

1 Walrasian Equilibria and Market Efficiency

Reading in Textbook: Chapter 3 in Stiglitz.

1.1 Motivation

When thinking about the role of government we have to consider a number of rather fundamental questions, such as:

- What should a government do?
- Why are some activities undertaken in the private sector and others in the public sector,
- What are the pros and cons of having the government intervene in a particular market?
- Assuming that the government should intervene in a particular market, how can we determine what a “good” intervention would be?

As is pointed out in the very beginning of the textbook, one way to start thinking about these very basic questions is to look at some facts from the real world. The first two chapters in the book are intended to give you a bit of background, and I think you should read these chapters. However, for now I think that it is sufficient that we agree on the following:

1. Government spending accounts for a big chunk of total economic activity. In sum, federal and local government spending about 30% of GDP in the US, and more than this in almost all other developed countries.
2. Certain goods (military defense, water and sewage, national parks, highway construction etc.) are almost exclusively provided by governments.
3. Other goods (education, mail delivery, policing) are provided by both the public and the private sector.
4. The government also provides a legal system.

5. Even for sectors like steel, autos, tomato growing, sugar etc. the government intervenes by providing explicit or implicit subsidies, tariff protection, and other regulatory measures that influence the market outcomes.
6. The government also redistributes income.

However, while the public sector is important, we rely mainly on the private sector for the production and distribution of most goods and services. Indeed, most economists would argue that a decentralized (=private) system has many virtues, and that one should be careful with government intervention in markets that work. This view is of course partially grounded in empirical observations (that is, comparing the Soviet Union and its satellites with the US and its European allies).

However, another reason for the belief that “one should not mess with market” is that there are appealing theoretical arguments for this view: the logic of the “invisible hand” pioneered by Adam Smith is the cornerstone of the most important model in economics- the *competitive* (or *Walrasian* or *neoclassical*) equilibrium model. My view is that, before even thinking about why government intervention can in some cases be justified, it is crucial to understand this model, which tells us that price taking behavior has certain advantages that seem hard for the public sector to replicate.

The *competitive/Walrasian/neoclassical* model is what for a quite long time has been the “benchmark model” in economics. The central assumptions are:

1. *Price taking behavior*; individual agents (consumer and firms) believe that their own actions have no influence on prices. Hence, a consumer takes the equilibrium prices as given and picks the best consumption plan given this equilibrium price. Similarly, firms take input and output prices as given and choose the production plan that maximizes profits.
2. *Market Clearing*; in equilibrium, prices are set so that supply=demand on all markets.

Notice that there is a bit of magic involved. There is no explicit mechanism in the model for how prices are formed. Intuitively, one would think that if the demand for a good should

exceed the supply, then the price ought to be adjusted upwards, whereas if there is excess supply, the demand should fall. Hence, it appears that only prices where the supply equals demand are stable, which is the loose justification of the market clearing assumption. Our intuition says that this is fine for markets where participants are “small”, but it is still an open question how accurate this intuition is.

A final remark before looking at the formal analysis is that there is a distinction between **partial equilibrium** and **general equilibrium** analysis. The supply-demand graphs from Econ 101 are prime examples of partial equilibrium analysis. These graphs can be instructive and useful, but here we seek to illustrate why competitive equilibria are economically efficient, and for this purpose it is more instructive to consider a general equilibrium setup, which means that all prices and quantities are determined simultaneously as a closed system.

1.2 The 2×2 Model of Pure Exchange

To begin with, we ignore production and consider the simplest non-trivial general equilibrium model possible. Suppose there are;

- 2 goods labeled 1 and 2. Quantities denoted by x_1, x_2
- 2 agents, A, B with preferences given by utility functions $u^A(x_1, x_2)$ and $u^B(x_1, x_2)$
- Agents live a one-period life. They wake up in the morning with *endowments* $e^A = (e_1^A, e_2^A)$ and $e^B = (e_1^B, e_2^B)$, which are quantities the two goods that the agents have before any trade
- I will denote consumption bundles $x^A = (x_1^A, x_2^A)$ and $x^B = (x_1^B, x_2^B)$ for Mr. A and Mrs. B .

1.2.1 The Consumer Choice Problem

The first thing to note here is that we have **not** specified any particular “dollar incomes” m^A, m^B . Instead, we will let trades be barter trades where one agent gives the other goods

in return for other goods. Indeed, there is no room for intrinsically useless pieces of paper in this or any other neoclassical equilibrium model. That is, as long as agents don't derive any pleasure from money (say as wallpaper) nobody would accept money unless money was explicitly backed by the right to purchase goods with it. Hence, the "income" of the consumer will be taken as the *value of the endowment*. The relevant maximization problem for consumer A is thus

$$\begin{aligned} & \max_{x_1, x_2} u^A(x_1, x_2) & (1) \\ \text{s.t. } & p_1 x_1 + p_2 x_2 \leq p_1 e_1^A + p_2 e_2^A \end{aligned}$$

and similarly for B (Just replace A with B). For comparison, the problem (1) is really just the generic utility maximization problem over apples and bananas from Econ 101, except that the "income" is endogenously determined as the value of the endowment $(p_1 e_1^A + p_2 e_2^A)$ instead of being an exogenous parameter.

1.2.2 Competitive Equilibria

The concept of a competitive equilibrium is one of the most important in economics:

Definition 1 *A competitive (Walrasian) equilibrium in the 2×2 pure exchange model is a price vector $p^* = (p_1^*, p_2^*)$ and consumption bundles $x^{A*} = (x_1^{A*}, x_2^{A*})$, $x^{B*} = (x_1^{B*}, x_2^{B*})$ satisfying:*

1. *The bundle consumed by each agent is the best affordable bundle given price vector p^* . That is x^{A*} solves the consumer choice problem (1) and x^{B*} solves the symmetric consumer choice problem for agent B given prices $(p_1, p_2) = (p_1^*, p_2^*)$.*
2. *Markets clear (feasibility).*

$$\begin{aligned} x_1^{A*} + x_1^{B*} &= e_1^A + e_1^B \\ x_2^{A*} + x_2^{B*} &= e_2^A + e_2^B \end{aligned}$$

1.3 Graphical Treatment

In later discussions it will be useful to distinguish between the parts in the definition of equilibrium that has to do with *feasibility* from the part that has to do with *optimizing behavior*.

Definition 2 *An allocation (a list of consumption bundles for each agent) is feasible if*

$$x_1^A + x_1^B \leq e_1^A + e_1^B$$

$$x_2^A + x_2^B \leq e_2^A + e_2^B$$

It is rather clear that *in equilibrium* (that is if we add optimal behavior as well) all resources must be used meaning that the more interesting feasible allocations are those where the resource constraints hold with equality.

Graphically any feasible allocation that uses all resources ($x_1^A + x_1^B = e_1^A + e_1^B$ and $x_2^A + x_2^B = e_2^A + e_2^B$) can be conveniently described as a point in a “box” as in figure 1. In the figure, the length of each side is the total resources of each good which immediately means that if we pick any point different from e in the box total consumption of each good will be equal to the total resources.

Now, optimal behavior is determined exactly as before. Given a price vector (p_1, p_2) we have that:

- The budget set for A consists of all (x_1, x_2) such that

$$p_1x_1 + p_2x_2 \leq p_1e_1^A + p_2e_2^A,$$

which are just all points *below* a line with slope $-\frac{p_1}{p_2}$ that goes through the endowment point e (note that when we look at it from the point of view of A the endowment e is located at (e_1^A, e_2^A) from the relevant origin in the southwest corner.

- The budget set for B consists of all (x_1, x_2) such that

$$p_1x_1 + p_2x_2 \leq p_1e_1^B + p_2e_2^B,$$

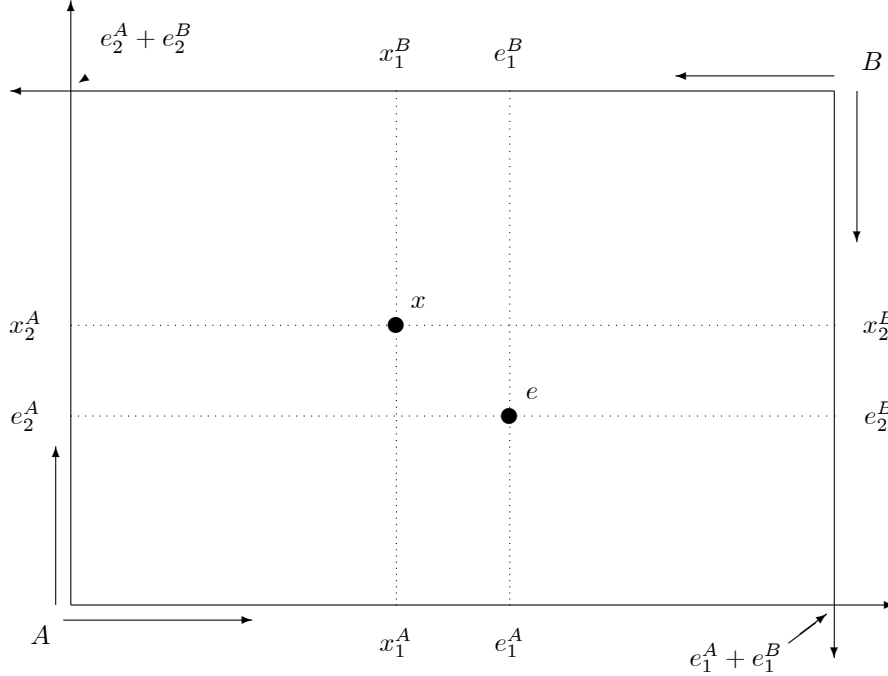


Figure 1: A Feasible Allocation in the Edgeworth box)

which are just all points *above* a line with slope $-\frac{p_1}{p_2}$ that goes through the endowment point e . That is, from the point of view of B the origin is in the northeast corner.

This is illustrated in figure 2. Observe that *there is absolutely no reason that the budget set must be in the set of feasible allocation*. In the picture this is indicated by the budget lines continuing across the edges in the box (but only for positive consumptions). The optimality requirement is then as usual graphically depicted as a tangency between the highest achievable indifference curve and the budget line.

Now, we can simply put the two pictures together in the box for some arbitrary prices (p_1, p_2) as in Figure 3. The way the picture is drawn we have that the net demand for good one of Mrs. B (i.e., what B wants to buy in addition to her endowment) exceeds the net supply of Mr. A for good 1. That is: B wants to buy more than A has to sell. Hence there is excess demand for good 1 : at the given prices the consumers want to consume more than is available in the market of good 1, so the market is **not** in equilibrium in Figure 3. The mirror image of this excess demand for good 1 is excess supply for good 2, but this is

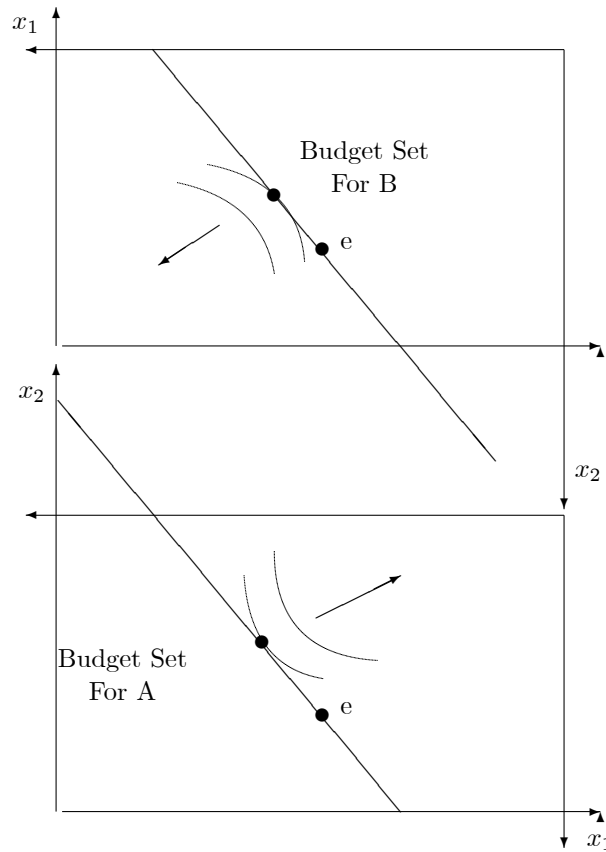


Figure 2: The Utility Maximization Conditions)

“automatic” given that we have excess demand for good 1 as will be discussed later.

So, how will an equilibrium look like in the box?

1. Allocation must be feasible \Rightarrow graphically this means that both agents choose “same point” in the Edgeworth box.
2. Both agents must choose the best bundle given the prices \Rightarrow the equilibrium must be such that both agents have a tangency between price line and indifference curve at equilibrium allocation.

An equilibrium can thus be depicted as in Figure 4 as a budget line that goes through the endowment which is such that both agents have a tangency with the price line at the same point.

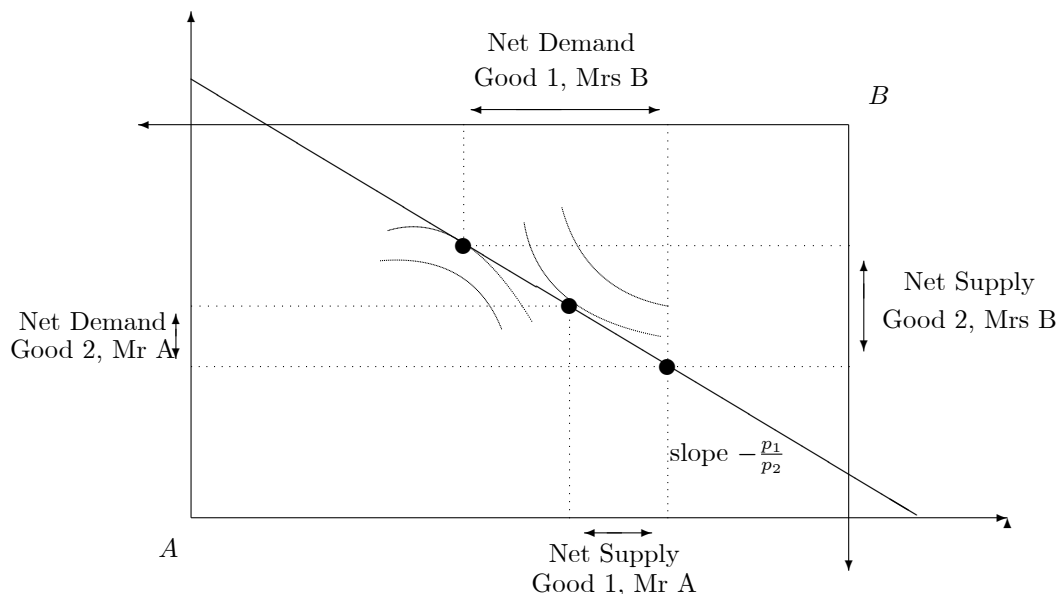


Figure 3: Example of Prices NOT Consistent with Equilibrium)

1.4 Greed is Good: Self Interest Leads to Efficient Allocations

Some examination of this picture reveals a rather remarkable property of competitive (Walrasian) equilibria. Given the equilibrium allocation x^* all bundles that are better for A are those to the northeast of the indifference curve intersecting x^* . Similarly, the bundles that are better for B are those to the southwest of the indifference curve intersecting x^* . This means

- THAT IT IS IMPOSSIBLE TO MAKE ONE PERSON BETTER OFF WITHOUT MAKING THE OTHER AGENT WORSE OFF
- True under much more general circumstances (more consumers, firms, goods, a time dimension, uncertainty...)

This important feature is emphasized in Figure 5 where the only difference from Figure 4 is that I've taken away all indifference curves not going through x^* .

An economist would say that the equilibrium outcome is *Pareto efficient*:

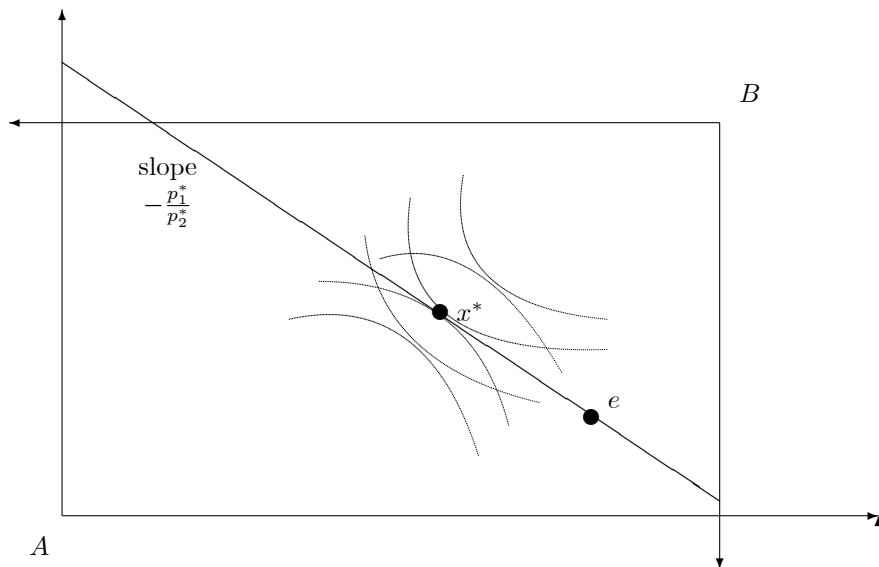


Figure 4: An Equilibrium in the Edgeworth Box)

Definition 3 *An allocation is Pareto efficient if it is feasible and if there is no other feasible allocation that makes both agents better off.*

Pareto efficiency is the concept of efficiency in economics. Indeed, economists usually just refer to it as “efficiency” and it is then commonly understood that

Clearly, allocations that are not Pareto efficient are undesirable. Then, there is a way to make all agents in the economy better off and if everyone is happier then that is clearly a better use of the resources.

Note that there is an infinite number of Pareto optimal allocations even in the simply 2×2 pure exchange model. To see this note that for **any** point such that there is a tangency between the indifference curves of the agents it is impossible to increase the happiness of one agent without making the other less happy. One can thus trace out the set of Pareto optimal allocations in the Edgeworth box as the set of tangencies as in Figure 6. The curve that connects all the Pareto optima is sometimes called the *contract curve*.

Important to note is:

1. Efficiency has nothing to do with distribution of resources.

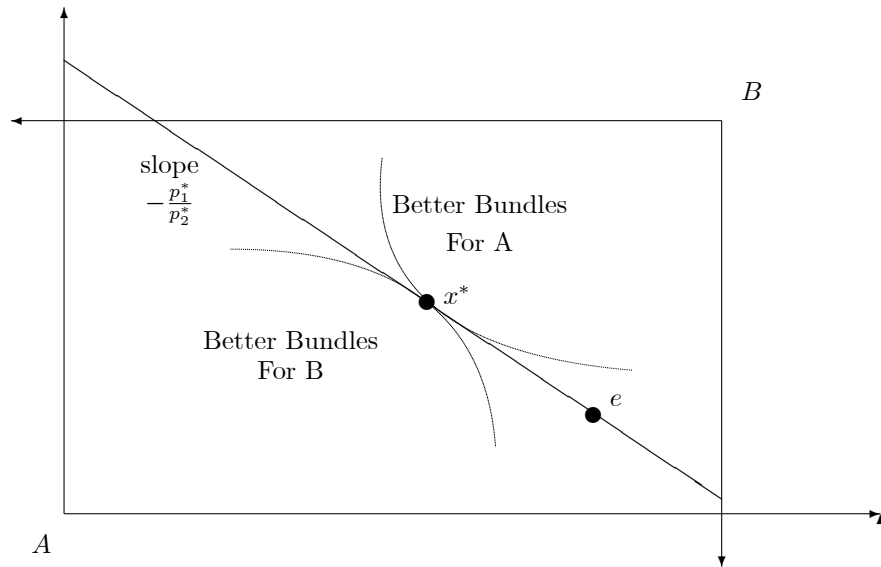


Figure 5: An Equilibrium is Pareto Efficient)

2. Equilibria depend on the initial distribution of resources, the notion of efficiency does not.
3. Despite potential issues about “fairness” the result that competitive equilibria are efficient may be thought of as a “greed is good” type of result. Indeed it is the basic reason for why economists are often very sceptical towards market interventions. Leaving the market alone (under the competitive assumptions which are loosely based

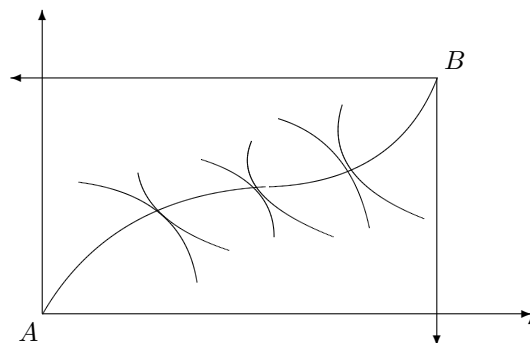


Figure 6: The contract Curve-All Efficient Allocations)

on ideas of many firms and many consumers) we have reasons to believe that the market outcome is at least approximately efficient. Messing with the market we may help some individuals or groups, but, as we'll see with more concrete examples of interventionist policies, efficiency is typically lost.

4. Later in the course we will analyze and discuss reasons for why the market may not produce Pareto efficient outcomes. In spite of the seeming generality of the result that equilibria are efficient (we have only considered the simplest exchange model, but it holds also when we have arbitrary numbers of goods and/or agents and production by firms...) there are lots of reasons why the market could produce inefficient equilibrium outcomes (public goods, externalities, informational issues, monopoly power....).

1.5 Walras Law

The graphs are instructive, but sometimes it is helpful to be able to actually compute an equilibrium. We note that given prices (p_1, p_2) , the aggregate demand is

$$\begin{aligned} x_1^A(p_1, p_2, p_1 e_1^A + p_2 e_2^A) + x_1^B(p_1, p_2, p_1 e_1^B + p_2 e_2^B) & \text{ for good 1} \\ x_2^A(p_1, p_2, p_1 e_1^A + p_2 e_2^A) + x_2^B(p_1, p_2, p_1 e_1^B + p_2 e_2^B) & \text{ for good 2} \end{aligned}$$

Where $x_1^A(\cdot)$, $x_2^A(\cdot)$, $x_1^B(\cdot)$ and $x_2^B(\cdot)$ are the regular demand functions you considered in the first half of the semester. Hence, we can *solve* for an equilibrium by solving

$$\begin{aligned} \underbrace{x_1^A(p_1, p_2, p_1 e_1^A + p_2 e_2^A) + x_1^B(p_1, p_2, p_1 e_1^B + p_2 e_2^B)}_{\text{aggregate demand for good 1 given prices } p_1, p_2} &= \underbrace{e_1^A + e_1^B}_{\text{resources of } x_1} \\ \underbrace{x_2^A(p_1, p_2, p_1 e_1^A + p_2 e_2^A) + x_2^B(p_1, p_2, p_1 e_1^B + p_2 e_2^B)}_{\text{aggregate demand for good 2 given prices } p_1, p_2} &= \underbrace{e_2^A + e_2^B}_{\text{resources of } x_1}, \end{aligned}$$

for (p_1, p_2) . At a first glance, this looks promising. Two equations in two unknowns. But

$$\begin{aligned} \max_{x_1, x_2} u^J(x_1, x_2) & \tag{2} \\ \text{s.t } p_1 x_1 + p_2 x_2 & \leq p_1 e_1^J + p_2 e_2^J \end{aligned}$$

and

$$\begin{aligned} & \max_{x_1, x_2} u^J(x_1, x_2) \\ \text{s.t. } & \frac{p_1}{p_2} x_1 + x_2 \leq \frac{p_1}{p_2} e_1^J + p_2 e_2^J \end{aligned} \tag{3}$$

are equivalent problems. Hence, we may normalize, for example by setting $p_2 = 1$ which gives the “system”

$$\begin{aligned} x_1^A(p_1, 1, p_1 e_1^A + e_2^A) + x_1^B(p_1, 1, p_1 e_1^B + e_2^B) &= e_1^A + e_1^B \\ x_2^A(p_1, 1, p_1 e_1^A + e_2^A) + x_2^B(p_1, 1, p_1 + e_1^B e_2^B) &= e_2^A + e_2^B \end{aligned}$$

That is, we get **two** equilibrium conditions and a **single** unknown. Luckily, it turns out that the two equilibrium conditions are equivalent. This is often referred to as Walras law (although sometimes the term Walras law is used for the fact that the “value of excess demand is zero”, which is the property that is used to prove the claim;

Proposition 1 *Suppose that (p_1, p_2) clears the market for good 1, that is*

$$x_1^A(p_1, p_2, p_1 e_1^A + p_2 e_2^A) + x_1^B(p_1, p_2, p_1 e_1^B + p_2 e_2^B) = e_1^A + e_1^B.$$

Then, the market for good 2 clears as well.

This comes directly from the fact that the budget constraint holds with equality for every agent *for any prices*. For simplicity of notation, let

$$\begin{aligned} m^A(p) &= p_1 e_1^A + p_2 e_2^A \\ m^B(p) &= p_1 e_1^B + p_2 e_2^B \end{aligned}$$

We know (because of optimization) that

$$\begin{aligned} p_1 x_1^A(p_1, p_2, m^A(p)) + p_2 x_2^A(p_1, p_2, m^A(p)) &= m^A(p) = p_1 e_1^A + p_2 e_2^A \\ p_1 x_1^B(p_1, p_2, m^B(p)) + p_2 x_2^B(p_1, p_2, m^B(p)) &= m^B(p) = p_1 e_1^B + p_2 e_2^B \end{aligned}$$

Summing we get (write out sums if you don’t like “ \sum ” signs)

$$p_1 \left(\sum_{J=A,B} [x_1^J(p_1, p_2, m^J(p)) - e_1^J] \right) + p_2 \left(\sum_{J=A,B} [x_2^J(p_1, p_2, m^J(p)) - e_2^J] \right) = 0$$

Since $p_1 > 0$ and $p_2 > 0$ it follows that if

$$\sum_{J=A,B} [x_1^J(p_1, p_2, m^J(p)) - e_1^J] = 0 \text{ (market for good 1 clears)}$$

then the equality above guarantees that

$$\sum_{J=A,B} [x_2^J(p_1, p_2, m^J(p)) - e_2^J] = 0 \text{ (market for good 2 clears)}$$

The economics behind these summations are actually straightforward. We begin by observing that agents will use their full budgets, which means that the value of the optimal demand given any price equals the value of the endowment for both agents. Summing over the agents, the value of the optimal demand for A + the value for the optimal demand for B must equal the value of the sum of the endowments. This means, *regardless of whether the price is an equilibrium price or not*, that the value of the excess demand/supply for good 1 + the value of the excess demand/supply for good 2 must be identical to zero, *regardless of whether the prices clear the market or not*.

1.6 Example 1: Calculating a Competitive Equilibrium Explicitly in the 2×2 Model

Assume that the agents have Cobb-Douglas preferences,

$$U^A(x_1, x_2) = a \ln x_1 + (1 - a) \ln x_2$$

$$U^B(x_1, x_2) = b \ln x_1 + (1 - b) \ln x_2,$$

and that the endowments are $e^A = (1, 0)$ and $e^B = (0, 1)$. In words, agent A is a seller of good 1 and a buyer of good 2 and agent B is the other way around. The relevant demands can therefore be calculated to be,

$$x_1^A(p, m^A(p)) = \frac{am^A(p)}{p_1} = \frac{a(p_1 \times 1 + p_2 \times 0)}{p_1} = a$$

$$x_1^B(p, m^B(p)) = \frac{bm^B(p)}{p_1} = \frac{b(p_1 \times 0 + p_2 \times 1)}{p_1} = b \frac{p_2}{p_1}.$$

So equilibrium requires that

$$\begin{aligned} x_1^A(p, m^A(p)) + x_1^B(p, m^B(p)) &= a + b \frac{p_2}{p_1} = 1 = e_1^A + e_1^B \Rightarrow \\ \frac{p_1^*}{p_2^*} &= \frac{b}{1-a} \end{aligned}$$

Plugging the relative price back into the demand expressions above we then have that the equilibrium allocation is

$$\begin{aligned} \langle x_1^A(p^*, m^A(p^*)), x_2^A(p^*, m^A(p^*)), x_1^B(p^*, m^B(p^*)), x_2^B(p^*, m^B(p^*)) \rangle \\ \langle a, b, 1-a, 1-b \rangle \end{aligned}$$

Notice that the more the other agent likes the good that the agent has in her endowment, the better off the agent is, simply reflecting that increased demand drives up the price, which is good for the seller of the good.

1.7 Example 2: A Representative Agent Example

A common way to write down general equilibrium model that are simple is to assume that all agents in the economy are identical clones of each other. This type of models are used a lot in macroeconomics (because it allows the choice problem for the individual to be quite rich) and are called representative agent models.

Consider a world populated with lots of agents consuming only apples. Each agent lives for two periods and has an apple tree that produces e units of apples in every period. There is a competitive market for borrowing and saving and r denotes the interest rate. All agents have identical preferences given by

$$u(c_1) + \delta u(c_2).$$

The choice problem for an individual is thus to decide how much to borrow or save. As we have seen before, we may write this either as

$$\max_{-\frac{e}{1+r} \leq s \leq e} u(e-s) + \delta u(e+s(1+r))$$

or as

$$\begin{aligned} & \max_{c_1, c_2} u(c_1) + \delta u(c_2) \\ \text{s.t. } & c_1 + \frac{1}{1+r}c_2 \leq e + \frac{1}{1+r}e. \end{aligned}$$

For our purposes, the first expression is simpler. The first order condition is simply

$$-u'(e - s) + \delta u'(e + s(1 + r))(1 + r) = 0$$

Now, assuming that the apples are non-storable we note that:

1. In equilibrium, r^* must be such that $s^* = 0$. If not, resources will not balance. That is, if $s < 0$ then all agents borrow, so total apple consumption in the first period exceeds total apple production. Symmetrically, if $s > 0$ all agents save, so total apple consumption in the second period exceeds what is available.
2. Hence, $s^* = 0$ must solve the optimization problem, and therefore satisfy the first order condition. We conclude that

$$-u'(e) + \delta u'(e)(1 + r^*) = 0$$

or

$$r^* = \frac{1 - \delta}{\delta}.$$

This is very very simple, but it is a useful theory of how the equilibrium interest rate is determined. It simply says that the equilibrium interest rate must be determined so that people are happy to consume what is available in every period, which boiled down to relation between the interest rate and the “discount factor” which measures how patient or impatient people are. For future reference, we observe that this worldview is not so easy to reconcile with one where people save to little for their retirement (which is a common claim by those in favor of compulsive savings programs such as social security).

1.8 Exchange Efficiency: Summary

So far we have dwelt mainly on what is Stiglitz labels “exchange efficiency”. The basic point is that, if the agents act as price takers and the price somehow is set to clear the market, then the outcome will be Pareto efficient. In terms of the Edgeworth box, it is more or less obvious that both agents must have indifference curves that are tangent to the (common) budget line. In the economics lingo this is usually expressed by saying that both agents must have *marginal rates of substitutions* that are equal to the relative price ratio. This is fine, but it is important to recall that the marginal rate of substitution (or the slope of an indifference curve) is just a fancy name for the rate that an agent is willing to trade one (small unit of a good) to another. The logic is therefore simply that the “internal terms of trade” must be equalized across agents. To understand this intuitively, it is best to think about the consequences if the slopes/MRS/internal rates of trades would differ. Then, one agent would be willing to trade, say, one banana for an apple. The other agent however would (for the slopes to differ) either be willing to give up more than a banana for an apple or more than an apple for a banana. In either case we have an inefficiency, as the two agents could find a trade that leads to a Pareto improvement.

1.9 Production Efficiency

An advantage with being careful about exchange efficiency is that the principles from a pure exchange economy carry over to a model with production without too much work.

Suppose that good 1 and good 2 are both produced from two (for easy graphical representation) inputs, and call these inputs land (L) and labor (N). A price taking profit maximizing firm in sector 1 will then seek to maximize its profit by solving

$$\begin{aligned} & \max_{y_1, L, N} p_1 y_1 - w_L L - w_N N \\ & \text{subj to } y_1 \leq f_1(L, N) \end{aligned}$$

where $f_1(\cdot)$ is the production function for good 1. The key observation for production efficiency is that regardless of which quantity y_1^* is part of the solution to this profit maximiza-

tion problem, **the profit maximizing quantity must be produced in the cheapest way possible**. Hence, if (y_1^*, L^*, N^*) is a solution to the profit maximization problem, then (L^*, N^*) is a solution to

$$\begin{aligned} & \max_{L, N} p_1 y_1^* - w_L L - w_N N \\ \text{subj. to } & y_1^* \leq f_1(L, N) \end{aligned}$$

or, since $\max_x -h(x) = \min_x h(x)$ and $p_1 y_1^*$ is a constant to the problem

$$\begin{aligned} & \min_{L, N} w_L L + w_N N \\ \text{subj. to } & f_1(L, N) \end{aligned}$$

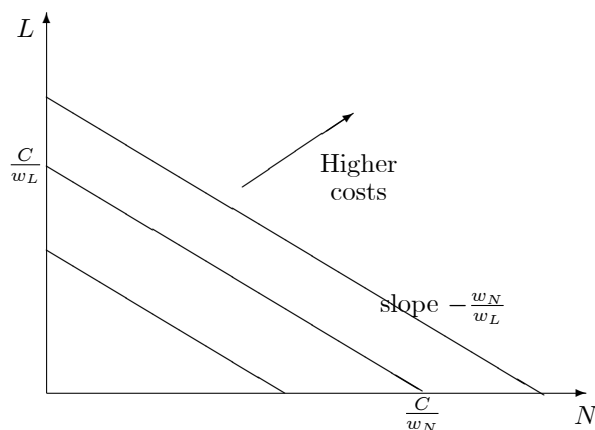


Figure 7: Constant Cost Curves (aka Isocosts)

We can draw this pretty much like we illustrate a utility maximization problem in an indifference curve graph. That is, first consider combinations of inputs (L and N) such that costs are the same. That is

$$\begin{aligned} C &= w_L L + w_N N \\ &\Leftrightarrow \\ L &= \frac{C}{w_L} - \frac{w_N}{w_L} N \end{aligned}$$

This is pretty intuitive. It just says that costs are kept constant along a straight line with slope given by the relative factor price. For example, say that $w_L = 2$ and $w_N = 4$, so that a

unit of labor is twice as expensive as a unit of land. Then, what the linear relationship says is simply that to keep costs constant we need to reduce the input of land with 2 units if we hire an additional unit of labor.

Next, consider combinations L, N such that output is constant. That is, (L, N) such that

$$y_1^* = f_1(L, N).$$

If the production function is linear, this will again just results in straight lines. However, this would be rather nasty (almost always this would lead to “corner solutions”) and textbooks have a tendency to shy away from anything inconvenient. Instead, it is usually assumed that the curves associated to the equation $y_1 = f_1(L, N)$ have the same nice convex shape as the generic textbook indifference curves. We then observe that the lowest costs to produce any particular output y_1^* must occur *at the line that touches the given “isoquant” (=curve of L and N that produce exactly y_1^* units of good 1) which is closest to the origin of the graph.* It is then graphically apparent that the solution must occur where there is a tangency between the isoquant and the isocost since otherwise it is possible to move towards lower cost levels and still produce the same output.

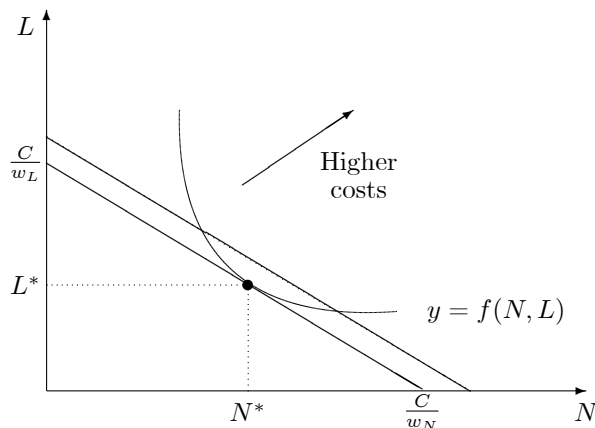


Figure 8: The Cost Minimization Problem

This makes perfect sense:

- The slope of the curve defined by the condition $y_1^* = f_1(L, N)$ (aka the marginal rate

of technical substitution) tells us how we can reduce the use of land if we increase the labor input by one small unit and keep output constant

That is, the slope tells us at which rate factors can be substituted.

- The relative factor price $\frac{w_N}{w_L}$ is the “rate at which factors can be exchanged in the market”.
- For efficiency (and profit maximization) in production the rate at which factors can be substituted must equal the relative factor price (otherwise we can produce more at a given cost).

At this point we can combine cost minimization in the two industries to get a graph which is virtually identical to the one illustrating the equilibrium in an exchange economy.

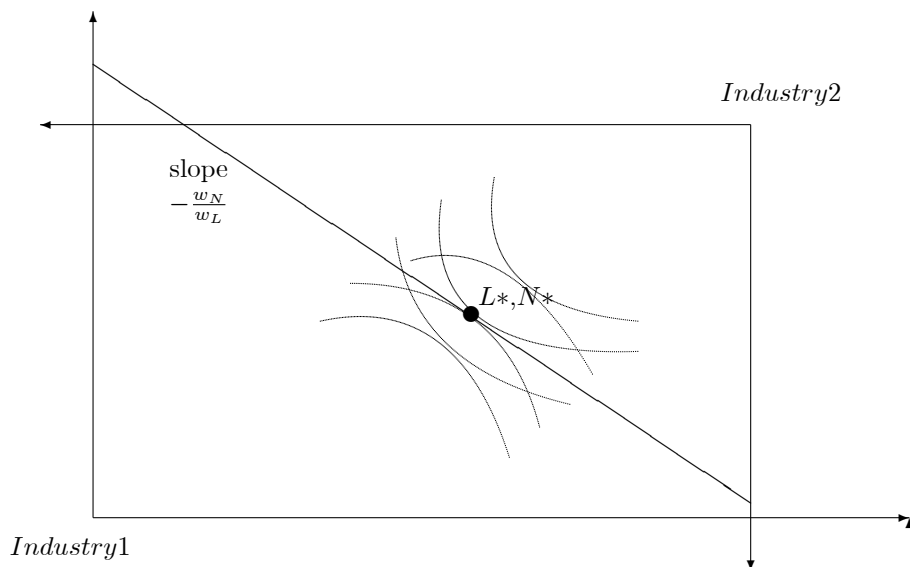


Figure 9: Competitive Factor Markets and Efficiency in Edgeworth Box)

1.10 Product Mix Efficiency

Exchange efficiency is purely about preferences (and endowments) and production efficiency is purely about technology (production or cost functions). The final efficiency criterion

combines consumer preferences and technological constraints. Tracing the Pareto frontier for productive efficiency we can generate a production possibilities frontier like the one illustrated in Fig 3.6 in book. Product mix efficiency then requires that goods are produced in such a way that the slope of all indifference curves coincide with the slope of the production possibilities frontier.