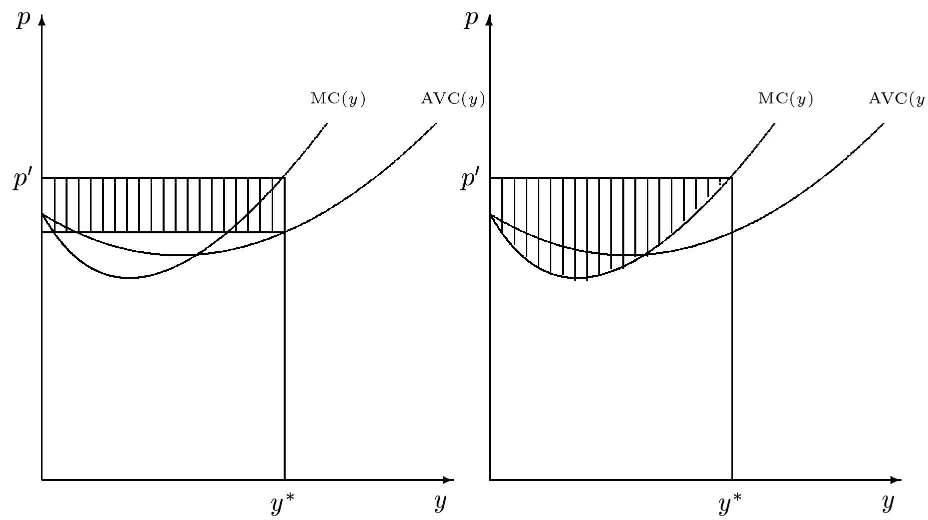

Figure 10: Two Equivalent Ways to Measure Producer Surplus

16.5 Example

Let the cost curve (i.e., the minimized cost function from some problem of cost minimization) be given by

$$C(y) = \frac{1}{2}y^2 + y + 1$$

and we have that:

- $AC(y) = \frac{C(y)}{y} = \frac{\frac{1}{2}y^2+y+1}{y} = \frac{1}{2}y + 1 + \frac{1}{y}$
- $AVC(y) = \frac{C_V(y)}{y} = \frac{\frac{1}{2}y^2+y}{y} = \frac{1}{2}y + 1$
- $AFC(y) = \frac{C(0)}{y} = \frac{1}{y}$
- $MC(y) = C'(y) = y + 1$

We can easily solve the firm supply problem explicitly in this example. The problem is

$$\max_{y \geq 0} py \Leftrightarrow C(y)$$

or, with the particular cost function

$$\max_{y \geq 0} py \Leftrightarrow \frac{1}{2}y^2 \Leftrightarrow y \Leftrightarrow 1$$

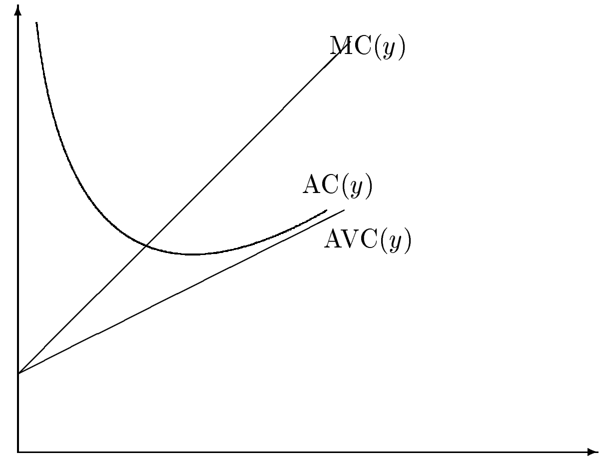

Figure 11: Cost Curves for $C(y) = \frac{1}{2}y^2 + y + 1$

The first order condition is

$$\begin{aligned} p \Leftrightarrow y \Leftrightarrow 1 &= 0 \Leftrightarrow \\ p &= \underbrace{y+1}_{\text{MC}} \end{aligned}$$

Solving this we get the candidate solution

$$y(p) = p \Leftrightarrow 1$$

Since the marginal costs are always increasing in y we don't need to worry about multiple solutions and this is reflected in the algebra above by the fact that solving the price equals marginal cost condition we get a unique solution for y. However, we do need to worry about the "shutting down constraint". If $y(p) \geq 0$, the candidate solution is $p \Leftrightarrow 1$ and the corresponding profit is

$$\begin{aligned} \pi(p) &= py(p) \Leftrightarrow c(y(p)) = \\ &= p(p \Leftrightarrow 1) \Leftrightarrow \frac{1}{2}(p \Leftrightarrow 1)^2 \Leftrightarrow (p \Leftrightarrow 1) \Leftrightarrow 1 \\ &= (p \Leftrightarrow 1)\left(p \Leftrightarrow \frac{1}{2}(p \Leftrightarrow 1) \Leftrightarrow 1\right) \Leftrightarrow 1 \\ &= \frac{(p \Leftrightarrow 1)^2}{2} \Leftrightarrow 1 \end{aligned}$$

to be compared with

$$\pi(0) = \Leftrightarrow 1$$

Under the assumption that $p \Leftrightarrow 1 > 0$ we have that $\pi(p) = \frac{(p-1)^2}{2} \Leftrightarrow 1 > \Leftrightarrow 1 = \pi(0)$, so in this case the solution to the interior first order condition is indeed the solution to the problem. Now, if $p < 1$, the first order condition $p = y + 1$ isn't satisfied for any $y \geq 0$. Intuitively it is rather clear that in this case $y(p) = 0$ is the optimal solution since the marginal cost for the first unit is 1 and the revenue for the first unit is $p < 1$. To see this formally, note that if $p < 1$ the profit satisfies

$$py \Leftrightarrow \frac{1}{2}y^2 \Leftrightarrow y \Leftrightarrow 1 < \Leftrightarrow \frac{1}{2}y^2 \Leftrightarrow 1 = \Leftrightarrow \frac{1}{2}y^2 \Leftrightarrow \pi(0) < \pi(0)$$

for all $y > 0$. Hence, the supply curve is

$$y(p) = \begin{cases} 0 & \text{for } p < 1 \\ p \Leftrightarrow 1 & \text{for } p \geq 1 \end{cases}$$

and the inverse supply curve can be plotted as in Figure 12

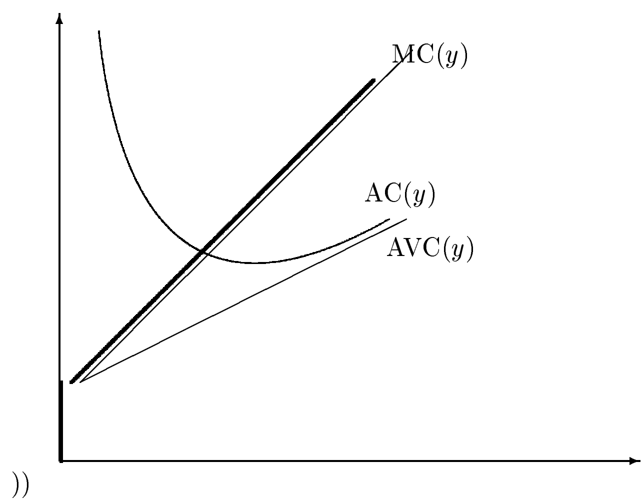

Figure 12: Firm Supply Given Cost Function for $C(y) = \frac{1}{2}y^2 + y + 1$

16.6 Supply with Constant Returns to Scale

An important special case is when there are constant returns to scale. We saw in two examples (fixed proportions and Cobb-Douglas with $a + b = 1$) that this leads to a cost function of the form

$$C(y) = Ky$$

and this is true in general for constant returns technologies. Now:

- $AC(y) = AVC(y) = \frac{Ky}{y} = K$
- $MC(y) = C'(y) = K$

So the firm supply is

$$y(p) = \begin{cases} 0 & \text{if } p < K \\ \text{anything} & \text{if } p = K \\ \text{not defined} & \text{if } p > K \end{cases}$$

That is, the supply curve is at the vertical axis for prices below K and then a horizontal line.

You may look back at what we said about why maximized profits must be zero in equilibrium with constant returns to scale. The bottom line here is that the equilibrium price can be determined from the cost side only (which includes stuff from the technology and factor prices).

16.7 Remark on a Figure in Varian

You may be confused when you read section 22.2 and look at Figure 22.1 in Varian. The discussion is actually sort of OK, but it may not be clear to you what the issue is and what the fuzz is about. The Figure and the section is meant to explain **why the assumption of a price taking firm makes some sense.** To do this Varian loosely describes a "game" where firms actually **sets prices.** We will look at this particular game towards the end of the course, but the point here is that if firms set prices and customers are free to choose whatever firm they want (and know what prices are posted everywhere), they'll buy at the lowest price. If you don't get it now, ignore it and go back to the section after we've discussed "Bertrand Competition".