OR 722 : Integer Programming

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Time and place: TBA

What is common in the following problems?

- Finding a "still life", an "oscillator" and a "spaceship" in Conway's game of life ?
- Planning the takeoff and landing of airplanes to minimize the congestion at airports?
- Finding the smallest number for which you cannot make change using given denominations $d_1, \ldots, d_k$?
- Partitioning the nodes of a graph into two equal size subsets to minimize the weight of the edges in between?
- Given n cities with pairwise distances, finding a tour of minimum total distance that visits all of them, and returns to the origin (the famous *traveling salesman problem*?
- Designing a network of minimum cost, where some nodes *must* be connected, and some others *may* or *may not* be connected – i.e. the Steiner-tree problem?

Answer: all these problems can be modeled and solved using **integer programming**.
Topics to be covered:

- "Usual" IP models in location, telecommunications ...

- and less usual ones:
  - How to find covering designs (think of it as buying the minimum number of lottery tickets so as to ensure at least 4 hits) ?
  - How to plan a party progressing from one place to another so that designated people spend at least a given amount of time together?
- The usage of modeling, and solution software – CPLEX and AMPL. There will be a strong emphasis on solving practical problems.
- What can and what cannot be modeled using IP – representation theory.
- Necessary background in linear programming: projection, Fourier-Motzkin elimination.
- Algorithms:
  - Branch-and-bound.
  - Cutting planes based on various geometric principles. "How many" cuts do we need to solve a problem?
  - Algorithms based on the "geometry of numbers": branching in "thin" directions, basis reduction methods.
  - Methods to solve systems of diophantine equations.
- Theory of IP:
  - Polyhedral theory for IP: what are "good" inequalities to use when solving an IP.
  - The complexity of IP: NP-hardness of various aspects.
  - Totally unimodular matrices: which IPs can be solved as linear programs?

Prerequisites, and course organization:

- STOR612 or equivalent. The knowledge of linear programming, especially duality theory is very important, and it is very useful to have seen *some* integer programming before.
- Grading: mostly based on homeworks. Possibly there will be class presentations as well.