Information Asymmetries
and
an Endogenous Productivity Reversion Mechanism

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Abstract

Several studies among recent empirical work have suggested that the systematic behavior of lending standards over the business cycles, with laxer standards applied during expansions and tighter standards applied during recessions, may be responsible for driving economic fluctuations. We build a dynamic screening model with informational asymmetry in credit markets that rationalizes these findings and generates endogenous (nontrivial) aggregate productivity fluctuations via the lending standards channel. When the economy is productive (productivity evolves endogenously), signals about entrepreneurs' quality become endogenously less informative, and consequently, screening out the bad projects becomes more costly. Separating contracts do not survive in equilibrium. Instead, contracts financing a mix of good and bad projects emerge. The composition effect due to both good and bad projects being implemented sets off a recession. The opposite happens at troughs.

JEL Codes: E32, E44, D24

I. Introduction

Several studies among recent empirical work have suggested that the systematic behavior of lending standards over the business cycles may be responsible for the reversion of trends in aggregate productivity. Asea and Blomberg (1998) use a panel data set of two million commercial and industrial loans to find that laxer lending standards occur during expansions and tighter standards occur during recessions, such behavior of lending standards considerably influencing dynamics of aggregate fluctuations. Lown and Morgan (2004) use a survey of loan officers and also find that lending standards varies systematically over the cycle. They also conclude that loan standards dominate loan rates in explaining variation in

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business loans and output in the time series, which is another observation motivating our work.[1] Berger and Udell (2004) document similar behavior of lending standards and test a hypothesis of suboptimal behavior of loan officers.

Consistent with these empirical findings, it is observed that delinquency rates (proportion of value of loans that are late) and loan charge-off rates (proportion of the value of loans written off) are high following the expansions and are low following the recessions (See Figure 1).

Note that financing of production through the financial institutions is widespread int he U.S. All non-corporate business, which represents roughly 1/3 of total business net worth in the U.S. since 1952, relies entirely on bank loans to finance its production. Corporate business, representing roughly 2/3 of the net worth, although considerably reduced its dependence on bank financing, today still holds over a quarter of its debt in bank loans and mortgages. As recent as mid-1970s, around 45% of its debt was attributed to bank loans and mortgages.

Because of the extent of the business subject to credit market frictions, these empirical facts have generated much concern among the policy makers. Alan Greenspan, speaking at the Chicago Bank Structure Conference in 2001, alarmingly stated that "The worst loans are made at the top of the business cycle...[and at the bottom] the problem is not making bad loans, it is not making any loans, whether good or bad...".[2]

It is useful to summarize the empirical facts as follows.

- Financing of production through financial institutions is widespread.
- Delinquency rates lag after the cycle.
- Loan standards are relaxed at the top of the cycle and tightened at the bottom.
- Lending standards (rather than rates) explain loan and output variation over time.

Motivated by these findings, we investigate the role of credit market frictions in generating a reversion of the aggregate productivity. Specifically, we examine the role of information asymmetry in credit markets for the process of the aggregate productivity reversal. We build a dynamic screening model with entrepreneurs of different types and competitive lenders. All agents are fully rational. When the economy is productive (productivity evolves endogenously), signals about entrepreneurs' quality become endogenously less informative. Consequently, screening out the bad projects becomes more costly. Contracts that try to do that simply do not survive in equilibrium. Instead, contracts financing a mix of good and

[1] It is important to make a distinction between the loan interest rate and lending standards, such as the size of the credit line extended, balance sheet variables of the borrower, time to loan maturity, etc. Both, standards and rates, can be used by the bank to screen borrowers.

[2] It is also observed from the data (see Figure 2) that the value of outstanding loans is procyclical. Certainly, the movement in the amount of available loanable funds must be a part of the reason. However, it appears that the behavior of lending standards may also contribute to the procyclicality of bank lending. To keep the model simple, we abstract from movement in the supply of loanable funds for now.

bad projects emerge. The composition effect due to both good and bad projects being implemented sets off a recession. The opposite happens at troughs.

Our model can give rise to endogenous (and non-trivial) fluctuations via the lending standards channel, with the strength of the entrepreneurs' signal of the quality of their project and thus the cost of screening evolving endogenously over the cycle. Aggregate productivity over this cycle changes due to the composition of types of projects being financed and implemented. This mechanism can also work in a model with exogenous shocks to productivity, but we do not have these in this paper.

Screening of entrepreneurs is done by requiring of partial loan repayment upon completion of the first stage of production. This payment can always be set high enough so that unproductive entrepreneurs cannot afford it. The payment, however, hurts productive entrepreneurs as it lowers their reinvestment into the second stage of production. There are two effects that along with competition in the banking sector deliver pooling equilibrium contracts at the top of the cycle and separating contracts with unproductive entrepreneurs unfinanced at the bottom. At peaks, when unproductive entrepreneurs generate higher income, it takes a higher payment to separate the bad entrepreneurs out, and hence the productive entrepreneurs are hurt more if there is separation. Moreover, pooling the unproductive and productive producers into the same contract does not hurt the best entrepreneurs as much at the peak as it would at the trough, because the default of the unproductive producer is less costly to the bank (since they recover more money after default) and hence lower loan interest is needed to ensure bank participation. Hence, competition in the banking sector ensures that unproductive entrepreneurs are able to obtain financing at the peak, thus sending the economy into a recession.

The rest of the paper is organized as follows. Section II overviews related literature. Section III introduces the general model and derives equilibrium contracts for given prices. Section IV is a special case of the general model. It is very simple, yet gets the message across. The dynamics of the model economy is derived. The general model's dynamics is derived in Section V.

II. Related Literature

Similar in spirit to our work is Dell'Ariccia and Marquez (2006). Although their focus is on generating a sequence of financial liberalization, a lending boom, deterioration of lending standards and finally a banking crisis as observed in many of the emerging economies, the idea that contracts that screen when the degree of informational asymmetry is severe and therefore screening costs are large, do not survive in equilibrium is similar. We have the screening costs endogenously evolving in our model.

A number of theoretical models illustrate potentially important interactions between credit market frictions and economic fluctuations. For the purpose of our discussion, we focus on two strands of related work. One strand argues that credit market imperfections amplify exogenous shocks and make them more persistent, which was already discussed above. Another strand of literature argues that credit market imperfections are responsible for a reversion in the productivity.

For example, in Bernanke and Gertler (1989), the assumption of the borrowers' balance sheets is a source of cycle amplification. Business upturns improve borrowers' net worth, which lowers agency costs of financing investment, increases investment and hence amplifies the upturn; vice versa, for downturns. Another example is Kiyotaki and Moore (1997), which assumes that loan payments cannot be enforced and hence only collateralized debt arises in equilibrium. A temporary shock that reduces a credit constrained firm's net worth reduces this firm's investment and hence makes it more credit constrained in the next period thus propagating the effect of a temporary shock. In Rampini (2004), entrepreneurial (productive) activity increases at peaks as agents are more willing to take on the risk thus resulting in a different amplification mechanism.

Suarez and Sussman (1997) generate a reversion mechanism that works through the effect of equilibrium prices on liquidity constraints. The model is a dynamic extension of the Stiglitz-Weiss (1982) model of lending under moral hazard in an overlapping generations model with three generations. During booms, old entrepreneurs sell high quantities and, as a consequence, prices are low and young entrepreneurs must finance a higher fraction of output externally. Because external financing generates excessive risk-taking booms are followed by high project failure rates. Though it delivers an endogenous reversion mechanism, the main channel through which this mechanism works - higher reliance on external financing at peaks - seems to be at odds with the data.

Reichlin and Siconolfi (2003) generalize the Rothschild and Stiglitz adverse selection problem to include moral hazard. Both safe and risky projects can be implemented. Entrepreneurs differ only in their return to implementing the risky project, with lower skilled entrepreneurs facing a higher fixed cost of implementing it. The safe project requires a zero fixed cost and hence yields higher expected returns. They embed this mechanism in an overlapping generations model where the opportunity cost of lending evolves endogenously. They show that endogenous cycles may arise: when loanable funds are high, equilibrium contracts are such that a large fraction of entrepreneurs engages into risky production, high setup costs decrease output and wages sending the economy into recession. This model relies heavily on risky projects being worse in expected terms than safe ones. In fact the opposite assumption (risky projects are better in expected terms) is made in Rampini (2004), and the informational friction delivers an amplification mechanism rather than a reversion mechanism.

III. Dynamic Model

A. Environment

Consider a model economy with two types of goods: consumption and capital. Time is discrete and indexed by $t = 0, 1, 2...$ The economy is populated with overlapping generations of entrepreneurs alive for two periods. When young, entrepreneurs are endowed with 1 unit of time and an ability to implement capital production projects. We assume they care only about consumption when old.

Consumption good is produced by an infinitely lived aggregate firm that employs labor and capital according to technology $Y_t = A_t K_t^{\beta} L_t^{1-\beta}$. Capital produced in $t-1$ can be used as an input in production in period t, upon completion of which it fully depreciates. We assume this firm has access to perfect credit markets and borrow at a risk free rate R_f. Hence, the aggregate firm can buy capital used in time t production (at price ρ_t) from young entrepreneurs. In period 0 the firm is endowed with K_0 capital and debt in the amount of $R_f \rho_0 K_0$. The aggregate firm behaves competitively, so the cost per unit of capital faced by the firm, $R_f \rho_t$, and the wage w_t are given by their marginal products $R_f \rho_t = A_t \beta k_t^{\beta-1}$ and $w_t = A_t (1-\beta) k_t^{\beta}$, where $k_t = K_t/L_t$.[3]

Each generation of entrepreneurs consists of two types, $i \in \{G, B\}$, of measures μ and $1-\mu$ respectively. The two types are not observable and differ in productivity of their capital production projects. Projects are implemented within a single period, but in two stages. A fixed cost in the amount of M units of consumption goods is required at each stage of production. Project i yields f_i units of capital good at the end of the first stage of production and $g_i + \frac{g_i}{M} s_i$ units of capital good at the end of the second stage, where s_i represents investment of funds beyond the fixed cost amount into the second stage of the project.[4] We assume that type G is more productive at each stage.

Assumption 1 *$f_G > f_B$ and $g_G > g_B$.*

There is a competitive banking sector that loans investment funds to the young entrepreneurs. Each period, banks are endowed with $2M\mu$ loanable consumption goods, exactly the fixed cost amount of implementing projects of all type G entrepreneurs.[5] A risk-free savings technology is available to the bank at rate R_f.

We consider contracts of the following form. Contracts are signed in the beginning of the entrepreneurs' young period. If an entrepreneur enters into a contract, he receives an amount M in the beginning of the first stage and another amount M in the beginning of the second stage, conditional on meeting a prepayment δ_t towards the loan balance. At the end of the period, the remaining loan repayment is $2MR_t - \delta_t$. Hence, a contract is characterized by (R_t, δ_t). We assume that production takes place whenever financing is obtained, due to the availability of a monitoring technology. We also assume limited liability, so that in case of default, fraction α of wealth is kept by the entrepreneur, while a fraction $1-\alpha$ is seized by the bank. We assume that default can take place only at the end of the second stage.[6]

Alternatively, we could assume that banks offer a credit line to entrepreneurs and contracts specify the credit limit and the interest rate on the borrowed funds. In the appendix, we prove that identical equilibrium outcomes are achieved under the assumption of contracts in the form of a credit line with the credit limit $2M - \delta_t$ and interest rate $\frac{2MR_t - \delta_t}{2M - \delta_t}$.

It is instructive to examine the cash flows for an entrepreneur who enters into a contract and pays in full. The timeline below illustrates that a time t young entrepreneur obtains M units of consumption good from the bank in the beginning of the first stage of production and invests it into the project. At the end

[3] Formally, taking prices as given, the firm solves $\max_{\{K_t, L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} A_t K_t^{\beta} L_t^{1-\beta} - R_f \rho_t K_t - w_t L_t + \rho_{t+1} K_{t+1} - \rho_{t+1} K_{t+1}$, where in period t the bank receives $\rho_{t+1} K_{t+1}$ from the bank and pays it to capital producers; it repays $R_f \rho_{t+1} K_{t+1}$ to the bank in the next period.

[4] Note that investing more than M into the first stage yields a zero return.

[5] This particular amount is assumed for analytical simlicity and is not crucial for the results.

[6] In other words, an entrepreneur defaulting in the middle of the period retains fraction $\alpha_0 = 0$ of his wealth. This assumption captures the idea that it is more difficult to steal or hide income during initial stages of production and is made only for analytical simplicity.

of the first stage he receives $\rho_{t+1}f_i$ in payment for his capital[7] and w_t as labor income from supplying 1 unit of time to the aggregate firm. He pays δ_t towards the loan balance upon completion of the first stage of production. He receives M units from the bank and invests into the second stage, along with his own income net of the loan payment $(w_t+\rho_{t+1}f_i-\delta_t)$ which transforms into capital at the rate of $\frac{g_i}{M}$. To ensure that any entrepreneur engaging in capital production reinvests all of his own funds into the second stage, we make the following assumption.

Assumption 2 $\frac{\rho_t g_B}{M} > 1$.

Note that in light of Assumption 1, this result also holds for type G. At the end of the period, the entrepreneur sells his capital $g_i+ (w_t + \rho_{t+1}f_i - \delta_t)\frac{g_i}{M}$ at price ρ_{t+1} and pays the remaining loan balance $2R_{t-1}M - \delta_{t-1}$ to the bank. In the beginning of the next period, the entrepreneur simply consumes his wealth.

t		$t+1$
YOUNG		OLD
$+M$ $-M$ $+\rho_{t+1}f_i+w_t$ $-\delta_t$	$+M$ $-M-(w_t+\rho_{t+1}f_i-\delta_t)$ $+\rho_{t+1}\left[g_i+(w_t+\rho_{t+1}f_i-\delta_t)\frac{g_i}{M}\right]$ $-(2MR_t-\delta_t)$	Consumption in the beginning of the period $\rho_{t+1}\left[g_i+\left(w_t+\rho_{t+1}f_i-\delta_t\right)\frac{g_i}{M}\right]-(2MR_t-\delta_t)$

Because entrepreneurs are productive for a single period only, two stages of production are necessary to allow the possibility of screening. In our case, the screening tool is the early loan payment. Type G can always afford a higher payment than type B. Screening would not be possible if capital output, or the income that its sales generate, occurred only at the end of the period. Note that for the same reason, we need that the aggregate firm can deliver the payment in the middle of the period for capital that is used in the next period's production of consumption goods.

Thus, we capture the idea that the lenders always have the ability to screen out the bad type. The costs of screening, which is incurred by productive entrepreneurs, varies endogenously with the state of the economy. We also capture the idea that screening is costly to entrepreneurs. Competition among the lenders yields the result that the screening instrument is not always applied. Precisely, it is used only when it is less harmful to type G.

Note that Assumption 2 and several assumptions made later in the paper depend on endogenous prices. In the appendix, we derive sufficient restrictions on parameters and the initial state (k_0) to guarantee that all of the assumptions made in the paper hold.

B. Entrepreneur' Behavior

For a given contract (δ_t, R_t) and prices (w_t and ρ_{t+1}), a time t young entrepreneur of type i chooses among the following options. These options and consumption levels associated with each are summarized below.

[7] The price of capital this entrepreneur receives for his capital is ρ_{t+1} to emphasize that this capital is used in period $t+1$.

(O1) Do not enter into the contract. Consume labor income w_t.
(O2) Enter into the contract, meet the payment δ_t, default. Consume $\alpha\rho_{t+1}\left[g_i + \left(w_t + \rho_{t+1}f_i - \delta_t\right)\frac{g_i}{M}\right]$.
(O3) Enter into the contract, meet the payment δ_t, pay in full. Consume $\rho_{t+1}\left[g_i + \left(w_t + \rho_{t+1}f_i - \delta_t\right)\frac{g_i}{M}\right] - (2MR_t - \delta_t)$.

Note that due to our assumption that default is not an option in the middle of the period, entrepreneur i will always choose O1 if $w_t + \rho_{t+1}f_i < \delta_t$, i.e., if he cannot afford the early payment.

We restrict our attention to equilibria along which the following assumptions are satisfied. This means that we only consider the set of parameters (including the initial condition of the economy) such that the following behavior is optimal. We derive the restrictions on the set of parameters in the appendix.

Assumption 3 *If type B enters into the contract, he chooses to default.*

Assumption 4 *If type G enters into the contract, he chooses to repay in full.*

Assumption 5 *Both types choose entering into a contract (δ_t, R_t) if they can afford δ_t, i.e., if $\delta_t < w_t + \rho_{t+1}f_i$.*

C. *Equilibrium contracts*

Next we characterize equilibrium contracts (δ_t, R_t) for some given prices, w_t and ρ_{t+1}. Our assumption of competition in the lending sector implies that there must be no profit-making opportunities.

The following proposition states the result that allows us to restrict attention to a single type of contracts being offered. It shows that a situation in which type-dependent contracts are offered and both types obtain financing cannot arise in equilibrium.

Proposition 1 *If for some given prices, w_t and ρ_{t+1}, positive measures of both types obtain financing, it must be the case that a single (pooling) contract is offered.*

We give the proof in the appendix. It is instructive, however, to discuss a sketch of this proof. Suppose that type-dependent contracts $\left(\delta_t^G, R_t^G\right) \neq \left(\delta_t^B, R_t^B\right)$ were offered and positive measures of each type self-selected those contracts. For types to self-select their contracts, it is always necessary that $0 \leq \delta_t^B \leq \delta_t^G$ and $R_t^B \geq R_t^G$. Two cases can be discussed. Case 1. $\delta_t^G = 0$. Then $R^G = R_f$ for contracts to be robust to cream skimming. For B not to select G's contract, it must be the case that $\delta_t^B = 0$. Then as B defaults, banks make negative profits. Case 2. If $\delta_t^G > 0$, then $R_t^G < R_f$ for contract $\left(\delta_t^G, R_t^G\right)$ to be robust to cream skimming. Then the zero profit condition means the banks have to require a high enough δ^B to compensate for the negative profit generated by contracts written for G. Such contracts cannot be sustained in equilibrium as there are profit making opportunities for a bank that offers a lower δ^B and attracts B.

In order to formally state the lender's maximization problem, we derive the zero profit condition for the lender in terms of R_t and δ_t (for given w_t and ρ_{t+1}).

Separating contracts. For $\delta_t > w_t + \rho_{t+1} f_B$, only type G can afford the payment, he enters into the contract by Assumption 5 and repays in full by Assumption 4. Type B stays out of the contract. Hence, the zero profit condition reduces to $R_t = R_f$.

Pooling contracts. For $\delta_t \leq w_t + \rho_{t+1} f_B$, both types enter the contract (by Assumption 5), type G repays in full while type B defaults (by Assumptions 4 and 3). Since types are not observable and only $2M\mu$ can be loaned out, measure μ^2 of entrepreneurs of type G and measure $(1-\mu)\mu$ of type B are financed. The lender's zero profit condition is then given by

$$\mu^2 2MR_t + (1-\mu)\mu \left[\delta_t + (1-\alpha)\rho_{t+1}\left(g_B + \left(w_t + \rho_{t+1} f_B - \delta_t\right)\frac{g_B}{M}\right)\right] = 2\mu M R_f,$$

where the right hand side represents the opportunity cost of loaning out the available funds, that is the risk-free earning on these funds. The left hand side represents the funds obtained as loan repayment. It consists of full repayment from measure μ^2 of type G entrepreneurs and δ_t collected as early repayment together with the fraction $1-\alpha$ of output ceased from measure $(1-\mu)\mu$ of type B entrepreneurs. This condition can be solved for R_t as a function of δ_t.

$$R_t = \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu}\left[\delta_t + (1-\alpha)\rho_{t+1}\left(g_B + \left(w_t + \rho_{t+1} f_B - \delta_t\right)\frac{g_B}{M}\right)\right]. \tag{1}$$

The general zero profit condition can be written as follows

$$R_{w,\rho}(\delta_t) = \begin{cases} \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu}\left[\delta_t + (1-\alpha)\rho_{t+1}\left(g_B + \left(w_t + \rho_{t+1} f_B - \delta_t\right)\frac{g_B}{M}\right)\right], & \text{if } \delta_t < w_t + \rho_{t+1} f_B \\ R_f, & \text{otherwise,} \end{cases} \tag{2}$$

where the subscript w, ρ serves to remind of the explicit dependence on prices for which current labor services and capital sell. We define the minimum level of payment unaffordable by type B as

$$\tilde{\delta}_{w,\rho} \equiv w_t + \rho_{t+1} f_B. \tag{3}$$

Note that $\tilde{\delta}_{w,\rho}$ increases in w_t and ρ_{t+1}. Indeed, as labor or capital income of type B increases, a higher payment is needed for separation.

The intercept of the zero profit function is

$$R_{w,\rho}(0) = \frac{R_f}{\mu} - \frac{(1-\mu)}{\mu 2M}(1-\alpha)\rho_{t+1}\left(g_B + (w_t + \rho_{t+1} f_B)\frac{g_B}{M}\right), \tag{4}$$

which decreases in ρ_{t+1} and w_t. Intuitively, the amount recovered from type B, who enters into contract when $\delta = 0$ by Assumption 5, increases in his income and hence a lower interest rate is needed to ensure zero profit for the lender.

To summarize the above discussion, for given prices (w_t and ρ_{t+1}), the equilibrium contracts (δ_t, R_t) are found by maximizing the utility of type G entrepreneur subject to the lender's zero profit.

$$\begin{aligned} \max_{\delta_t, R_t} \quad & \rho_{t+1}\left(g_G + \left(w_t + \rho_{t+1} f_G - \delta_t\right)\frac{g_G}{M}\right) - \left(2MR_t - \delta_t\right) \\ \text{s.t} \quad & \\ R_t \quad = \quad & R_{w,\rho}\left(\delta_t\right), \end{aligned} \tag{5}$$

where $R_{w,\rho}\left(\delta_t\right)$ is given in (2). We also make an assumption that for $\delta_t < w_t + \rho_{t+1} f_B$, (so type B enters), raising δ by one unit and hence effectively lowering reinvestment for type B by 1 unit actually raises the total collection from type B.

Assumption 6 $1 > (1-\alpha)\,\rho_{t+1}\frac{g_B}{M}$.

This implies that the segment of $R_{w,\rho}\left(\delta_t\right)$ corresponding to pooling contracts (i.e. $\delta_t < \tilde{\delta}_{w,\rho}$) is downward sloping. As δ_t increases, total collection from defaulting agents increase and a lower R_t is needed to ensure zero profit.[8]

In the following lemma, we show that in case of pooling contracts, R_t is higher than the risk-free rate. Intuitively, this is true because type B enters into the contract and defaults, so it takes a higher than the risk-free interest rate to ensure zero profits for the lender. The proof is in the appendix.

Lemma 1 *If $\delta_t \leq w_t + \rho_{t+1} f_B$, then $R_t > R_f$.*

For given prices, we illustrate the lender's maximization problem by depicting the indifference curves of type G together with the zero profit condition for the bank (Figure 3). It is useful to define the following critical slope,

$$CS \equiv -\frac{R_{w,\rho}(0) - R_f}{\tilde{\delta}_{w,\rho}} < 0. \tag{6}$$

This slope is negative due to Lemma 1. Whether pooling or separating contracts emerge in equilibrium depends on how steep CS is relative to the slope of type G's indifference curves, which is what we derive next. Because type G always repays, his total consumption is given by $\rho_{t+1}\left[g_i + \left(w_t + \rho_{t+1} f_i - \delta_t\right)\frac{g_i}{M}\right]$ $-\left(2MR_t - \delta_t\right)$. Then the indifference curves describing his trade-off between R and δ, given current prices, have a slope

$$ICS \equiv -\frac{\rho_{t+1}\frac{g_G}{M} - 1}{2M} < 0. \tag{7}$$

The slope is negative due to Assumption 2. Also note the slope becomes steeper as ρ_{t+1} rises. Intuitively, if capital sells for a higher price, the cost of forgone investment associated with δ rises, and for a given rise in δ, type G requires a greater compensation in the form of a reduction in R.

[8] This assumption actually guarantees that every point on the zero profit condition is robust to profit making opportunities from a bank that seeks to attract type B. This is not necessary though, as we require that only an equilibrium contract is robust to such opportunity.

From Figure 3, it is obvious that a pooling contract $(\delta^*, R^*) = (0, R_{w,\rho}(0))$ is offered if $ICS < CS$ and a separating contract $(\delta^*, R^*) = (0, R_{w,\rho}(0))$ is offered otherwise. The latter case is illustrated in the figure. The following proposition summarizes this result.

Proposition 2 *If* $\frac{1-\rho_{t+1}\frac{g_G}{M}}{2M} \geq -\frac{R_{w,\rho}(0)-R_f}{\tilde{\delta}_{w,\rho}}$, *then a separating contract is offered,* $(\delta^*, R^*) = \left(\tilde{\delta}_{w,\rho}, R_f\right)$, *where* $\tilde{\delta}_{w,\rho}$ *and* $R_{w,\rho}(0)$ *are given in* (3) *and* (4). *Otherwise, a pooling contract is offered,* $(\delta^*, R^*) = (0, R_{w,\rho}(0))$.
Proof. *Obvious from the above discussion.* ■

IV. Simplified Mechanism

Up to this point we have derived equilibrium contracts for some given prices w_t and ρ_{t+1}. We have not mentioned the dynamics of output and capital, which is the main focus of this paper. This is because in general, ρ_{t+1} which matters for entrepreneurs' decisions today, will depend on k_{t+1}, while k_{t+1} depends on current decision making outcomes. This makes the study of evolution of capital/output difficult. If, however, we assume the presence of a productivity externality due to the size of the economy, the main mechanism of the paper remains, but the equilibrium dynamics simplifies substantially. Hence, it is instructive to discuss the simplified mechanism here to convey the main message and obtain nice intuition for the reversion mechanism. The general case is presented in section VI.

A. Prices and Equilibrium Contracts

Suppose $A_t = K_t^{\gamma}$, $\gamma + \beta = 1$. Then

$$\begin{aligned} \rho_t &= \beta A_t K_t^{\beta-1} L_t^{1-\beta} = \beta K_t^{\gamma+\beta-1} L_t^{1-\beta} = \beta, \\ w_t &= (1-\beta) A_t \frac{K_t^{\beta}}{L_t^{\beta}} = (1-\beta) \frac{K_t^{\beta+\gamma}}{L_t^{\beta}} = (1-\beta) k_t. \end{aligned}$$

Note that the capital rental rate is independent of the stock of capital, which is the key to simplifying the analysis. The only state variable is k_t. We restate the analysis of Section IV, which was conducted for some given prices w_t and ρ_{t+1}, in terms of the state variable k_t. Time subscripts are dropped in the remainder of the section.

Assumption 1 remains as above. Assumption 2 reduces to $\frac{\beta g_B}{M} > 1$.

Consumption associated with each strategy can be written in terms of the state variable. Option (O1), that is, not engaging in production, yields $(1-\beta)k$. Option (O2), entering into the contract to default, yields $\alpha\beta[g_i + ((1-\beta)k + \beta f_i - \delta)\frac{g_i}{M}]$. Finally, option (O3), entering into the contract to repay, yields $\beta[g_i + ((1-\beta)k + \beta f_i - \delta)\frac{g_i}{M}] - (2MR - \delta)$. Again, if entrepreneur i cannot afford δ, i.e., if $(1-\beta)k + \beta f_i < \delta$, he will choose O1.

Assumptions 3, 4, 5 that ensure that type B enters into the contract to default, type G enters into the contract to repay, and both types enter into the contract if they can afford δ, respectively, are stated in the appendix.

The lender's zero profit function simplifies to

$$R_k(\delta) = \begin{cases} \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu}\left[\delta + (1-\alpha)\beta\left(g_B + ((1-\beta)k + \beta f_B - \delta)\frac{g_B}{M}\right)\right], & \text{if } \delta < \tilde{\delta}_k \\ R_f, & \text{otherwise,} \end{cases} \tag{8}$$

where

$$\tilde{\delta}_k = (1-\beta)k + \beta f_B \tag{9}$$

is the minimum separating repayment level.

A few things can be pointed out about the shape of this function. First of all, the slope of the zero profit function in the interval $\delta \in [0, \tilde{\delta}_k]$, i.e., for δ such that pooling contracts are offered, is given by

$$R_k'(\delta) = -\left(\frac{1-\mu}{\mu}\right)\frac{1}{2M}\left[1 - (1-\alpha)\beta\frac{g_B}{M}\right],$$

which is independent of the economy's state k. Note that this slope is negative in light of assumption 6, which is simplified to $1 > (1-\alpha)\beta\frac{g_B}{M}$. It states that recovering one unit of consumption good from type B as a part of early payment δ is better than recovering fraction $(1-\alpha)$ of return to that unit from type B project. Hence, as δ increases in the interval $\left[0, \tilde{\delta}_k\right]$, a lower R is needed to ensure zero profit for the lender, because amount recovered per unit loaned out to type B increases, while the mix of the two types remains unchanged. Also, the intercept of the zero profit condition is given by

$$R_k(0) = \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu}\left[(1-\alpha)\beta\left(g_B + ((1-\beta)k + \beta f_B)\frac{g_B}{M}\right)\right], \tag{10}$$

which decreases in k. In fact, we see from equation (8) that the entire segment of the function shifts down as the state of the economy k increases. Intuitively, the amount recovered from type B increases in k, due to their labor income, and hence a lower interest is needed to ensure zero profit.

Consumption of type G entrepreneurs is given by $\beta[g_i + ((1-\beta)k + \beta f_i - \delta)\frac{g_i}{M}] - (2MR - \delta)$. So, the trade-off between R and δ that he faces is given by the slope of his indifference curves, $ICS \equiv -\frac{\beta\frac{g_G}{M} - 1}{2M}$. Note that the slope in independent of k.

Proposition 2, which specifies the contracts offered in equilibrium, simplifies to the following.

If $\frac{1-\beta\frac{g_G}{M}}{2M} \geq -\frac{R_k(0) - R_f}{\tilde{\delta}_k}$, then a separating contract is offered, $(\delta^*, R^*) = \left(\tilde{\delta}_k, R_f\right)$, where $\tilde{\delta}_k$ and $R_k(0)$ are given by (9) and (10). Otherwise, a pooling contract is offered, $(\delta^*, R^*) = (0, R_k(0))$.

Note that because $\frac{R_k(0) - R_f}{\tilde{\delta}_k}$ decreases in k while the slope of indifference curves does not depend on k, there is a threshold level of capital stock $\bar{k}$ given by the unique solution to $\frac{\beta\frac{g_G}{M} - 1}{2M} = \frac{R_{\bar{k}}(0) - R_f}{\tilde{\delta}_{\bar{k}}}$,[9] such that

[9] There is an analytical solution $\bar{k} = \left(\frac{(1-\mu)(2MR_f - g_B(1-\alpha)\beta)}{\mu(\beta\frac{g_G}{M} - 1) + \beta\frac{g_B}{M}(1-\mu)(1-\alpha)} - \beta f_B\right)/(1-\beta)$.

for $k > \bar{k}$, a pooling contract $(\delta^*, R^*) = (0, R_k(0))$ arises in equilibrium, while for $k \leq \bar{k}$, a separating contract $(\delta^*, R^*) = ((1-\beta)k + \beta f_B, R_f)$ arises.

There are two effects that along with competition in the banking sector deliver a pooling equilibrium for $k > \bar{k}$ and a separating one for $k < \bar{k}$. First, with high k, unproductive entrepreneurs earn higher labor income, so a higher $\tilde{\delta}_k$ is needed for separation. Recalling that δ is costly for type G as it lowers the rate of reinvestment into the second stage of production, it follows that separation is costlier for larger k. Hence, for $k > \bar{k}$, type G will prefer to pay a higher than risk-free loan rate to increase its reinvestment rate. The second effect can be explained as follows. Pooling both types into the same contract does not hurt type G as much when k is high, because the default of the unproductive producers is less costly to the bank and hence a lower loan interest, $R_k(0)$, is needed to ensure bank participation.

Figure 4 illustrates the equilibrium contract determination for two arbitrary states of the economy, k_L and k_H, where $k_L < \bar{k} < k_H$. Since $\frac{R_{k_H}(0)-R_f}{\tilde{\delta}_{k_H}} < \frac{\beta\frac{g_G}{M}-1}{2M} < \frac{R_{k_L}(0)-R_f}{\tilde{\delta}_{k_L}}$, a pooling contract is offered for k_H and a separating contract is offered for k_L.

B. Dynamics

We have up to this point discussed the type of contracts offered for a given state of the economy. We now move on to the discussion of how the economy's state evolves through time. We derive the transition function, which has two segments, each corresponding to the type of equilibrium contracts.

For $k \leq \bar{k}$, we established that separating contracts with $\delta^* = \tilde{\delta}_k = (1-\beta)k + \beta f_B$ arise and thus only type G (of measure μ) engages in production of capital. Each entrepreneur of type G produces f_G in the first stage of production and sells it at price β, he also earns wage $(1-\beta)k$. Type G then meets the early payment $\tilde{\delta}_k$ and reinvests the remaining funds $(1-\beta)k_t + \beta f_G - \tilde{\delta}_k$ into the project, which yields $g_G + [(1-\beta)k_t + \beta f_G - \tilde{\delta}_k]\frac{g_G}{M} = g_G + \beta(f_G - f_B)\frac{g_G}{M}$ units of capital, where we substituted for $\tilde{\delta}_k$. Hence, the next period capital stock is given by

$$k_S(k) = \mu\left(f_G + g_G + \beta(f_G - f_B)\frac{g_G}{M}\right), \tag{11}$$

where the subscript serves to emphasize that separating contracts facilitate this segment of the transition function.

For $k > \bar{k}$, we found that pooling contracts arise with $\delta^* = 0$ and thus both types participate in production of capital. Measure μ^2 of type G and measure $(1-\mu)\mu$ of type B obtain financing, the next period capital stock is then given by

$$k_P(k) = \mu^2\left(f_G + g_G + ((1-\beta)k + \beta f_G)\frac{g_G}{M}\right) + (1-\mu)\mu\left(f_G + g_B + ((1-\beta)k + \beta f_B)\frac{g_B}{M}\right). \tag{12}$$

Note that there is crowding out of type G here.

The capital transition function for this simple mechanism is then given by

$$k'(k) = \begin{cases} k_S(k) & \text{if } k \leq \bar{k} \\ k_P(k) & \text{otherwise} \end{cases} . \tag{13}$$

Note that $k_S'(k) = 0$ for the following reason. On the range of k leading to separation, any additional unit of capital, which translates into a rise of labor income, is paid to the bank to keep the separation viable. Consequently, none of it is reinvested to affect the next period's capital stock. We also have $k_P'(k) > 0$, because on the range of k leading to pooling contracts, $\delta^* = 0$, hence, every additional unit of labor income is reinvested and unambiguously (due to Assumption 2 augments capital production.

The following assumption ensures that when both types engage in production, an extra unit of capital, which translates into an extra $1-\beta$ units of input into the second stage of production, results in less than one unit of additional capital. This assumption ensures that there is no perpetual growth in this economy.

Assumption 7 $k_P'(k) = (1-\beta)\,\mu\left(\mu\frac{g_G}{M} + (1-\mu)\frac{g_B}{M}\right) < 1.$

How does $k_S(\bar{k})$ relate to $k_P(\bar{k})$? On the one hand, under separation, all of the productive entrepreneurs (entire measure μ) engage in production. Unproductive agents do not produce and do not crowd out type G from getting financed. On the other hand, resources must be allocated to keep separation viable, so each one of type G producers invests less in production. In this paper we focus on the case where the composition effect dominates the per agent production effect, in other words, where $k_S(\bar{k}) > k_P(\bar{k})$. So as type B get pooled into the mix, the total capital output suffers despite the fact that more resources are invested in its production (as no resources are used for separation).

Assumption 8 $k_P(\bar{k}) < k_S(\bar{k})$.

Note also that since $k_S'(k) = 0$, setting $k_P(\bar{k}) < k_S(\bar{k})$ is sufficient to guarantee that for $k \leq \bar{k}$, we have $k_P(k) < k_S(k)$.

With all the previous assumptions, equilibrium dynamics can be summarized as follows:
Case 1. $\bar{k} < k_P(\bar{k}) < k_S(\bar{k})$. The capital stock converges to $k_{ss} = k_P(\bar{k})$. See Figure 5.
Case 2. $k_P(\bar{k}) < \bar{k} < k_S(\bar{k})$. The capital stock exhibits cycles, not necessarily trivial. See Figure 6.
Case 3. $k_P(\bar{k}) < k_S(\bar{k}) < \bar{k}$. The capital stock converges to $k_{ss} = k_S(\bar{k})$. See Figure 7.

Case 2 is the focus of this paper. When capital is high, pooling contracts are offered such that both bad and good entrepreneurs choose to produce under the contract, good entrepreneurs choose to repay and bad entrepreneurs default at the end of the period. Because bad entrepreneurs participate in production thus crowding out some type G producers, the next period's capital stock is lower. When capital is low enough, the cost of separation is also low, type G prefers paying the cost of separation, the composition effect dominates, that is, even though each type G produces less (as the reinvestment rate is lower), all available funds are channeled towards type G projects, which generates a high capital stock in the next period and so on.

What drives such dynamics is that when capital is high, unproductive entrepreneurs generate higher labor income, so it takes a higher payment to separate the good entrepreneurs out and hence the productive entrepreneurs are hurt more by separation. Moreover, pooling the unproductive and productive producers into the same contract does not hurt the best entrepreneurs as much when k is high, because the default of the unproductive producers is less costly to the bank and hence a lower loan interest is needed to ensure bank participation.

V. No externalities

A. Prices and Equilibrium Contracts

We now consider the standard case with no externalities, that is $A_t \equiv A$. Then competition in the consumption goods sector implies $\rho(k') = A\beta k'^{\beta-1}$ and $w(k) = A(1-\beta)k^{\beta}$. The main change is that the price of capital ρ is no longer a constant, as in the previous section. We will show that although it introduces complications in the analysis of the model's dynamics, the main mechanism remains. In addition, it gives rise to an indeterminacy region. For all k in that region, tomorrow's level of capital k' depends on the aggregate firm's forecast of what type of equilibrium will emerge in the current period. If the aggregate firm (buyer of today's capital production) believes that separating contracts will arise and lots of capital will be produced thus paying a low price for capital, then such belief will be self-fulfilling. The reverse is also true.

The aggregate firm's belief regarding total capital production in the current period affect the price for which the capital is sold. Through this effect on the price, the belief k', then enters into the decision making of young entrepreneurs. We refer to (k, k') as the state of the economy. Given such state, we can infer equilibrium outcomes. We require that beliefs k' must be consistent with the equilibrium outcome.

With this in mind, the critical repayment to have separation (δ), and the interest rate $R_{k,k'}(0)$, implied by the banking zero profit condition with $\delta = 0$, are given by

$$\begin{aligned} \tilde{\delta}_{k,k'} &= w(k) + \rho(k')f_B, \\ R_{k,k'}(0) &= \frac{R_f}{\mu} - \frac{1-\mu}{2M\mu}(1-\alpha)\rho(k')(g_B + (w(k) + \rho(k')f_B)\frac{g_B}{M}). \end{aligned}$$

In order to determine if pooling or separating contracts are established for a given state (k, k'), we determine, as in the previous section, the "critical slope" (CS) and the slope of the indifference curve (ICS):

$$CS = -\frac{R_{k,k'}(0) - R_f}{\tilde{\delta}_{k,k'}}, \quad ICS = \frac{1 - \rho(k')\frac{g_G}{M}}{2M}.$$

Since we already established in section III that there is a pooling equilibrium iff $ICS \leq CS$, we obtain

$$\kappa(k,k') = \begin{cases} \kappa_p(k,k') & \text{if } \frac{1-\rho(k')\frac{g_G}{M}}{2M} < -\frac{R_{k,k'}(0)-R_f}{\tilde{\delta}_{k,k'}} \\ \kappa_s(k,k') & \text{otherwise} \end{cases} \tag{14}$$

where

$$\kappa_p(k,k') = \mu^2[f_G + g_G + (w(k) + \rho(k')f_G)\frac{g_G}{M}] + (1-\mu)\mu[f_B + g_B + (w(k) + \rho(k')f_B)\frac{g_B}{M}], \tag{15}$$

$$\kappa_s(k,k') = \mu[f_G + g_G + \rho(k')(f_G - f_B)\frac{g_G}{M}] \tag{16}$$

represent total capital produced at the end of the current period in case of pooling and separating contracts arising in equilibrium, respectively. In other words, the state of the economy (k,k') determines what type of contracts are offered and consequently how much capital is produced over this period. Note that beliefs k' enter these functions through the price $\rho(k')$.

It is instructive to go through some intuition. Fix k. For a very low k', and hence a high $\rho(k')$, the income upon the sale of capital produced in the first stage is high. This means that the minimum separating payment $(\tilde{\delta}_{k,k'})$ is high and $R_{k,k'}(0)$ is low (as more wealth is recovered upon default of type B). The critical slope is then very flat. The slope of type G's indifference curve, on the other hand, is steep because of the high $\rho(k')$ (the opportunity cost of every unit of early payment is high), so a greater drop in R is required to compensate for a unit rise in early payment. With CS flat and ICS steep, type G prefers a pooling contract. Note that as beliefs k' rise, the slope of the indifference $\frac{1-\rho(k')\frac{g_G}{M}}{2M}$ rises (flattens), while the critical slope $-\frac{R_{k,k'}(0)-R_f}{\delta_{k,k'}}$ declines (steepens).[10] This implies that there exists (for a fixed k) a cutoff $\tilde{k}'_k$, defined as a solution of

$$\frac{1-\rho(k')\frac{g_G}{M}}{2M} = -\frac{R_{k,k'}(0)-R_f}{\delta_{k,k'}}. \tag{17}$$

For beliefs k' below this cutoff level, pooling equilibria will arise leading to production of $\kappa_s(k,k')$ units of capital produced, and for beliefs k' above this level, separating equilibria will arise leading to $\kappa_s(k,k')$ units of capital produced. Hence, (14) simplifies to

$$\kappa(k,k') = \begin{cases} \kappa_p(k,k') & \text{if } k' < \tilde{k}'_k, \\ \kappa_s(k,k') & \text{if } k' \geq \tilde{k}'_k. \end{cases} \tag{18}$$

We derive restrictions on beliefs k' that can arise in equilibrium, in particular, we require that in accordance with the rational expectations hypothesis, equilibrium beliefs are consistent with equilibrium outcomes, i.e.,

$$\kappa(k,k') = k'. \tag{19}$$

[10] Indeed, it is obvious from the expressions above that $R_{k,k'}(0)$ rises in k' and declines in k, while $\delta_{k,k'}$ rises in k and declines in k'.

We define the transition function (or correspondence) $k'(k)$ as the solution to equation (19). In the next subsection we establish the existence of a solution to equation 19, and we examine what type of dynamics emerges in this case.

B. Dynamics

The equilibrium dynamics depend crucially on the equation $\kappa(k,k') = k'$. The left hand side of the equation gives the future level of capital as a function of current capital k and beliefs about future capital k'. These beliefs in turn have to be equal to the actual level of future capital k'.

It is useful to examine $\kappa(k,k')$. Using the expressions (15) and (16), we derive the following comparative statics results

$$\frac{\partial\kappa_p(k,k')}{\partial k'} \leq 0 \tag{20}$$

$$\frac{\partial\kappa_s(k,k')}{\partial k'} \leq 0 \tag{21}$$

$$\frac{\partial\kappa_p(k,k')}{\partial k} \geq 0 \tag{22}$$

$$\frac{\partial\kappa_s(k,k')}{\partial k} = 0 \tag{23}$$

$$\frac{d\tilde{k}'_k}{dk} \geq 0. \tag{24}$$

We obtain $\frac{d\tilde{k}'_k}{dk} \geq 0$ by differentiating equation (17) implicitly.[11] The intuition for the first two results comes through the effect of reinvestment. As k' increases, capital prices decline and so do reinvestment rates and consequently capital production. Result (22) also comes from higher reinvestment, all of the extra k is reinvested, as $\delta = 0$. In case of separating, the extra k is paid to the bank to keep separation viable, and hence the result (23). Finally, with higher k, w is higher and CS flattens because both the separating δ increases and the intercept of the zero profit condition declines (more income is recovered from B so a lower interest is needed to ensure zero profit). Hence, it takes a higher k' (lower ρ) to make the slope of the indifference curves of type G flatter.

Figure 8 illustrates $\kappa(k,k')$ as a function of beliefs k' (for a fixed k). It consists of two segments: for low beliefs k' and hence high capital price, the cost of forgone investment associated with the early payment is high, so for a given rise in δ, type G requires a greater compensation in the form of a reduction in R, that is, indifference curves are steeper. The critical slope, on the other hand, is flatter for low belief k'.

[11] We obtain

$$-\frac{\rho'(k')\frac{g_G}{M}}{2M}\frac{d\tilde{k}'_k}{dk} = -\left[\frac{\left(\frac{dR_{k,k'}(0)}{dk'}\frac{d\tilde{k}'_k}{dk}+\frac{dR_{k,k'}(0)}{dk}\right)\delta_{k,k'} - R_{k,k'}(0)\left(w'(k)+\rho'(k')f_B\frac{d\tilde{k}'_k}{dk}\right)}{\delta^2_{k,k'}}\right],$$

$$\frac{d\tilde{k}'_k}{dk}\left[-\frac{\rho'(k')\frac{g_G}{M}}{2M}+\frac{\frac{dR_{k,k'}(0)}{dk'}}{\delta_{k,k'}}-\frac{\rho'(k')f_B R_{k,k'}(0)}{\delta^2_{k,k'}}\right] = \frac{w'(k)R_{k,k'}(0)}{\delta^2_{k,k'}}-\frac{\frac{dR_{k,k'}(0)}{dk}}{\delta_{k,k'}}.$$

Because $\rho'(k') < 0$, $\frac{dR_{k,k'}(0)}{dk'} > 0$ and $\frac{dR_{k,k'}(0)}{dk} < 0$, the result follows directly.

Pooling equilibrium arises. The reverse is true for high k'. Both segments are downward sloping, as with a higher k' and hence a lower price of capital, reinvestment is lower and hence the total production is also lower.

We first make two technical assumptions (that are equivalent to assumptions 7 and 8 in the previous section). The first guarantees there is no perpetual growth, while the second guarantees the existence of equilibrium, as will be shown in the next lemma.

Assumption 9 $\frac{\partial \kappa_p(k,k')}{\partial k} < 1$.

Assumption 10 $\tilde{k}'_{k_{hi}} \geq \kappa_p(k_{hi}, \tilde{k}'_{k_{hi}})$, *where* k_{hi} *is the solution to* $\kappa_s(k, \tilde{k}'_k) = \tilde{k}'_k$.

Note that k_{hi} represents the highest k for which a separating equilibrium exists. Figure 9 is a sketch of $\kappa(k_{hi}, k')$. It illustrates Assumption 10 graphically. In light of comparative statics results $(22) - (24)$, Assumption 10 above guarantees that for all $k < k_{hi}$, $\kappa_s\left(k, \tilde{k}'_k\right) \geq \kappa_p\left(k, \tilde{k}'_k\right)$, i.e., there is a jump up in $\kappa(k, k')$ at $k' = \tilde{k}'_k$. This in turn implies existence of an equilibrium (existence of at least one intersection of $\kappa(k, k')$ with the forty five degree line. As k increases beyond k_{hi}, there is no need for the jump to be up to guarantee existence. The next lemma guarantees the existence of an equilibrium path, but also shows the possibility of indeterminacy.

Lemma 2 *If Assumption* 10 *is satisfied, then, for a fixed* k*, there are at least one and at most two solutions to* $\kappa(k, k') = k'$.

Proof. The fact that there are at most two solutions is direct in light of results (20) and (21), i.e., $\frac{\partial \kappa_p(k,k')}{\partial k'} \leq 0$ and $\frac{\partial \kappa_s(k,k')}{\partial k'} \leq 0$. Existence comes from the fact that both segments of $\kappa(k, k')$ decrease in k' together with Assumption 10. ■

Figures 10-12 illustrate that for a given level of capital k, three potential situations may arise. Either a separating equilibrium, a pooling one or both of them, depending on the belief. In that last case we have indeterminacy. In light of results (22), (23) and (24), i.e., $\frac{\partial \kappa_p(k,k')}{\partial k} \geq 0$, $\frac{\partial \kappa_s(k,k')}{\partial k} = 0$, and $\frac{dk'_k}{dk} \geq 0$, we deduce the shape of the transition function $k'(k)$. From these comparative statics results and Figures 12-14, it is clear that as k increases, we go from a region with a separating equilibrium to one where a separating and a pooling equilibria coexist, and then to one where only the pooling equilibrium exists. For a particular economy, low levels of capital can generate any of these three cases, but as k goes up, we switch to the next region in the way described before, never the reverse.

Denoting by k_{li} the solution to $\kappa_p(k, \tilde{k}'_k) = \tilde{k}'_k$, i.e. the minimum k for which pooling equilibrium emerges. Figure 13 illustrates the definition of k_{li}, which is symmetric to the earlier definition of k_{hi} (Figure 9). Note for this k, a separating equilibrium can also emerge (by result 21 and Assumption 10). Subscript li stands for the lower bound of the indeterminacy region. We already defined the notation for the solution to $\kappa_s(k, \tilde{k}'_k) = \tilde{k}'_k$ as k_{hi}. Then the subscript hi stands for the higher bound of the indeterminacy region. We can write the transition correspondence as

$$k'(k) = \begin{cases} k_S(k) & \text{if } k \leq k_{li} \\ \{k_S(k), k_P(k)\} & \text{if } k \in \{k_{li}, k_{hi}\} \\ k_P(k) & \text{if } k \geq k_{hi} \end{cases} \tag{25}$$

where $k_S(k)$ satisfies $\kappa_s(k, k_S(k)) = k_S(k)$ and $k_P(k)$ satisfies $\kappa_p(k, k_P(k)) = k_P(k)$.

Figure 14 illustrates a possible transition correspondence. Because $\kappa_s(k, k')$ is independent of k, we have that $k_S(k)$ is a constant function. Recall the intuition here is that every additional capital is used to keep separation viable. Now consider k_{hi}. At this point, due to Assumption 10, $k_P(k_{hi}) < k_S(k_{hi})$. Finally, we can show that $k_P(k)$ is increasing in k, due to results (22) and (20).[12] Note that these results also imply that $k_P(k) < k_S(k)$ for all $k \in [k_{li}, k_{hi}]$.

In the appendix we derive parametric restrictions that guarantee that all of the assumptions hold. We verified that the set of parameters satisfying these restrictions is non-empty.

VI. Conclusion

There is a widespread disagreement with regard to the importance of information frictions in credit market in driving economic fluctuations. We propose a novel reversion mechanism that is consistent with several empirical findings outlined above. We build a dynamic screening model with fully rational entrepreneurs of different types and competitive lenders. When the economy is productive (productivity evolves endogenously), signals about entrepreneurs' quality become endogenously less informative. Consequently, screening out the bad projects becomes more costly. Contracts that try to do that simply do not survive in equilibrium. Instead, contracts financing a mix of good and bad projects emerge. The composition effect due to both good and bad projects being implemented sets off a recession. The opposite happens at troughs.

Our model can give rise to endogenous (and non-trivial) fluctuations via the lending standards channel, with the strength of the entrepreneurs' signal of the quality of their project and thus the cost of screening evolving endogenously over the cycle. Aggregate productivity over this cycle changes due to the composition of types of projects being financed and implemented. This mechanism can also work in a model with exogenous shocks to productivity.

In the future we aim to embed our reversion mechanism within the general equilibrium model with infinitely lived agents and assess its importance quantitatively. Overall, there is still very little quantitative work present in this discussion, with an exception of Carlstrom and Fuerst (1997) and (1998).

VII. Appendix

Proof of Proposition 1

[12] Indeed, implicitly differentiating $\kappa_p(k, k_P(k)) = k_P(k)$ gives

$$\begin{aligned} \frac{d\kappa_p(k,k')}{dk} + \frac{d\kappa_p(k,k')}{dk'} k'_P(k) &= k'_P(k), \\ k'_P(k) &= \frac{d\kappa_p(k,k')}{dk} \Big/ \left(1 - \frac{d\kappa_p(k,k')}{dk'}\right) \geq 0. \end{aligned}$$

Suppose instead type-dependent contracts $\left(\delta_t^G, R_t^G\right) \neq \left(\delta_t^B, R_t^B\right)$ were offered and positive measures of each type self-select those contracts. Then it must be the case that $0 \leq \delta_t^B \leq \delta_t^G$, otherwise, type B would select $\left(\delta_t^G, R_t^G\right)$ as the interest rate is irrelevant for their decision (by Assumption 3) while the early payment is costly (by Assumption 2).

Case 1. If $\delta_t^G = 0$, then $R_t^G = R_f$ for contract $\left(\delta_t^G, R_t^G\right)$ to be robust to cream skimming. For B to pick $\left(\delta_t^B, R_t^B\right)$ it is necessary that $\delta_t^B = 0$. For G to pick $\left(\delta_t^G, R_t^G\right)$, it is necessary that $R_t^B > R_t^G$. By Assumption 3 Type B prefers to default. He also chooses contract $\left(\delta_t^B, R_t^B\right)$. So, his payoff from default (which is the same under both contracts here), must be greater than paying in full under both contracts. Since paying in full yields greater utility under $\left(\delta_t^G, R_t^G\right)$, it must be the case that

$$\begin{aligned}
\alpha\rho_{t+1}\left[g_B + \left(w_t + \rho_{t+1}f_B - 0\right)\frac{g_B}{M}\right] &> \rho_{t+1}\left[g_B + \left(w_t + \rho_{t+1}f_B - 0\right)\frac{g_B}{M}\right] - \left(2MR_t^G - 0\right), \text{ i.e.,} \\
2MR_t^f &> (1-\alpha)\,\rho_{t+1}\left[g_B + \left(w_t + \rho_{t+1}f_B\right)\frac{g_B}{M}\right],
\end{aligned}$$

which means the bank makes negative profits from lending to type B. Because the bank makes zero profits lending to type G, overall profit is negative. Such contracts cannot be sustained in equilibrium.
Case 2. If $\delta_t^G > 0$, then $R_t^G < R_f$ for contract $\left(\delta_t^G, R_t^G\right)$ to be robust to cream skimming. For B to pick $\left(\delta_t^B, R_t^B\right)$, it is necessary that $\delta^B \leq \delta^G$. For G to pick $\left(\delta_t^G, R_t^G\right)$, it is necessary that $R^B \geq R^G$. Then the zero lenders' profit Assumption is given by

$$\mu^2 2MR_t^G + (1-\mu)\mu\left[\delta_t^B + (1-\alpha)\rho_{t+1}\left(g_B + \left(w_t + \rho_{t+1}f_B - \delta_t^B\right)\frac{g_B}{M}\right)\right] = 2\mu MR_f,$$

Replacing R_t^G with R_f yields a strict inequality

$$\delta_t^B + (1-\alpha)\rho_{t+1}\left(g_B + \left(w_t + \rho_{t+1}f_B - \delta_t^B\right)\frac{g_B}{M}\right) > 2MR_f,$$

which means that there is a profit opportunity for the lender, as it can offer type B a contract with a lower δ^B and still make positive profit. Such contracts cannot be sustained.

Proof of Lemma 1

If $\delta_t \leq w_t + \rho_{t+1}f_B$, then by Assumption 3, Type B chooses O2 over O3, that is, O2 yields higher consumption,

$$\begin{aligned}
\alpha\rho_{t+1}[g_B + \left(w_t + \rho_{t+1}f_B - \delta_t\right)\frac{g_B}{M}] &> \rho_{t+1}\left(g_B + \left(w_t + \rho_{t+1}f_B - \delta_t\right)\frac{g_B}{M}\right) - \left(2MR_t - \delta_t\right), \text{ i.e.,} \\
2MR_t - \delta_t &> (1-\alpha)\,\rho_{t+1}\left(g_B + \left(w_t + \rho_{t+1}f_B - \delta_t\right)\frac{g_B}{M}\right).
\end{aligned}$$

Now rearranging the expression for R_t given by (5) and then using the above inequality gives the result,

$$\begin{aligned}
\mu 2MR_t + (1-\mu)\left[\delta_t + (1-\alpha)\rho_{t+1}\left(g_B + \left(w_t + \rho_{t+1}f_B - \delta_t\right)\frac{g_B}{M}\right)\right] &= 2MR_f, \\
\mu 2MR_t + (1-\mu)\left[\delta_t + \left(2MR_t - \delta_t\right)\right] &> 2MR_f, \\
R_t &> R_f.
\end{aligned}$$

Credit Line as the Alternative Form of Contracts

Identical equilibrium outcomes are achieved under the assumption of contracts in the form of a credit line with the credit limit $2M - \delta_t$ and interest rate $\frac{2MR_t - \delta_t}{2M - \delta_t}$.

In the first stage the entrepreneur borrows M and starts production (he does not borrow more since it would not be productive to do so). In the second stage he taps the credit line for the remaining $M - \delta$ and complements it with his own savings, so his disposable income is $M - \delta + \rho f_i + w$, exactly as in the paper. Finally, for this credit line he repays, in the last stage $2MR - \delta$, so the effective interest rate is $\frac{2MR - \delta}{2M - \delta}$.

Note that this analysis holds true if the bank can monitor the reinvestment in the second stage, not only of the amount tapped in the credit line, but also of at least $\rho f_B + w$ extra.

Summary of the Assumptions

We now proceed to discuss the parametric assumptions which guarantee that assumptions (3)-(5) are satisfied on an equilibrium path. Since these assumptions are inequalities that depend on the actual values of k_t on the equilibrium path, we need to find bounds for that level. We define $\underline{k}$ and $\overline{k}$ such that if $k_0 \in [\underline{k}, \overline{k}]$, $k_t \in [\underline{k}, \overline{k}]$ for all t. Certainly, there are many ways to define such $\underline{k}$ and $\overline{k}$. We define our interval as follows. For any economy (both the simple model and the general model), there is a minimum level of capital at which pooling equilibria occur. We denote it by $\underline{k}_P$.[13] Then, if we define $\underline{k} = k_P(\underline{k}_P)$ and $\overline{k} = k_S(\underline{k}_P)$ we see that for any $k_0 \in [\underline{k}, \overline{k}]$, k_t remains in that range.

► Assumption 1 is parametric,

$$f_G > f_B, \quad g_G > g_B.$$

► Assumption 2 can be written as $\frac{\rho(k) g_B}{M} \geq 1$ for all t. Since $\rho(k)$ is decreasing, we can make the parametric requirement

$$\frac{\rho(\bar{k}) g_B}{M} \geq 1. \tag{26}$$

* For the simple case it becomes

$$\frac{\beta g_B}{M} \geq 1.$$

► Assumption 3, i.e., B defaults when he enters. This must hold only whenever type B entrepreneurs can afford to enter ($\delta \leq \rho(k') f_B + w(k)$), and for any interest rate $R \geq R_f$. In fact, if for some interest rate (and any delta) this does not hold, the pooling contract could be offered at the risk-free interest rate, and

[13] In section 5, $\underline{k}_P = \tilde{k}$. In section 6, $\underline{k}_P = k_{li}$

would be preferred by type G. Noticing the most restrictive case is when $R = R_f$ and $\delta = \rho(k')f_B + w(k)$, we can write the constraint as

$$\begin{aligned}
\alpha\rho(k')[g_B + (w(k) + \rho(k')f_B - \delta)\frac{g_B}{M}] &\geq \rho(k')[g_B + (w(k) + \rho(k')f_B - \delta)\frac{g_B}{M}] - (2MR_f - \delta) \\
\alpha\rho(k')g_B &\geq \rho(k')g_B - 2MR_f + w(k) + \rho(k')f_B \\
w(k) + \rho(k')f_B + (1-\alpha)\rho(k')g_B &\leq 2MR_f
\end{aligned}$$

Since $\rho(k)$ is decreasing we set

(27) $$w(\bar{k}) + \rho(\underline{k})f_B + (1-\alpha)\rho(\underline{k})g_B \leq 2MR_f.$$

* In the simple case, this can be written as

$$(1-\beta)\,\bar{k} + \beta f_B + (1-\alpha)\beta g_B \leq 2MR_f.$$

► Assumption 4, i.e., G chooses to repay in full, must be stated twice, once for $\delta = 0$ (pooling contracts) and once for $\delta = w(k)+\rho(k')f_B$ (separating contracts). We do not need to check for $\delta \in (0, w(k)+\rho(k')f_B)$, if the constraint is not satisfied in that range, then the type G entrepreneurs are not willing to repay, and by assumption 3 nobody is willing to repay. If that is the case, that level of δ is simply not feasible. Since such a level is never selected in equilibrium anyway, the dynamics do not change.

Case 1 (pooling)

$$\begin{aligned}
\alpha\rho(k')[g_G + (w(k) + \rho(k')f_G - 0)\frac{g_G}{M}] &\leq \rho(k')[g_G + (w(k) + \rho(k')f_G - 0)\frac{g_G}{M}] - \big(2MR_{k,k'}(0) - 0\big) \\
(1-\alpha)\rho(k')[g_G + (w(k) + \rho(k')f_G)\frac{g_G}{M}] &\geq 2MR_{k,k'}(0)
\end{aligned}$$

Using the definition of $R_{k,k'}(0)$ we obtain

$$\begin{aligned}
2MR_f \leq\ & (1-\alpha)\rho(k')[\mu[g_G + (w(k) + \rho(k')f_G)\frac{g_G}{M}] \\
& +(1-\mu)[g_B + (w(k) + \rho(k')f_B)\frac{g_B}{M}]]
\end{aligned}$$

This can be written in parametric form as

$$\begin{aligned}
2MR_f \leq\ & (1-\alpha)\rho(k_P(\bar{k}))[\mu[g_G + (w(\underline{k}) + \rho(k_P(\bar{k}))f_G)\frac{g_G}{M}] \\
& +(1-\mu)[g_B + (w(\underline{k}) + \rho(k_P(\bar{k}))f_B)\frac{g_B}{M}]]
\end{aligned}$$

* In the simple case, it reduces to

$$\begin{aligned}
2MR_f \leq\ & (1-\alpha)\beta[\mu[g_G + (w(\underline{k}) + \beta f_G)\frac{g_G}{M}] \\
& +(1-\mu)[g_B + (w(\underline{k}) + \beta f_B)\frac{g_B}{M}]].
\end{aligned}$$

Case 2 (separating)

In case of a separating equilibrium ($\delta = w(k) + \rho(k')f_B$), the assumption becomes

$$\begin{aligned} \alpha\rho(k')[g_G + \rho(k')(f_G - f_B)\frac{g_G}{M}] &\leq \rho(k')[g_G + \rho(k')(f_G - f_B)\frac{g_G}{M}] - \left(2MR_f - w(k) - \rho(k')f_B\right) \\ (1-\alpha)\,\rho(k')[g_G + \rho(k')(f_G - f_B)\frac{g_G}{M}] &\geq 2MR_f - w(k) - \rho(k')f_B \end{aligned}$$

We set

$$(1-\alpha)\,\rho(\bar{k})[g_G + \rho(\bar{k})(f_G - f_B)\frac{g_G}{M}] \geq 2MR_f - w(\underline{k}) - \rho(\bar{k})f_B$$

Note that this assumption is not overly restrictive as the only consistent belief is $k' = \bar{k}$ (regardless of k).

* For the simplest case we have

$$(1-\alpha)\,\beta[g_G + \beta(f_G - f_B)\frac{g_G}{M}] \geq 2MR_f - (1-\beta)\,\underline{k} - \beta f_B.$$

► Assumption 5, both participate when they can afford δ. We need to specify a condition for B under pooling and for G under both, pooling and separating.

Case 1. B, pooling

We need that, for any $\delta \in [0, w(k) + \rho(k')f_B)$

$$\alpha\rho(k')[g_B + (w(k) + \rho(k')f_B - \delta)\frac{g_B}{M}] \geq w(k)$$

Since this condition is the most restrictive when $\delta = w(k) + \rho(k')f_B$, we get

$$\alpha\rho(k')g_B \geq w(k)$$

We set (here we use $k \in \left[\underline{k}, \bar{k}\right], k' \in \left[\underline{k}, k_P\left(\bar{k}\right)\right]$)

$$\alpha\rho(\kappa_P(\bar{k}))g_B \geq w(\bar{k}).$$

* In the simplest case, this reduces to

$$\alpha\beta g_B \geq (1-\beta)\,\bar{k}$$

Case 2. G, pooling

Notice first that the utility of a type G entrepreneur in a pooling equilibrium is the utility of repaying, and this is bigger than the utility of defaulting (by assumption 4). Since this utility of defaulting is bigger than the utility of a type B entrepreneur that defaults (since type G is more productive in both subperiods), and this utility is in fact bigger than $w(k)$, the reservation utility, by case 1, we can conclude without extra assumptions that agent 1 always participates when there is a pooling equilibrium.

Case 3. G, separating equilibrium. The critical condition becomes

$$\begin{aligned} \rho(k')[g_G + \rho(k')(f_G - f_B)\frac{g_G}{M}] - \left(2MR_f - w(k) - \rho(k')f_B\right) &\geq w\,(k) \\ \rho(k')[g_G + \rho(k')(f_G - f_B)\frac{g_G}{M} + f_B] &\geq 2MR_f \end{aligned}$$

We set

$$\rho(\bar{k})[g_G + \rho(\bar{k})(f_G - f_B)\frac{g_G}{M} + f_B] \geq 2MR_f \tag{28}$$

Note that this assumption is not overly restrictive as the only consistent belief possible is $k' = \bar{k}$ (regardless of k).

* In the simplest case, this becomes

$$\beta[g_G + \beta(f_G - f_B)\frac{g_G}{M} + f_B] \geq 2MR_f.$$

▶ Assumption 6, in the simplified model is $1 > (1-\alpha)\,\beta\frac{g_B}{M}$.

▶ Its equivalent in the general model is $1 > (1-\alpha)\,\rho_{t+1}\frac{g_B}{M}$. We require that

$$1 > (1-\alpha)\,\rho\left(\underline{k}\right)\frac{g_B}{M}.$$

▶ Assumption 7, in the simplified model, is already parametric, $(1-\beta)\,\mu\left(\mu\frac{g_G}{M} + (1-\mu)\,\frac{g_B}{M}\right) < 1$.

▶ Its equivalent in the general model, Assumption 9 requires that

$$\frac{\partial\kappa_p(k,k')}{\partial k} = \mu\left[\mu\frac{g_G}{M} + (1-\mu)\frac{g_B}{M}\right]w'(k) < 1,$$

which is implied by

$$\kappa_P'(k) = \mu\left[\mu\frac{g_G}{M} + (1-\mu)\frac{g_B}{M}\right]A(1-\beta)\beta\underline{k}^{\beta-1} < 1. \tag{29}$$

▶ Assumption 8, in the simplified model, is already parametric.

▶ Its equivalent in the general model is Assumption 10 , $\tilde{k}'_{k_{hi}} \geq \kappa_p(k_{hi}, \tilde{k}'_{k_{hi}})$, where k_{hi} is the solution to $\kappa_s(k, \tilde{k}'_k) = \tilde{k}'_k$ and $\tilde{k}'_k$ solves $\frac{1-\rho(k')\frac{g_G}{M}}{2M} = -\frac{R_{k,k'}(0)-R_f}{\delta_{k,k'}}$. It is also parametric.

References

Asea, Patrick K. and Brock Blomberg (1998), "Lending cycles," *Journal of Econometrics*, Elsevier, vol. 83(1-2), pages 89-128.

Bernanke, Ben and Gertler, Mark. (1989), "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79(1), pages 14-31.

Berger, Allen N. and Gregory F. Udell (2003), "The institutional memory hypothesis and the procyclicality of bank lending behavior," *Journal of Financial Intermediation,* vol. 13, issue 4, October 2004, Pages 458-495.

Carlstrom, Charles and Fuerst, Timothy, (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, 87(5), pages 893-910.

Dell'Ariccia, Giovanni and Robert Marquez (2006), "Lending Booms and Lending Standards," *Journal of Finance*, vol. 61(5), pages 2511-2546, October.

Kiyotaki, Nobuhiro & Moore, John, (1997), "Credit Cycles," *Journal of Political Economy*, 105(2), pages 211-48.

Lown, Cara and Morgan, Donald P. (2006), "The Credit Cycle and the Business Cycle: New Findings Using the Loan Officer Opinion Survey," *Journal of Money, Credit and Banking*, 2006, vol. 38, issue 6, pages 1575-1597.

Rampini, Adriano (2004), "Entrepreneurial Activity, Risk, and the Business Cycle", *Journal of Monetary Economics*, 51(3), pages 555,573.

Reichlin, Pietro and Siconolfi, Paolo, (2004), "Optimal Debt Contracts and Moral Hazard Along the Business Cycle", *Economic Theory*, 24(1), pages 75-109.

Suarez, J. and O. Sussman, (1997), "Endogenous Cycles in a Stiglitz-Weiss Economy", *Journal of Economic Theory*, 76, 47-71.

Stiglitz J. and A. Weiss (1981), "Credit Rationing in Markets with Imperfect Information", *American Economic Review*, 71, 393-410.

FIGURE 1.

Charge-off and delinquency rates lag after the cycle.

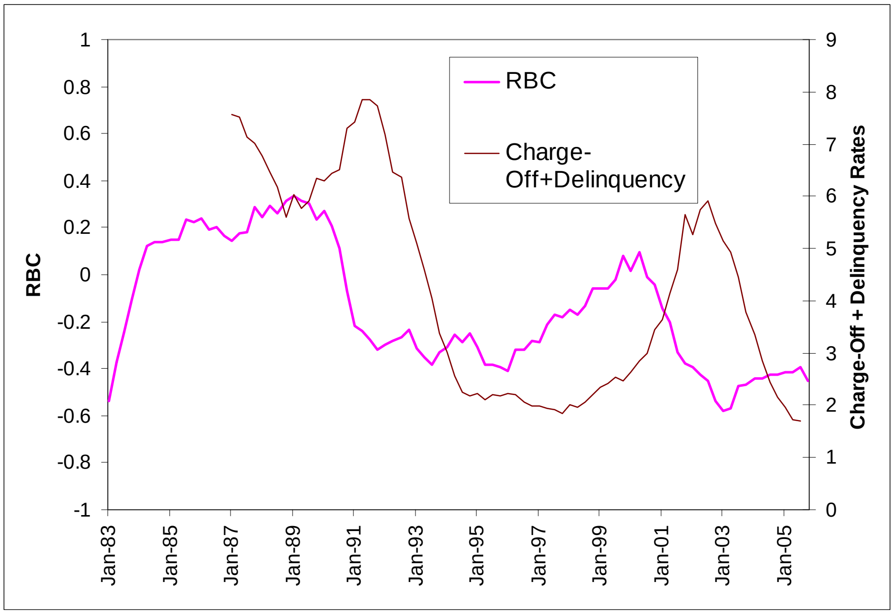

FIGURE 2.

Value of outstanding loans and mortgages is procyclical.

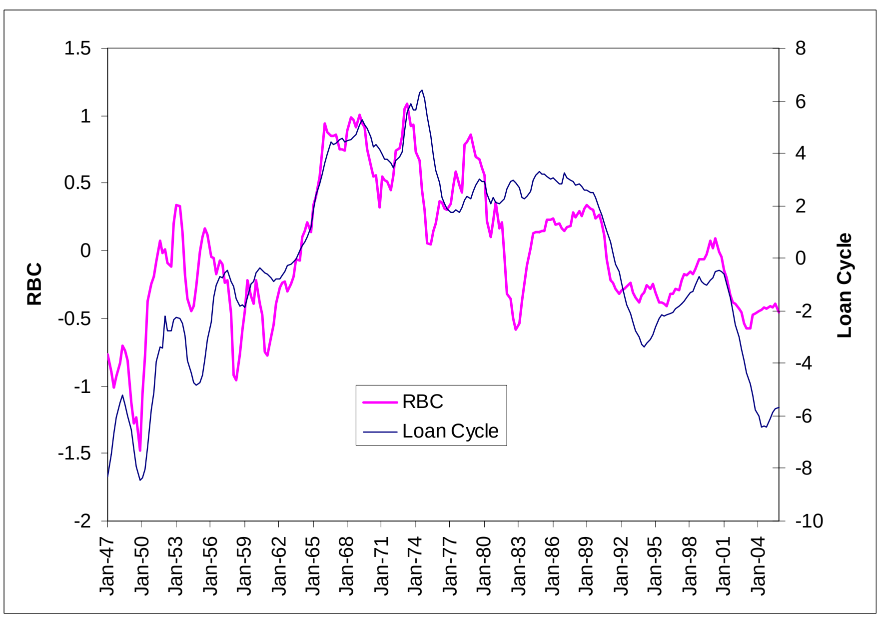

Figure 3.

General Model. Equilibrium Contract Determination (for given w_t and ρ_{t+1}).

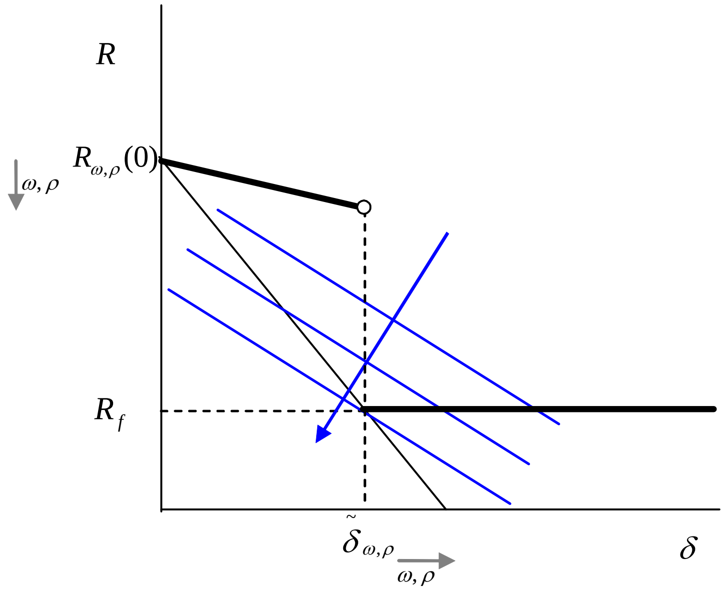

Figure 4.

Simplified Model. Equilibrium Contract Determination, 2 states, $k_H > k_L$.

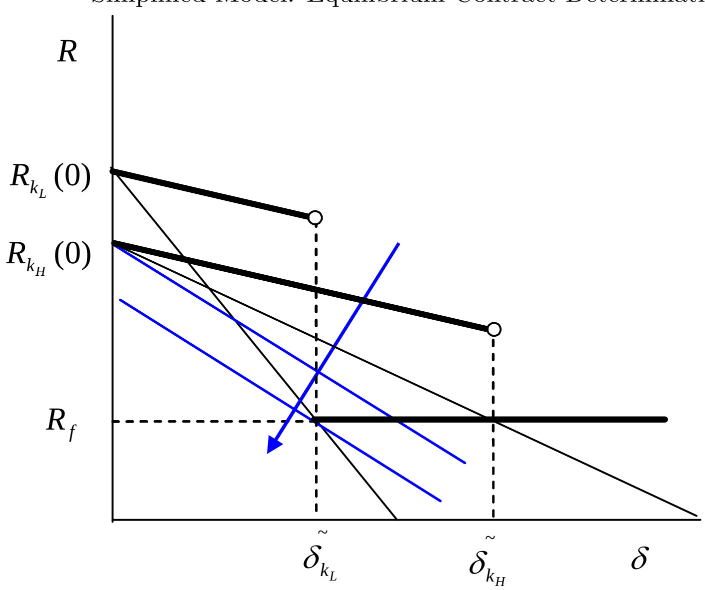

FIGURE 5.
Transition function $k'(k)$ in the simplified model,
$\bar{k} < k_P(\bar{k}) < k_S(\bar{k})$, only pooling contracts emerge in the long run.

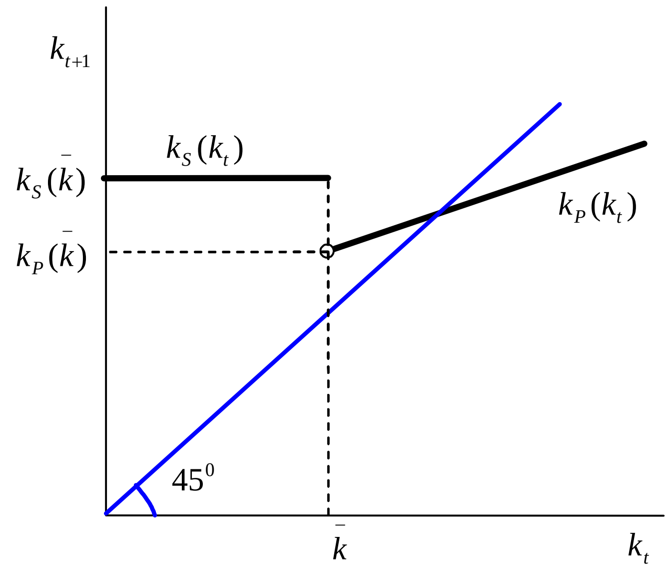

FIGURE 6.
Transition function $k'(k)$ in the simplified model,
$k_P(\bar{k}) < \bar{k} < k_S(\bar{k})$, endogenous cycles.

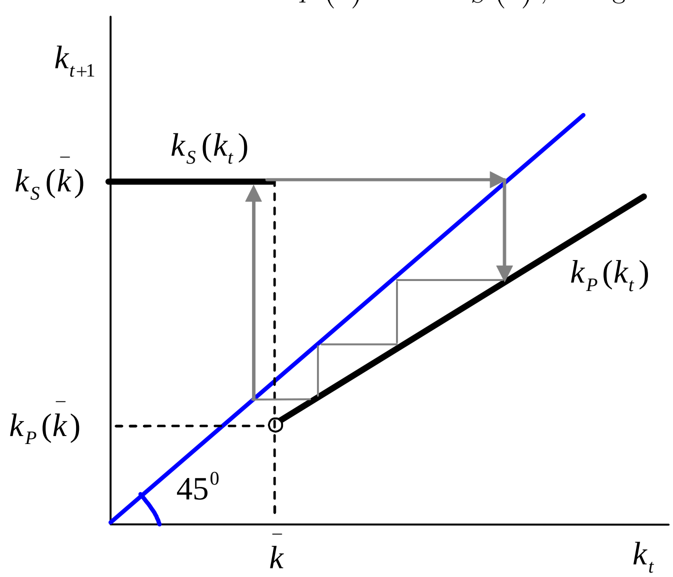

FIGURE 7.
Transition function $k'(k)$ in the simplified model,
$k_P(\bar{k}) < k_S(\bar{k}) < \bar{k}$, only separating contracts emerge in the long run.

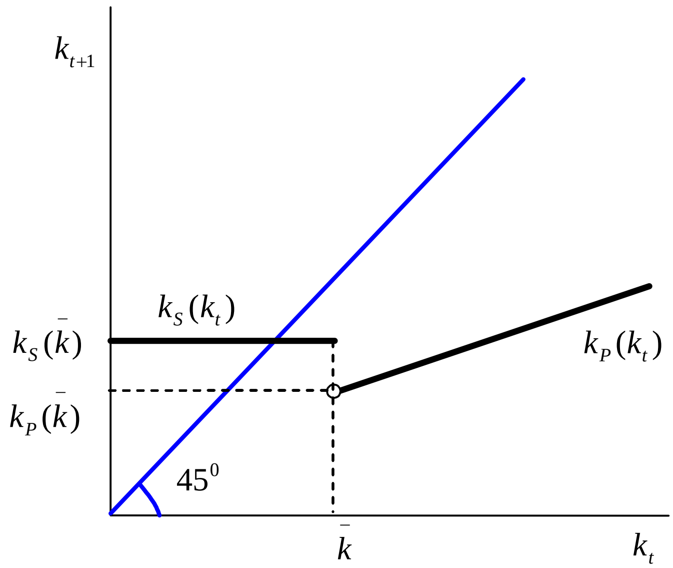

FIGURE 8.
General Model. Illustration of $\kappa(k, k')$, for a fixed k,
$\tilde{k}'_k$ is defined as a solution to $\frac{1-\rho(k')\frac{g_G}{M}}{2M} = -\frac{R_{k,k'}(0)-R_f}{\delta_{k,k'}}$.

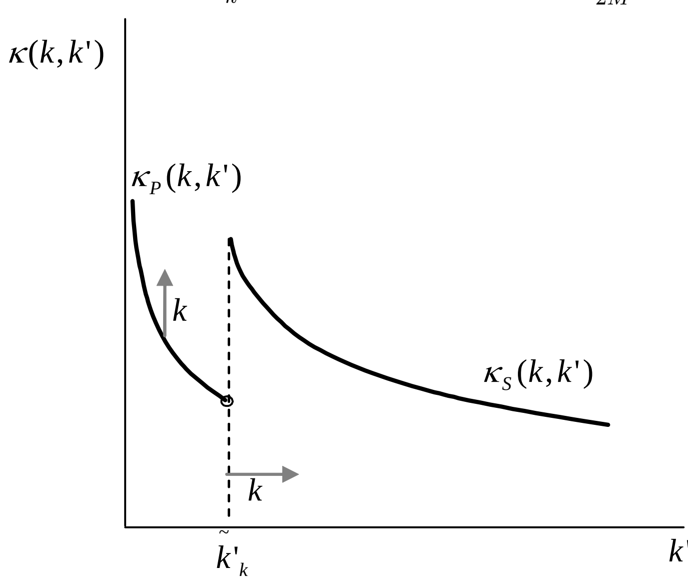

Figure 9.

Illustration of Assumption 10, i.e., $\tilde{k}'_{k_{hi}} \geq \kappa_p(k_{hi}, \tilde{k}'_{k_{hi}})$, where k_{hi} is the max capital for which a separating contract can emerge, defined as a solution to $\kappa_s(k, \tilde{k}'_k) = \tilde{k}'_k$.

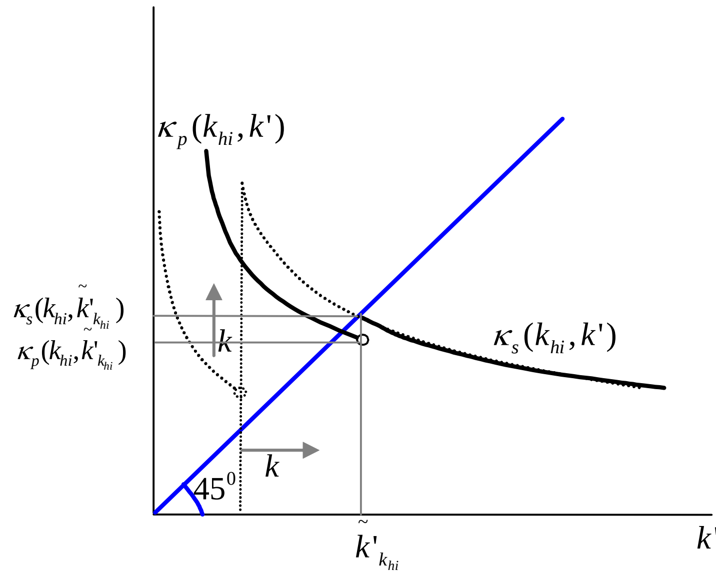

Figure 10.

General Model. Determination of $k'(k_L)$, for some low level k_L.

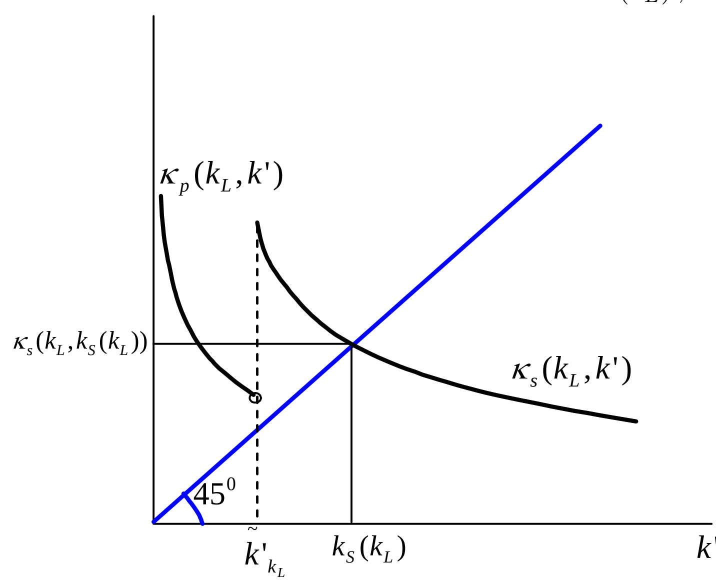

Figure 11.

General Model. Determination of $k'(k_M)$, for some level k_M in the region of indeterminacy.

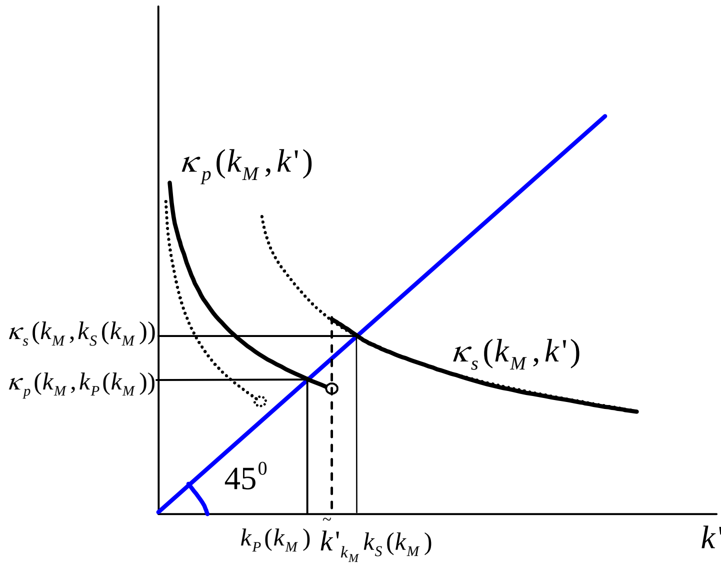

Figure 12.

General Model. Determination of $k'(k_H)$, for some high level k_H.

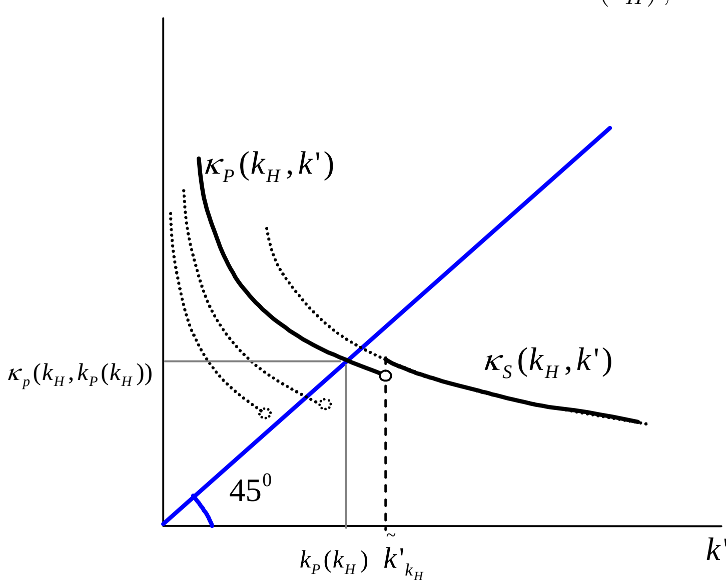

FIGURE 13.

k_{li} is the min capital for which a pooling contract can emerge, defined as a solution to $\kappa_p(k, \tilde{k}'_k) = \tilde{k}'_k$.

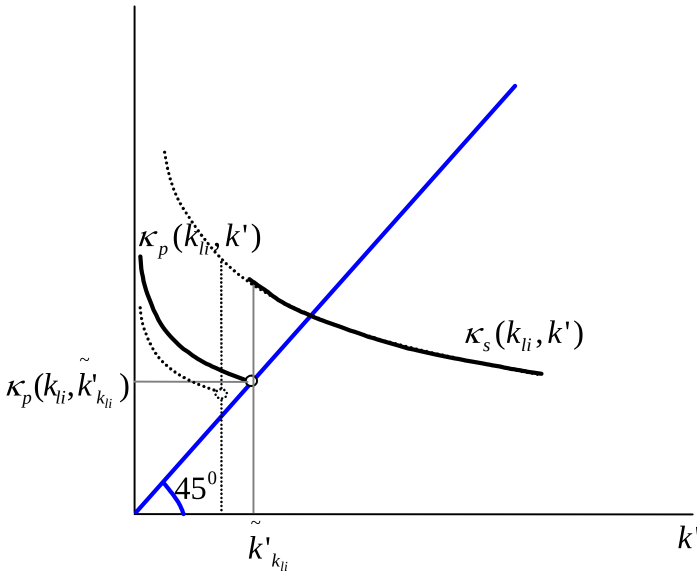

FIGURE 14.

General Model. Transition Correspondence.