

## 14 Monopoly

*Varian: Read the chapter on Monopoly. You may also want to go back and review elasticities.*

Up to this point we have studied firms that take all prices as given. While this may be a reasonable approximation in highly competitive markets (which we think of as markets where a large number of firms produce some homogenous good), price taking doesn't make much sense in markets where there is some obvious "market power", such as Microsoft, Terasen Gas, Coca Cola, and pharmaceutical companies producing patented drugs.

To analyze monopoly pricing we will study a (partial equilibrium) model where a single firm is the only actor in the industry apart from consumers. The consumers will play a passive role; their behavior is summarized by a demand function (which we think of as generated by optimizing consumers) for the good. We will proceed as follows:

1. Study the standard textbook monopoly pricing model.
2. Make the observation that it is no longer obvious that the pricing scheme should be a single price per unit sold. Some examples of various forms of price discrimination will be given
3. Study a "durable good monopoly", where the point is that market power is eroded because the monopolist starts to "compete with itself".

### 14.1 A Profit Maximizing Monopolist

Like a competitive firm, we assume that the monopolist strives to generate as large profits as possible. That is, it seeks to maximize

$$py - C(y).$$

However, unlike a competitive firm, a rational monopolist understands that it can only sell what the consumers are willing to buy and, since it operates alone in the market, the firm can set both price and the level of production. We let  $D(p)$  denote the (direct) demand

for the product and note that the monopolist can sell at most  $D(p)$  units at price  $p$ . The optimization problem for the monopolist is thus

$$\begin{aligned} & \max_{p,y} py - C(y) \\ \text{subj to } & y \leq D(p). \end{aligned}$$

Clearly,  $y = D(p)$  in a solution since otherwise the sales could be increased without lowering the price, so we may rewrite the problem as

$$\max_p pD(p) - C(D(p)),$$

where we have price as the choice variable. Alternatively we may invert the demand (view it as in the pictures where we have  $y$  as the independent variable and  $p$  as the dependent variable). Then if  $p(y)$  is the *inverse demand* we can write the problem as

$$\max_y p(y)y - C(y),$$

### 14.1.1 Inverse and Direct Demand

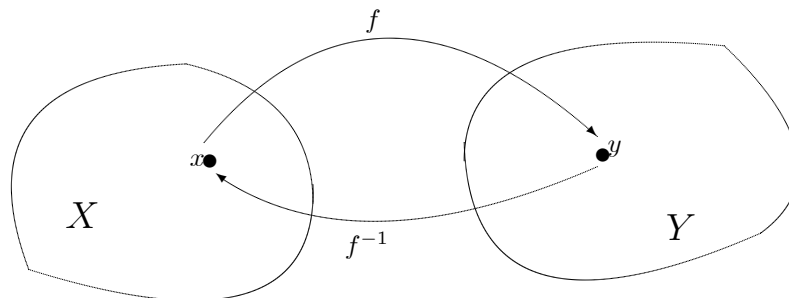


Figure 1: The Inverse of a Function

Mathematically speaking, if  $y = D(p)$  is the direct demand function, the *inverse of D* is

a function  $p = D^{-1}(y)$  such that

$$D^{-1}\left(\underbrace{D(p)}_{\substack{\text{quantity} \\ \text{demanded} \\ \text{at price } p}}\right) = p.$$

The concept is illustrated for a general function  $f$  in Figure 1. The point is that if the function  $f$  takes  $x$  to a value  $y$ , then the inverse takes the value  $y$  back to the value  $x$  we started with. This must be so for each  $x$  and each  $f(x)$ , so the inverse reverts the operation of the original function  $f$ . As an example, say that

$$D(p) = a - bp$$

Then, to find the inverse we only need to solve out  $p$  as a function of the quantity. That means that

$$\begin{aligned} y &= a - bp(y) \Leftrightarrow \\ p(y) &= \frac{a}{b} - \frac{1}{b}y = A - By \end{aligned}$$

where  $A = \frac{a}{b}$  and  $B = \frac{1}{b}$

## 14.2 The Optimality Condition

Both versions of the problems are equivalent, so in principle it doesn't matter which one to solve. However, it turns out that it is somewhat more convenient to work with

$$\max_y p(y)y - C(y),$$

The first order condition is

$$\begin{aligned} p'(y)y + p(y) - c'(y) &= 0 \Leftrightarrow \\ p'(y)y + p(y) &= c'(y) \end{aligned}$$

To interpret the condition note that  $p(y)y$  is the revenue the monopolist gets if selling  $y$  units. Hence

- $\frac{d}{dy}p(y)y = p'(y)y + p(y)$  is the *marginal revenue*. In words, the additional revenue the monopolist gets for a small extra unit of output
- $C'(y)$  is just the *marginal cost*.
- Thus, the optimality condition is the rather natural condition that the additional revenue from the last small unit outweighs the cost.

Observe that the condition  $p = C'(y)$  has the *exact same interpretation*. The difference is that a competitive firm treats price as given, while the monopolist picks price/quantity combinations on a downward sloping demand curve. Hence the trade-off is different for the monopolist. If the monopolist sells few units it can charge a high price, but the more units it sells the lower price it must charge. When it lowers the price to get one additional unit sold it must lower the price for all units, so the loss in revenue associated with selling one more unit will be higher the more units the monopolist sells. This logic is most easily understood in an example with constant marginal cost and a linear demand.

### 14.3 Example: Linear Demand

Let

$$p(y) = A - By$$

$$c(y) = cy$$

so the problem is

$$\max_y \underbrace{(A - By)}_{p(y)}y - cy$$

The first order condition is

$$\begin{aligned} -By + A - By - c &= 0 \\ &\Leftrightarrow \\ y^* &= \frac{A - c}{2B} \end{aligned}$$

Note that  $(A - By)y$  is the revenue, so

$$A - 2By$$

is the marginal revenue. The problem and its solution can thus be depicted in a graph as in

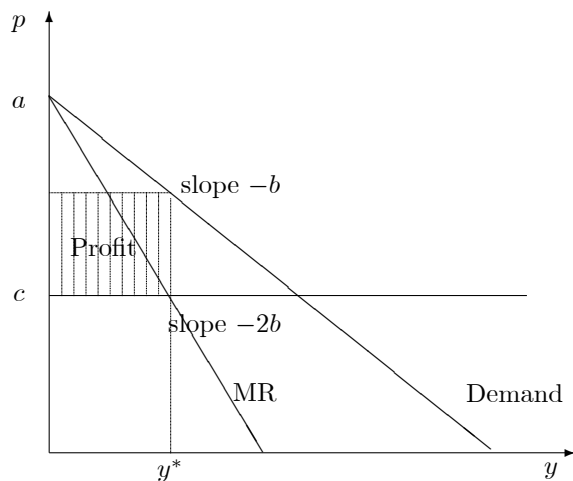


Figure 2: A Profit Maximizing Monopolist

Figure 2, where the demand curve as well as the marginal revenue is drawn. To some of you it may be geometrically obvious that the solution is to set the quantity halfway in between 0 and the quantity where the demand curve intersects the horizontal line at height  $c$ . Solving  $c = A - By$  we get  $y = \frac{A-c}{B}$ , so  $y^*$  is indeed this middle point. To understand geometrically that this is the solution to the profit maximization problem, observe that the monopolist aims to make the rectangle representing profits as large as it can.

- If the quantity sold is small, the profit is a high “thin” rectangle and there are few units to lose revenues from if the quantity is increased/price decreased.
- If the quantity is close to  $A - By$ , then profits is a low “fat” rectangle and there are many units to gain revenues from if quantity is decreased/price increased

It may be visually clear that the profit is the higher with  $y^*$  at the midpoint between 0 and  $A - By$  and one can also show it by pure algebra (no derivatives involved) or elementary (=pretty hard) geometric reasoning. Intuitively:

- Additional revenue from extra unit due to increased sales is approximately

$$p(y + 1) = A - B(y + 1) \approx A - By$$

- Loss in revenue in terms of lower price on **all** units is

$$(p(y + 1) - p(y)) y = (A - B(y + 1) - A + By) y = -By$$

so the change in revenue is approximately  $a - 2By$ .

## 14.4 Elasticity

The basic trade-off for a monopolist is that:

- If the price is high, the profit per unit is high but it sells only a few units.
- If the price is low, the profit per unit is low, but it sells many units.

The outcome of this trade-off is as we saw a “compromise” between quantity sold and profit per unit. Clearly, it is important for this trade-off how responsive demand is to changes in the price.

- The most obvious measure of responsiveness to price is simple  $D'(p)$ , the derivative of the demand function.
- However, for several reasons (that I will not detail) economists usually measure responsiveness of demand in terms of *elasticities*.

The elasticity of a demand function  $D(p)$  is given by

$$e(p) = \frac{D'(p)p}{D(p)} = \frac{dy}{dp} \frac{p}{y} = \frac{\frac{dy}{y}}{\frac{dp}{p}}$$

The reason why this is convenient is that the **elasticity is a good approximation of the percentage change in quantity demanded over the percentage change in price.**

This can be seen by observing that for a small change in the price

$$D(p') \approx D(p) + D'(p)(p' - p)$$

and putting this into the definition of the demand elasticity we get

$$e(p) \approx \frac{(D(p') - D(p))p}{D(p)(p' - p)} = \frac{\overbrace{D(p') - D(p)}^{\Delta y}}{\underbrace{D(p)}_{\Delta p} \underbrace{(p' - p)}_p} = \frac{\frac{\Delta y}{y}}{\frac{\Delta p}{p}}$$

The main usefulness with the elasticity compared to the slope of the demand function is that it is a unit-free measurement that makes it easy for economists to compare empirical findings for different markets with each other and sometimes even get a sense for whether firms have monopoly power or not.

## 14.5 The elasticity of a linear demand function.

Let

$$\begin{aligned} D(p) &= a - bp \\ \Rightarrow D'(p) &= -b \end{aligned}$$

the elasticity is thus

$$e(p) = \frac{D'(p)p}{D(p)} = \frac{-bp}{a - bp}$$

Now, since elasticities are negative and it is easier to think about positive numbers we typically convert elasticities to absolute values.

1. If  $|e(p)| = 1 \Rightarrow \frac{bp}{a-bp} = 1 \Leftrightarrow bp = a - bp \Leftrightarrow p = \frac{a}{2b}$
2.  $|e(p)| > 1 \Rightarrow \frac{bp}{a-bp} > 1 \Leftrightarrow bp > a - bp \Leftrightarrow p > \frac{a}{2b}$
3.  $|e(p)| < 1 \Rightarrow \frac{bp}{a-bp} < 1 \Leftrightarrow bp < a - bp \Leftrightarrow p < \frac{a}{2b}$
4.  $\frac{bp}{a-bp} \rightarrow 0$  as  $p \rightarrow 0$
5.  $\frac{bp}{a-bp} \rightarrow \infty$  as  $p \rightarrow \frac{a}{b}$  since  $bp \rightarrow a$  and  $a - bp \rightarrow 0$  as  $p \rightarrow \frac{a}{b}$ .

DRAW!

## 14.6 Elasticities and The Monopolist Problem

For this it is convenient to use the version of the monopoly problem with the direct demand, i.e., we solve

$$\max_p pD(p) - C(D(p))$$

The first order condition is

$$D(p) + pD'(p) - C'(D(p))D'(p) = 0.$$

The interpretation of this the same as in the previous version of the first order condition, but it is a little harder to see:

- $D(p) + pD'(p)$  is the change in revenue from a slight change in the *price*
- $C'(D(p))D'(p)$  is the change in cost from a slight change in the price

Hence the condition has the same interpretation in terms of marginal revenue being equalized to marginal cost (it is the same problem, so it should). The first order conditions can be reorganized as

$$\begin{aligned} D(p) + pD'(p) &= C'(D(p))D'(p) \Leftrightarrow \text{/divide by } D'(p)/ \\ \frac{D(p)}{D'(p)} + p &= C'(D(p)) \Leftrightarrow \\ p \left( \frac{D(p)}{D'(p)p} + 1 \right) &= C'(D(p)) \Leftrightarrow \left/ \frac{D(p)}{D'(p)p} = \frac{1}{e(p)} \right/ \\ p \left( \frac{1}{e(p)} + 1 \right) &= C'(D(p)) \end{aligned}$$

Now this tells us that

$$\frac{1}{e(p)} + 1 > 0$$

since otherwise the marginal revenue is negative. Translating the negative elasticity to an absolute value this means that

$$-\frac{1}{|e(p)|} + 1 > 0$$

or,  $|e(p)| > 1$ . We can then rearrange the first order condition once more as

$$p = \left( \frac{|e(p)|}{|e(p)| - 1} \right) C'(D(p)),$$

which is often referred to as a “markup pricing” formula. The point with the expression is that we see that price must exceed the cost. However, since the monopolist picks the elasticity when picking a point on the demand curve we should keep in mind that the elasticity in general varies (recall the linear example).

## 14.7 The Inefficiency of Monopoly Pricing

Consumer(s) value the last unit produced at the monopoly price  $p^m$ . However, the monopolist only gives up the marginal cost  $C'(y^m)$  if producing an extra unit, which the consumer would value at  $p^m$ . Due to this “wedge” between the marginal cost and the willingness to pay for the last unit all agents could be made happier if, in addition to the units traded at monopoly price, the monopolist and the consumer(s) could trade some additional units at a lower price. You may note that the problem is the difference between the marginal cost and the price the consumers pay, so a tax on a good sold by a competitive market (which in the constant returns case would imply that the equilibrium price must be  $c + t$ ) creates an inefficiency for exactly the same reason.

### 14.7.1 Justifying “Deadweight Loss Tringles”

Often times the distortion is quantified in a graph using the “deadweight loss”, which is illustrated for the case of a constant marginal cost in Figure 3. The intuitive idea is that the demand curve gives the willingness to pay for the marginal unit, the that the distance between the demand and the marginal cost curve gives the “dollars lost” for that unit not being produced. To really make sense of this we’d have to do more careful “welfare analysis”. However if utility is quasi-linear then it is rather easy to demonstrate that this is a valid way to proceed. Suppose  $U(x, y) = x + v(y)$  and that the price of good  $x$  is normalized to one.

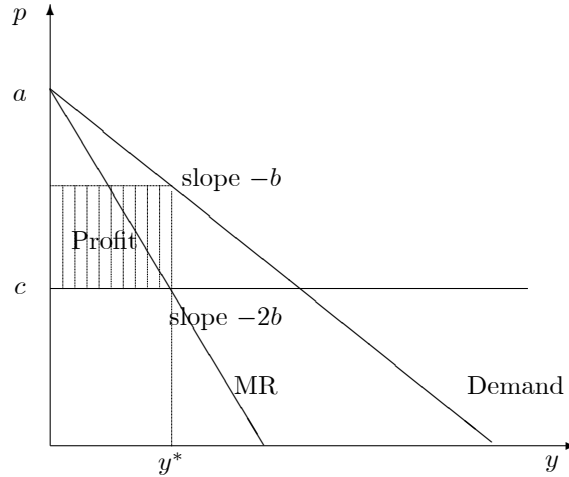


Figure 3: The Deadweight Loss of a Monopoly

The problem for a utility maximizing consumer is then

$$\begin{aligned} \max x + v(y) \\ \text{s.t } x + py \leq m \end{aligned}$$

Plugging in the constraint and optimizing we get the first order condition

$$v'(y) = p$$

This condition *defines the inverse demand function for y* (as long as  $m$  is large enough so as to guarantee that the solution is interior), that is

$$p(y) = v'(y)$$

Observe that

$$v(y) - v(0) = \int_0^y v'(y)dy = \int_0^y p(y)dy = \text{Area under inverse demand,}$$

so since

$x + v(y)$  is the utility of consuming  $(x, y)$

$m + v(0)$  is the utility of consuming  $x = m$  and  $y = 0$

we have that the consumer is happier if consuming  $(x, y)$  than  $(m, 0)$  if and only if

$$x + v(y) \geq m + v(0) \Leftrightarrow$$

$$\text{Area under inverse demand} = v(y) - v(0) \geq m - x$$

**Thus, the area under the inverse demand tells you how many units of the other good the consumer would be willing to give up for  $y$  units of good  $y$  if facing an “all-or-nothing choice”.**

## 14.8 Price Discrimination

The basic reason why a monopoly (charging a uniform price) is inefficient is that even though it would be beneficial for society, the monopolist doesn't want to increase output since this would reduce the profits earned for all the units sold at the monopoly price.

However, if:

1. The monopolist can make the price contingent on quantity, and
2. “Individualize” price/quantity offers (in case of more than one consumer),

then this trade-off disappears. This is illustrated in Figure 4 where the horizontal line indicates the (constant) marginal cost of production. If the market would be competitive, then we know that the equilibrium price would equal the marginal cost and that there would be no distortion. The associated demands are given marginal cost pricing are given by  $y_1^*, y_2^*, \dots, y_n^*$  in the figure.

The standard monopoly analysis would be to add up all the demand curves and then set a price so that the profits are maximized (given that the price is the same for all customers and the same no matter how much each customer buys).

However, let  $A_i$  be the area under the demand curve in between 0 and  $y_i^*$  for agent  $i$  and suppose gives the following “offer” to each agent:

- Consumer 1 can pay  $A_1$  and get  $y_1^*$  or pay nothing and get nothing (and this is a non-negotiable offer)

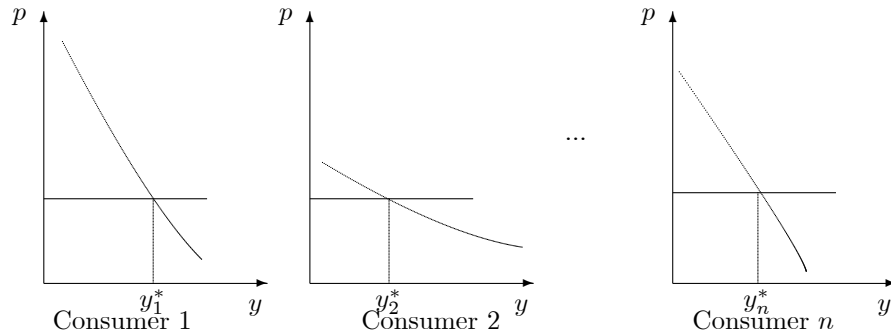


Figure 4: Perfect Price Discrimination

- Consumer 2 can pay  $A_2$  and get  $y_2^*$  or pay nothing and get nothing (and this is a non-negotiable offer)

and so on for all consumers in the economy. All consumers would be willing to buy (strictly so if the monopolist sweetens the deal with a penny off)! Note,

- **This is fully efficient!** It is impossible to increase consumer surplus without reducing the profit for the monopolist (which presumably has some owner or owners who would have to decrease the consumption if the profits would be reduced).
- We say that the monopolist **fully extracts the surplus from the consumers** by using the individualized “take it or leave it-offers”.
- The form of price discrimination considered in this example is referred to as **perfect price discrimination**.

## 14.9 Is Perfect Price Discrimination Reasonable?

Most economists would probably say that perfect price-discrimination is unrealistic in most circumstances. There are several reasons for this. For example:

1. The monopolist is assumed to know the demand function (that is know preferences) for each and every individual in the economy. While firms probably have a rather good idea about the aggregate demand, this seems rather unreasonable.

2. Even if the monopolist could see who has what demand, the monopolist must somehow prevent consumers from trading with each other.

However, the important insight from the example with perfect price discrimination is that when we think about a monopolist **there is no a priori reason to restrict attention to trading mechanisms where the producer sets a price and customers decide how much to buy**. Explicitly introducing the considerations above (the reasons why perfect price discrimination is not possible) into the formal model of a profit maximizing monopolist we can allow more clever pricing schemes than a fixed per unit price equal for all customers. Depending on details of the model we can then make sense of several pricing practices that we do observe in the real world. For example:

1. If the monopolist can not see which consumer is high demand and which is a low demand consumer, the monopolist can still capture some extra profits compared to a uniform price by **non-linear pricing**, which means that the pricing scheme involves **quantity discounts**.
2. We can also make sense of student and senior citizen discounts by thinking of being a student/senior citizen as an imperfect measure of willingness to pay.
3. Sometimes the quantity dimension may be replaced by a **quality** dimension (think of demand for airline tickets) and here the same type of logic that rationalizes non-linear pricing can be used to rationalize why airlines want to make economy seats really crummy in order to make business travellers to “self-select” into expensive but nice seats with caviar and champagne served.

## 14.10 Justifications of the Standard Monopoly Model

**Optional:** Which problem that is the more appropriate description of monopoly pricing behavior depends on what may appear as “tiny details”. As just mentioned various forms of price discrimination can be rationalized as ways to increase the profits for the monopolist: there are monopoly pricing models that can

explain student and senior citizen discounts, bundling (of for example browsers and operating systems), frequent flyer programs, how a monopolist can sometimes spend extra money to make goods less useful, etc.<sup>1</sup>

Because of this plethora of models, it is of some interest to ask under what circumstances the standard model described in the beginning of the section makes sense. There are essentially two ways to justify that model:

1. Arbitrage possibilities: if consumers can trade with each other, this makes it very hard to price discriminate. As an example, suppose that the per unit price would be decreasing in the number of units sold. Then, a single consumer has an incentive to buy a large quantity of the good (at a low price per unit) and resell to consumers that are facing higher prices per unit if buying directly from the monopolist. Note: whether this is possible or not depends on the good in question. Airline tickets, cable TV services, electricity are examples of non-transferable goods, and for these goods it is either impossible or very hard/costly for the consumers to do arbitrage.
2. Unit demands and uncertainty about preferences. One can then show that, if the monopolist cannot observe the preferences of the consumer, then the optimal pricing mechanism is applying the standard monopoly pricing formula, where the demand function is given by

$$D(p) = 1 - F(p) = 1 - \Pr[v \leq p],$$

that is,  $F$  is the cumulative distribution over possible valuations for the object.

## 14.11 A Durable Good Monopolist

Many goods are durable: cars, refrigerators, light bulbs, art, computers etc. all last for some time. This “durability” introduces some quite interesting issues concerning the market power (or lack thereof) for a monopolist. Intuitively, the main ideas are as follows:

- Suppose that some customers purchase the product at date  $t$ . The crucial observation is that, the customers that bought the product tend to be those with high valuations.

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<sup>1</sup>An example of the last practice was when the 486 chip was introduced. A cheaper, lower quality, variant, 486S, was also available. This was produced by taking a regular 486 chip and disabling certain functions.

- But then, at date  $t + 1$  the monopolist has an incentive to lower its price to capture some profit also from the customers with lower valuations.
- But then,...., the problem is that all customers will look forward to the future decrease in price, which erodes the monopoly power at date  $t$ .

This is referred to as a “time inconsistency problem”, which is similar to Birgitta’s dilemma in the battle of the sexes game discussed in game theory. If Birgitta could credibly commit to always go to the Opera, then she would get a higher payoff than in the backwards induction solution. Similarly, the durable good monopolist would do better if it could commit never to decrease its price.

The main result in the literature is that, if the monopolist can change its price often enough, then almost all customers will pay a price close to the marginal cost. That is, the competition against future incarnations of itself makes the monopoly power go away almost completely. This type of analysis is too advanced for the scope of this course. Instead, we will show something weaker: that the monopoly profit is always less than commitment to the static monopoly price in both periods. This we can do in a model with only two periods by applying the backwards induction procedure previously discussed.

### 14.11.1 Model

1. There are two periods, indexed by  $t = 1, 2$
2. Both consumers and the monopolist discounts future utility and profits respectively at a common discount factor  $\delta < 1$
3. Monopolist sells an indivisible good
4. The (direct) demand is given by  $D(p) = 1 - p$ . For interpretation, it is a good idea to think of  $D(p)$  as the probability that the valuation for the good is less than  $p$ .
5. For simplicity, we let the constant marginal cost be  $c = 0$ .

### 14.11.2 Static Benchmark (Commitment)

First consider the case where the monopolist commits to never changing the price. Since consumers discount their utilities, nobody will then buy anything at time  $t = 2$ . The best (common) price the monopolist can charge is then the solution to

$$\max_p p(1-p)$$

This problem has solution  $p^* = \frac{1}{2}$ , and the associated quantity sold is  $q^* = D(p^*) = 1 - p^* = \frac{1}{2}$ , so the monopolist makes a profit  $\pi^* = p^*q^* = \frac{1}{4}$  if it somehow can promise never to change the price in the future (we have not shown it, but this is actually the best the monopolist can do if it is able to commit to prices in both periods before any sales are made).

### 14.11.3 Selling in Both Periods (Non-Commitment)

We will solve the problem by applying the backwards induction procedure that we discussed in the section on game theory. To implement this, we must figure out what happens in the second period after any history of play. A history can be described as *either* a quantity sold in the first period *or* a price. One can proceed either way, but I find it somewhat easier to work with the quantity;

Consider the second period, and assume that in the first period the monopolist sold  $q_1$  units. That means that customers with valuations above  $q_1$  have left the market already, so the second period inverse demand (as a function of the arbitrary  $q_1$ ) is

$$D_2(p, q_1) = \underbrace{1 - q_1}_{\text{New intercept}} - q_2$$

Second period problem is thus

$$\max_{q_2} q_2(1 - q_1 - q_2),$$

which has a solution (which depends on  $q_1$ ) given by

$$\begin{aligned} q_2(q_1) &= \frac{(1 - q_1)}{2} \\ p_2(q_1) &= \frac{(1 - q_1)}{2}. \end{aligned}$$

The second period profit is thus

$$\pi_2(q_1) = \frac{(1 - q_1)^2}{4}$$

Now, in the first period a consumer with willingness to pay  $v$  will buy the product if

$$\begin{aligned} v - p_1 &\geq 0 \text{ and} \\ v - p_1 &\geq \delta(v - p_2) \end{aligned}$$

The price in the second period will be below the price in the first period, so we can ignore the first inequality. Now, if  $q_1$  is the quantity sold, it will be bought by the  $q_1$  agents with the highest valuation, so the agents with valuation

$$1 - q_1 \leq v \leq 1$$

are the ones that buy in the first period, and

$$(1 - q_1) - p_1 = \delta((1 - q_1) - p_2),$$

since the agent with valuation  $v = 1 - q_1$  must be indifferent between buying in period 1 and buying in period 2. But we know that when the monopolist re-optimizes in the second period, then

$$p_2(q_1) = \frac{(1 - q_1)}{2},$$

so, the indifference condition becomes

$$\begin{aligned} (1 - q_1) - p_1 &= \delta((1 - q_1) - p_2(q_1)) = \delta\left((1 - q_1) - \frac{1 - q_1}{2}\right) \\ &= \frac{\delta}{2}(1 - q_1) \Rightarrow \\ p_1 &= \left(1 - \frac{\delta}{2}\right)(1 - q_1). \end{aligned}$$

The first period profit maximization problem for the monopolist is thus

$$\max_{q_1} q_1(1 - q_1) \left(1 - \frac{\delta}{2}\right) + \delta \frac{(1 - q_1)^2}{4},$$

which gives first order condition

$$\begin{aligned}
(1 - 2q_1) \left(1 - \frac{\delta}{2}\right) - \frac{\delta}{2} (1 - q_1) &= 0 \\
&\Leftrightarrow \\
q_1 \left(2 - \delta - \frac{\delta}{2}\right) &= 1 - \frac{\delta}{2} - \frac{\delta}{2} = 1 - \delta \\
q_1^* &= \frac{2(1 - \delta)}{4 - 3\delta}
\end{aligned}$$

Now, when we have the solution in terms of the first period quantity we notice:

1. Previously, we expressed the second period price, quantity sold and profit as

$$\begin{aligned}
q_2(q_1) &= \frac{1 - q_1}{2} \\
p_2(q_1) &= \frac{1 - q_1}{2} \\
\pi_2(q_1) &= \left(\frac{1 - q_1}{2}\right)^2.
\end{aligned}$$

By evaluating these expressions at  $q_1^*$  we get the values that will be chosen after the best first period choice as

$$\begin{aligned}
q_2^* &= q_2(q_1^*) = \frac{1 - q_1^*}{2} = \frac{1 - \frac{2(1-\delta)}{4-3\delta}}{2} = \frac{4-3\delta-2(1-\delta)}{4-3\delta} = \frac{1}{2} \frac{2 - \delta}{4 - 3\delta} \\
p_2^* &= p_2(q_1^*) = \frac{1 - q_1^*}{2} = \frac{1}{2} \frac{2 - \delta}{4 - 3\delta} \\
\pi_2^* &= \pi_2(q_1^*) = \left(\frac{1 - q_1^*}{2}\right)^2 = \frac{1}{4} \left(\frac{2 - \delta}{4 - 3\delta}\right)^2
\end{aligned}$$

2. Next, we observe that the indifference condition for buying in first versus second period was

$$p_1 = \left(1 - \frac{\delta}{2}\right) (1 - q_1)$$

so, the price charged at period 1 is

$$\begin{aligned}
p_1^* &= \left(1 - \frac{\delta}{2}\right) \left(1 - \frac{2(1 - \delta)}{4 - 3\delta}\right) \\
&= \left(\frac{2 - \delta}{2}\right) \left(\frac{4 - 3\delta - 2(1 - \delta)}{4 - 3\delta}\right) = \frac{(2 - \delta)^2}{2(4 - 3\delta)}.
\end{aligned}$$

3. Hence, the discounted value of the monopolists profit is (I write it as a function of  $\delta$  as that is a key parameter)

$$\begin{aligned}
\Pi^*(\delta) &= \pi_1^* + \delta\pi_2^* = p_1^*q_1^* + \delta p_2^*q_2^* \\
&= \underbrace{\frac{(2-\delta)^2}{2(4-3\delta)}}_{p_1^*} \underbrace{\frac{2(1-\delta)}{4-3\delta}}_{q_1^*} + \delta \underbrace{\frac{1}{4} \left( \frac{2-\delta}{4-3\delta} \right)^2}_{\pi_2^*} \\
&= \frac{1}{4} \left( \frac{2-\delta}{4-3\delta} \right)^2 [4(1-\delta) + \delta]
\end{aligned}$$

4. Observe that

$$\begin{aligned}
\Pi^*(0) &= \frac{1}{4} \left( \frac{2}{4} \right)^2 4 = \frac{1}{4} \\
\Pi^*(1) &= \frac{1}{4} \left( \frac{2-1}{4-3} \right)^2 [4(1-1) + 1] \\
&= \frac{1}{4} \left( \frac{1}{1} \right)^2 [1] = \frac{1}{4},
\end{aligned}$$

that is:

- when  $\delta = 0$ , nobody cares about the next period so the problem becomes like the static problem
- when  $\delta = 1$ , the monopolist charges  $\frac{(2-1)^2}{2(4-3)} = \frac{1}{2}$  (or higher) in the first period, so nobody actually buys anything in the first period. The second period is then just like the only period in the static model.
- For intermediate values of  $\delta$  we have that  $0 < \delta^2 < \delta$ , implying that

$$\left( \frac{2-\delta}{4-3\delta} \right)^2 [4(1-\delta) + \delta] = \frac{(2-\delta)^2}{4-3\delta} = \frac{4-4\delta+\delta^2}{4-3\delta} < \frac{4-4\delta+\delta}{4-3\delta} = 1$$

Hence

$$\Pi^*(\delta) = \frac{1}{4} \left( \frac{2-\delta}{4-3\delta} \right)^2 [4(1-\delta) + \delta] < \frac{1}{4}.$$

That is, the option to sell to those customers who don't buy at time 1 unambiguously hurts the monopolist.

#### 14.11.4 Discussion

The conclusion that not selling in the second period is better for the monopolist is often considered counter intuitive: flexibility, which is often times considered valuable, is bad. The point is that consumers are not going to be fooled. If the monopolist will do what is optimal in the second period, potential buyers will understand this, so the monopolist is better off if it can somehow commit not to change the price in the second period even though the second period incentive obviously is to change the price to sell more units.