Content-based image retrieval using Legendre chromaticity distribution moments

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Abstract: It is a well-known fact that the direct storing and comparison of the histogram for the purpose of content-based image retrieval (CBIR) is inefficient in terms of memory space and query processing time. It is shown that the set of Legendre chromaticity distribution moments (LCDM) provides a compact, fixed-length and computation effective representation of the colour contents of an image. Only a small fixed number of compact LCDM features need to be stored to effectively characterise the colour content of an image. The need to store the whole chromaticity histogram is circumvented. Consequently the time involved in database querying is reduced. It is also shown that LCDM can be computed directly from the chromaticity space without first having to evaluate the chromaticity histogram.

1 Introduction

With the rapid growth of the internet, online collections of images are more common and are increasingly becoming larger. Hence, there is a need for tools to efficiently manage, organise and navigate through them [1]. Content-based image retrieval (CBIR) generally refers to the method of retrieving images from image databases based on the visual properties such as dominant colour, colour distribution, textures, shapes, objects and motion of images. Queries in CBIR can be based on: example images [2], sketches and drawings [3], colour and texture patterns [1], camera and object motion [4], semantics [5–8], and other graphical information.

The colour content of an image is an important element in content-based image retrieval. Research on the utilisation of colour for image retrieval include the original work by Swain and Ballard [9], Photobook [10], IBM's QBIC Project [1], VisualSEEK [11], coherence vector [12], colour correlograms [13], work on vector wavelets by Albuz *et al.* [14], human perceptual colour clustering by Gong *et al.* [15], Han *et al.*'s work on fuzzy colour histogram [16], chromaticity moments by Paschos *et al.* [2], and frequency layered colour indexing by Qiu *et al.* [17]. In particular, Swain and Ballard [9] proposed a simple yet novel image indexing method which is based on chromaticity histogram. The histogram is a valuable cue for image indexing. This is due to the fact that, histograms are invariant to translation and rotation about an axis perpendicular to an image plane, and vary only slowly under change of perspective, change in scale and also occlusion. However, storing the histogram directly as a feature vector for the purpose of image database indexing is not practical, the reason for which follows. In the case of colour indexing, chromaticity histograms are created by partitioning the chromaticity space into uniform regions and then assigning the total number of pixels with chromaticity values within each particular region to the related histogram bin. If the number of bins per dimension is Q, the length of the feature vector based on the chromaticity histogram is therefore Q^2. Hence, if $Q = 10$, the length of the feature vector is $Q^2 = 100$. Clearly this is impractical both in terms of storage space and computation time.

Our main aim in this paper is to present a method to model the image chromaticity characteristics by using a small number of features for efficient image databases indexing. We show that the chromaticity histogram does not need to be constructed to achieve this. Hence, the chromaticity space is not quantised into histogram bins as in previous methods. In our method, we capture the characteristics of the colour contents of an image by using Legendre chromaticity distribution moments (LCDM) directly from the chromaticity space. LCDM serve as a set of low-level features which can be easily incorporated into more complex schemes to work together with other features such as texture descriptors and shape descriptors to reduce the occurrence of false positives and to capture image semantic contents.

Representing the chromaticity distribution by LCDM makes similarity comparison using simple distance functions more efficient. Stricker and Orengo [18] show that a slight shift in the histogram may cause false negatives and false positives when L_1 and L_2 metrics are used as distance measures, respectively. LCDM are global descriptors which serve as a measure of correlation in the chromaticity space. Hence, when using LCDM as feature vectors for similarity comparison, we are in fact comparing the correlation of the chromaticity distribution instead of the chromaticity distribution itself. We conjecture that the correlation of the chromaticity distribution is less likely to change when compared with the chromaticity distribution itself. This, together with the fact that only a small amount of LCDM (typically less than 10) is needed for effective indexing, makes colour-content comparison with simple distance functions feasible and effective; this is especially important when the size of the image databases is large [19].

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This research is motivated by Teague's work on Legendre moments [20] and Paschos *et al.*'s work on chromaticity moments [2]. Teague introduced the notion of orthogonal moments and, in particular, the set of Legendre moments. The set of Legendre moments is well-known for its capability to represent signal functions in compact form with minimal information redundancy. We show that instead of using regular moments [21] as is done in Paschos *et al.*'s method, the utilisation of the set of Legendre moments as feature vectors in image database indexing gives better retrieval performance. There are a few significant distinctions in our proposed method as compared with previous methods:

- We use Legendre moments instead of regular moments. This is due to the fact that Legendre moments [20, 22, 23] belong to the class of orthogonal moments and aremore compact in terms of information energy. Consequently the chromaticity distribution can be represented more efficiently by a small number of LCDM. This has a direct impact on both the retrieval time and storage space needed to save the feature vectors of the image database.
- In our method, the chromaticity space is not quantised as in other methods such as [2, 9, 16, 18, 24]. We show that LCDM can be computed from the chromaticity space without first quantising it.
- We show that LCDM can be obtained directly from the chromaticity space without having first to construct the histogram.
- In Paschos *et al.*'s [2] work on chromaticity moments, regular moments of both the trace and distribution of the chromaticity space are used as features for image database indexing. With LCDM it is sufficient to consider only the chromaticity distribution. Additionally, we use the opponent chromaticity space instead of the CIE (Commission Internationale de l'Eclairage) chromaticity space as in [2].
- Mandal *et al.* [25] proposed the use of Legendre moments, but only with histogram of image grey-level values. For this case, the problem of quantisation naturally does not occur due to the fact the grey-level values are already quantised (they are integer values). This is different from the case of colour indexing, where the chromaticity values are often noninteger values. For example, the RGB value (100, 200, 30) corresponds to value (−0.3030, 0.3636) in the opponent chromaticitiy space. Hence, this makes obtaining the frequency count of each chromaticity value more difficult and 'binning' need to be performed beforehand.

Experimental studies by utilising an image database consisting of images from the SIMPLIcity image database [8] shows that LCDM is comparable in terms of retrieval accuracy to the full chromaticity histogram. Moreover, the time needed to index the image database is relatively small because the set of LCDM is computationally inexpensive. Experimental studies also validate the fact that the time needed to retrieve an image from the database is relatively small. This is due the small-feature vector length and is direct result of information compactness of LCDM.

2 Proposed method

A brief theory on Legendre moments [20] is first given and we then show how the Legendre moments can be modified to obtain the set of LCDM.

2.1 Legendre moments

The Legendre moments of order $(m+n)$ are defined as

$$\lambda_{mn} = \frac{(2m+1)(2n+1)}{4}\int_{-1}^{1}\int_{-1}^{1} P_m(x)P_n(y)f(x,y)dxdy \quad (1)$$

where $m,\ n = 1,\ 2,\ 3, \ldots, \infty$. The nth-order Legendre polynomials is defined as

$$P_n(x) = \sum_{j=0}^{n} a_{nj}x^j = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n \quad (2)$$

Legendre polynomials up to the sixth order are as follows:

$$P_0(x) = 1$$

$$P_1(x) = \frac{1}{2}x$$

$$P_2(x) = \frac{1}{2}(3x^2-1)$$

$$P_3(x) = \frac{1}{2}(5x^3-3x)$$

$$P_4(x) = \frac{1}{8}(35x^4-30x^2+3)$$

$$P_5(x) = \frac{1}{8}(63x^5-70x^3+15x)$$

$$P_6(x) = \frac{1}{16}(231x^6-315x^4+105x^2-5)$$

The set of Legendre polynomials $\{P_n(x)\}$ form a complete orthogonal basis set on the interval $[-1, 1]$

$$\int_{-1}^{1} P_n(x)P_m(x)dx = \frac{2}{2n+1}\delta_{nm} \quad (3)$$

This property of Legendre polynomials is important to ensure information compactness and minimal information redundancy of the Legendre moments. The relation between Legendre polynomials and monomials $\{x^n\}$, which forms the kernel of regular moments, is given in the Appendix (Section 6). The orthogonality, and hence uncorrelatedness, of Legendre polynomials is the primary reason we choose Legendre moments in place of regular moments for the characterisation of the chromaticity space. By the orthogonality principle, the image function $f(x, y)$ can be written as an infinite series of expansion in terms of Legendre polynomials over the square $[-1 \le x, y \le 1]$

$$f(x,y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\lambda_{mn}P_m(x)P_n(y) \quad (4)$$

where the Legendre moments $\{\lambda_{mn}\}$ are computed over the same square. If only Legendre moments of order $m+n \le D$ are given, the function $f(x, y)$ can be approximated by a continuous function which is a truncated series

$$\hat{f}(x,y) = \sum_{m=0}^{D}\sum_{n=0}^{m}\lambda_{m-n,n}P_{m-n}(x)P_n(y) \quad (5)$$

If only Legendre moments of order $m \le m_{max}$ and $n \le n_{max}$ are given, then we have

$$\hat{f}(x,y) = \sum_{m=0}^{m_{max}}\sum_{n=0}^{n_{max}}\lambda_{m,n}P_m(x)P_n(y) \quad (6)$$

Very often, depending on the nature of the application, finite orders of Legendre moments are sufficient to characterise the $f(x, y)$.

2.2 Opponent chromaticity space

The most common representation for digital images is the RGB colour space. The direct utilisation of the (R, G, B) triplet for image indexing is unreliable due to its susceptibility to change of brightness. Hence, the (R, G, B) triplet are mapped to brightness independent chromaticities prior to indexing

$$r = \frac{R}{R+G+B} \qquad g = \frac{G}{R+G+B} \qquad b = \frac{B}{R+G+B} \tag{7}$$

Since $b = 1 - r - g$, the two-dimensional chromaticities (r, g) is sufficient to describe the colour content of the image. To avoid the case of $R = G = B = 0$, which will cause an undefined division, we utilise the transformations $R \to R + \delta$, $G \to G + \delta$, $B \to B + \delta$, and modify the equation to

$$r = \frac{R+\delta}{R+G+B+3\delta} \qquad g = \frac{G+\delta}{R+G+B+3\delta} \qquad b = \frac{B+\delta}{R+G+B+3\delta} \tag{8}$$

where δ is an arbitrary small number. The brightness independence property is not affected since $R, G, B \gg \delta$. In our case, we choose $\delta = 0.01$.

In colour research it is well known that a primal encoding of colour in RGB co-ordinates is less uniform than an opponent encoding [26]. The opponent colour space has three axes: white-black, yellow-blue and red-green axes. Each axis is mutually uncorrelated and so conveys independent information. The opponent chromaticity space

$$(rg, yb) = \left(r - g, \frac{r}{2} + \frac{g}{2} - b\right) \tag{9}$$

is more uniform and so can be more efficiently characterised by the Legendre moments (see following subsection). Note that the white-black axis is not used since it encodes brightness information and will not be included in the chromaticity histogram. It is straightforward to show that $-1 \le rg \le 1$ and $-1 \le yb \le 1$.

2.3 Legendre chromaticity distribution moments (LCDM)

Conventionally, chromaticity space is divided in uniform regions and the pixel counts are assigned to the related histogram bins. However, we take a different approach and formulate LCDM in such a way that it can be computed directly from the chromaticity space, i.e. without having first to construct the chromaticity histogram. Consider an $M \times N$ colour image $I(i, j) = [R(i, j), G(i, j), B(i, j)]$ where $i = 0, 1, \ldots, M-1, j = 0, 1, \ldots, N-1$. Mapping the RGB triplets to the opponent chromaticity space, we have $I(i, j) = [rg(i, j), yb(i, j)]$ or more concisely $I(k) = [rg(k), yb(k)]$, where $k = 0, 1, \ldots, MN-1$. If we define the distribution of its chromaticity space as $C(x, y)$, we have

$$C(x, y) = \text{number of pixels with chromaticity value } (x, y) = \sum_{k=0}^{MN-1} \delta(x, rg(k))\delta(y, yb(k)) \tag{10}$$

where $\delta(u, v)$ is the Kronecker delta

$$\delta(u, v) = \begin{cases} 1 & u = v \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Now, we define LCDM λ^c_{mn} as the Legendre moments of $C(x, y)$, with $(m + n)$ being the order. Using the discrete form of (1) (see Appendix, Section 6.2), substituting $f(x, y)$ with $C(x, y)$, and letting $(x_k, y_k) \in \{(x, y)|C(x, y) \ne 0\}$, we have

$$\begin{aligned}\lambda^c_{mn} &= A_{mn} \sum_k P_m(x_k)P_n(y_k)C(x_k, y_k) \\ &= A_{mn} \sum_k \sum_{l=0}^{MN-1} P_m(x_k)P_n(y_k)\delta(x_k, rg(l))\delta(y_k, yb(l)) \\ &= A_{mn} \sum_{l=0}^{MN-1} P_m(rg(l))P_n(yb(l))\end{aligned} \tag{12}$$

where $A_{mn} = (2m+1)(2n+1)/MN$. The LCDM is calculated directly using the last equation of (12) and there is no histogram construction process. Generally, the lower order and LCDM represents slow-changing components in the chromaticity space while the higher order represents the fast changing components. In our experiment, λ^c_{00} is left out because its value is a constant regardless of the image. In the following Section, the performance of LCDM is evaluated.

3 Experimental results

Experiments are performed to gauge the performance of LCDM. Results for chromaticity histogram (HIST) [9] and chromaticity moments (CM) [2] where appropriate. Both HIST and CM are modified to work in the opponent chromaticity space.

3.1 Image database

The image database are taken from the COREL photograph data set, which is used in [8]. The database consists of 1000 colour images which are stored in JPEG format with size 384 × 256 or 256 × 384. The images can be divided into ten categories, each consisting of 100 images, as listed in

Table 1: Categories of images

ID	Category name
1	African people and villages
2	Beaches
3	Buildings
4	Buses
5	Dinosaurs
6	Elephants
7	Flowers
8	Horses
9	Mountains and glaciers
10	Food

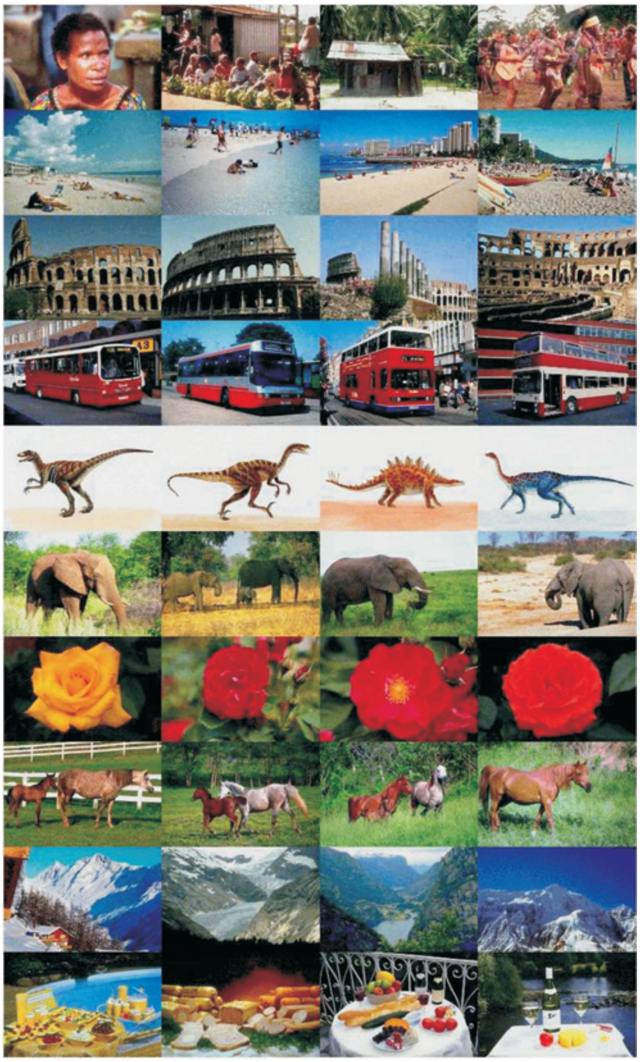

Fig. 1 *Samples images from database*

Table 1. Some sample images are shown in Fig. 1. Most categories simply include images containing the specific objects. Images in 'African people and villages' class are those depicting African people in various activities in their village. The 'Buildings' class refers to a variety of buildings of different architectural styles. The 'Beach' class refers to scenery of coasts. We assume that the images in each category have similar chromaticity distribution and colour descriptors such as LCDM, HIST and CM are used to extract the characteristics of chromaticity distributions and are then compared for image retrieval. Each image is rescaled to a standard size of 128 × 128 before its chromaticity contents are extracted.

3.2 Method of evaluation

For each image the set of LCDM, HIST and CM are calculated, resulting in a feature vector $\mathbf{V} \in \mathcal{R}^L$, where L is the length of the feature vector. The feature vectors for all the images are stored and when a query image is presented, the feature vector of the query image is compared to that of the images in the database. A retrieved image is considered a match if it belongs to the same category of the query image. To gauge the retrieval accuracy we use the recall–precision–fallout measure as proposed in [27].

We denote the query image as I_{query} and the respective retrieved image as $I_{retrieved}$. For each query image I_{query}, Q images $I^{(k)}_{retrieved}$, $k = 1, 2, \ldots, Q$, with the smallest distances $d(I_{retrieved}, I_{query})$ are retrieved. The distance function

$$d(I_{retrieved}, I_{query}) = \sum_{k=1}^{L} \left| [V_k]_{I_{query}} - [V_k]_{I_{retrieved}} \right| \quad (13)$$

is utilised throughout the experiments. The retrieved images are denoted in such a way that

$$d(I^1_{retrieved}, I_{query}) \le d(I^2_{retrieved}, I_{query}) \le \cdots \le d(I^Q_{retrieved}, I_{query}) \quad (14)$$

If we let R be number of the correctly matched images, that is

$$R = \left| \{ I^{(k)}_{retrieved} : \text{class}(I^{(k)}_{retrieved}) = \text{class}(I_{query}), \quad k = 1, 2, \ldots, Q \} \right| \quad (15)$$

where $|A|^1$ denotes the cardinal number pertaining to set A and $\text{class}(I^{(k)}_{retrieved})$ the category of the image $I^k_{retrieved}$. (The symbol $|\cdot|$ is used to represent both the absolute value and also the cardinal number. Its actual meaning should be clear from the context.) The recall rate can then be defined as

$$r(I_{query}) = \frac{R}{N_{class}} \quad (16)$$

the precision as

$$p(I_{query}) = \frac{R}{Q} \quad (17)$$

and the fallout rate as

$$f(I_{query}) = \frac{Q - R}{N_{total} - N_{class}} \quad (18)$$

where N_{class} is the number of images of the category where I_{query} is taken from, and N_{total} the total number of images in the database. Assuming $N_{total} - N_{class} > N_{class} > Q$, we have

$$0 \le r \le \frac{Q}{N_{class}} < 1, \quad 0 \le p \le 1, \quad 0 \le f \le \frac{Q}{N_{total} - N_{class}} < 1 \quad (19)$$

An accurate retrieval basically means high values for recall rate and precision and low value for fallout rate.

3.3 Lengths of feature vectors, L

The length of the feature vector for each image is crucial and directly determines the retrieval time needed for each query. The length of the feature vector for HIST, CM and LCDM are Q^2, $(D+1)$ $(D+2)$ and $D(D+3)/2$, respectively. Just a small number of LCDM are needed to characterise the major colour content of an image. Experimental support is given subsequently. Hence, by utilising LCDM as feature vectors, a significant amount of storage space can be saved, especially when compared with HIST. The storage space needed is reduced by a factor of $k = 2Q^2/[D(D+3)]$. In addition, the smaller length of feature vector also means that the time needed for similarity comparison is less. The values of the lengths of the feature vectors are summarised in Table 2. Table 3 shows how the length of the feature vector using HIST increases dramatically with the increasing of Q. It can be seen that the

Table 2: Lengths of feature vectors of LCDM, HIST and CM

	Length of feature vector, L	Typical value
LCDM	$P(P+3)/2$	5, $P=2$
HIST	Q^2	100, $Q=10$
CM	$(P+1)(P+2)$	12, $P=2$

Table 3: Length of feature vector, L_{HIST}, for HIST

Bins, Q	2	4	8	16	32	64
L_{HIST}	4	16	64	256	1024	4096

feature length can easily get out of control. Table 4 shows how the feature lengths of LCDM and CM change with D. The feature length of CM is approximately two times that of LCDM for each particular value of D.

Table 4: Length of feature vector, L_{LCDM} and L_{CD} for LCDM and CDM

Maximum order, D	1	2	3	4	5	6	7	8	9	10
LCDM	2	5	9	14	20	27	35	44	54	65
CM	6	12	20	30	42	56	72	90	110	132

3.4 Retrieval results

An example of the retrieved images is shown in Fig. 2. Note that we have chosen $D=2$ for the case of LCDM. It can be seen that most of the retrieved images have similar chromaticity distribution as the query image. Some mistaken retrievals are partly due to the close similarity of colour content between different categories. For example, 'Beaches' and 'Mountains and glaciers' have close colour content. The blue colour of the water matches the skies of mountains and glaciers, the white clouds above the beach and the

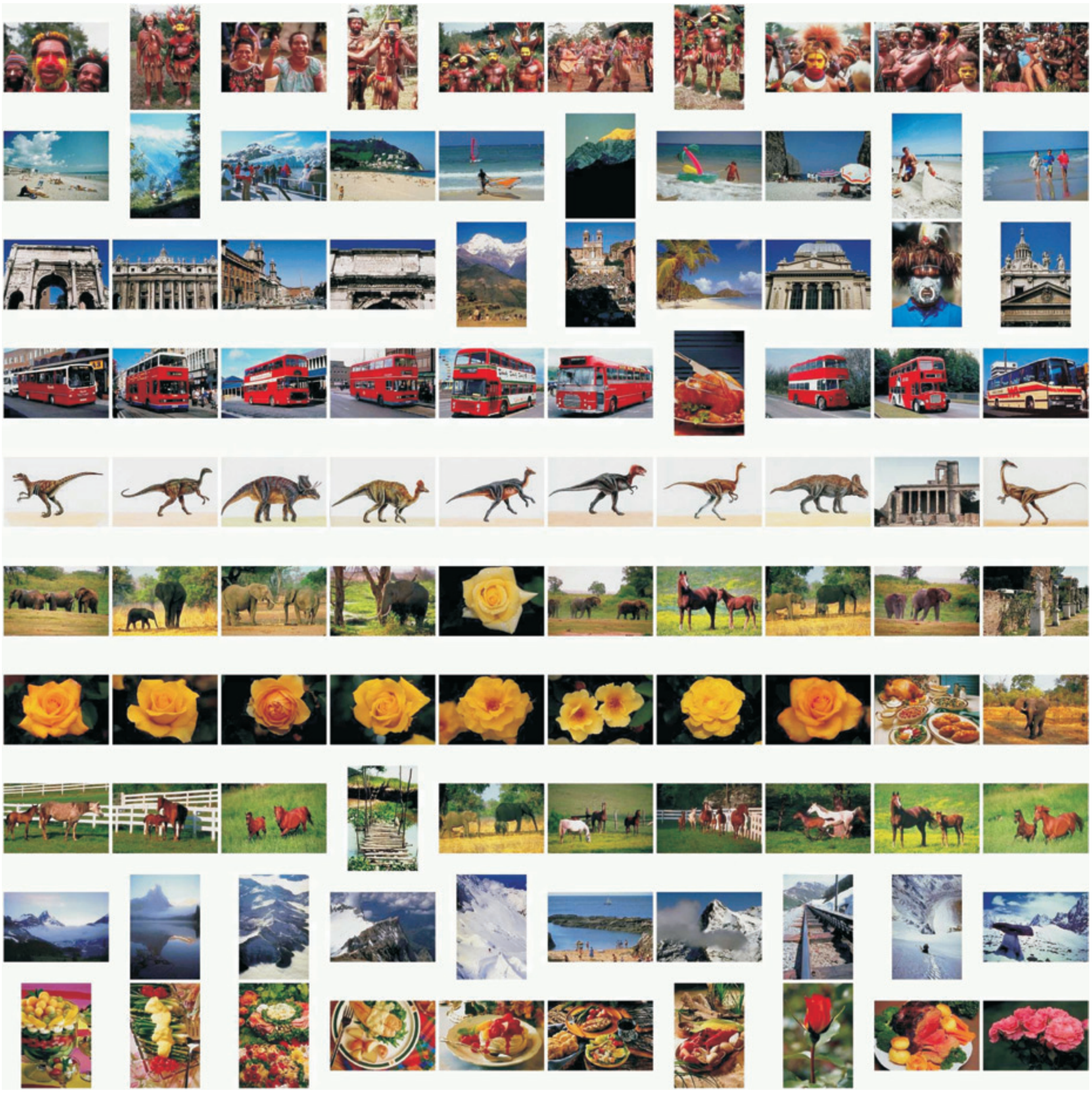

Fig. 2 *Examples of retrieved images using LCDM with $D=2$, average recall rate, precision and fallout rate are 0.082, 0.82 and 0.002, respectively*

Top 10 retrieved images for each category are shown; first image on left is query image

Table 5: Average recall rate, precision and fallout rate of LDCM for various values of D

D	1	2	3	4	5	6	7	8	9	10
Recall rate ($\times 10^{-1}$)	0.530	0.820	0.760	0.830	0.810	0.820	0.800	0.770	0.760	0.760
Precision	0.530	0.820	0.760	0.830	0.810	0.820	0.800	0.770	0.760	0.760
Fallout rate ($\times 10^{-2}$)	0.522	0.200	0.267	0.189	0.211	0.200	0.222	0.256	0.267	0.267

yellowish sand match the snow of the glaciers. For images of categories 'Elephants' and 'Horses', the colours of green grass are often similar and when under certain lighting conditions and viewing angles, the colours of the skins of the elephants and the horses are very close. It is therefore understandable that such misclassifications happen and even in these cases, LCDM still function properly to extract the major colour contents of the images, though not in a strict semantic sense. The average recall rate, precision and fallout rates for different values of D are tabulated in Table 5 of LCDM are tabulated in Table 5. It can be observed that the retrieval precision of LCDM is quite good even at a small value of D, particularly $D = 2$ with a precision of 0.82. We conjecture that $D = 2$ is good enough for achieving good retrieval results and at the same time maintaining a short feature vector for fast retrieval.

3.5 Comparison with chromaticity histogram

Chromaticity histogram (HIST) [9] is one of the most commonly used method to capture the colour content of an image. To measure the similarity between histograms, the L_1 distance function similar to (13) is employed. For simplicity, we denote this method HIST. The histogram quantisation levels are $Q = 1, 2, \ldots, 8$. The feature lengths resulting from these quantisation are of the same order as LCDM for $D = 1, 2, \ldots, 10$, i.e., when $Q = 8$, $L_{\mathrm{HIST}} = 64$ and when $D = 10$, $L_{\mathrm{LCDM}} = 65$. This makes comparison between the two methods feasible. The comparison results are shown in Fig. 3. It can be observed that for the same feature length, LCDM performs better than HIST by giving consistently higher values of precision.

3.6 Comparison with chromaticity moments

Paschos *et al.* proposed to use regular moments to capture the characteristics of the colour contents of images. This is achieved by obtaining the regular moments of both the trace and the distribution of the chromaticity space, which we abbreviate as CM. Here comparisons are made our

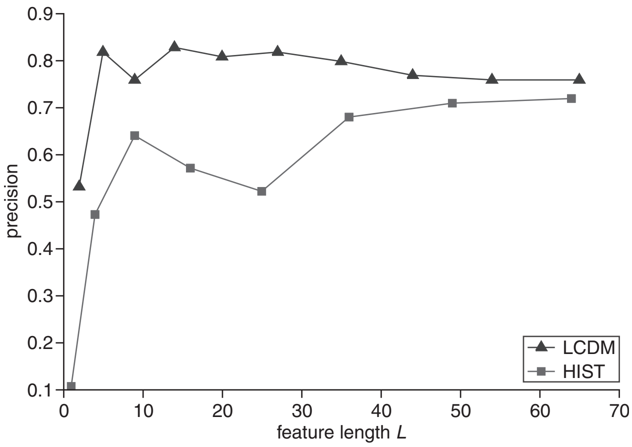

Fig. 3 *Comparison of retrieval precision of LCDM and HIST*

proposed method and CM. Different quantisation levels $Q = 8, 16, 32, 64, 128$ are used to obtain the values for CM, and we denote them as CM-8, CM-16, CM-32, CM-64 and CM-128, respectively. The results are shown in Fig. 4 for LCDM and CM with $D = 1, 2, \ldots, 10$. It can be observed that LCDM outperforms CM in all cases. Moreover, for each value of D the number of features for CM is two times that of LCDM. Figure 5 shows another graph of the results with respect to the feature length L.

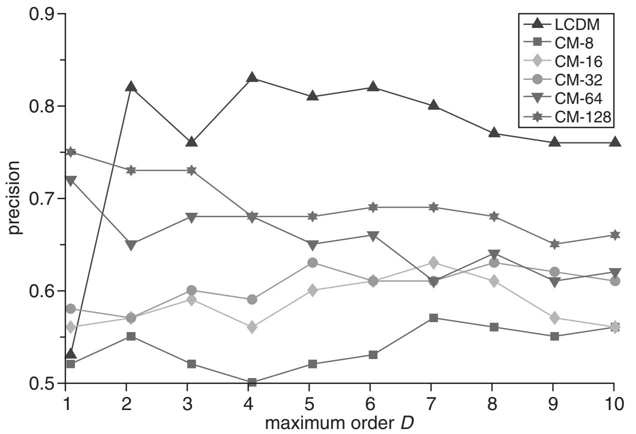

Fig. 4 *Comparison of retrieval precision of LCDM and HIST with respect to maximum order D*

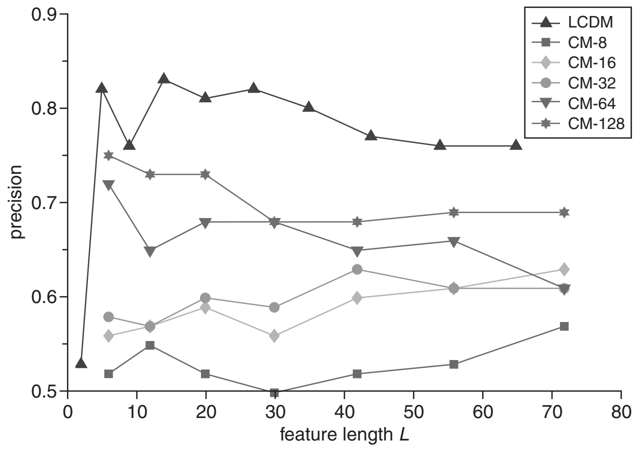

Fig. 5 *Comparison of retrieval precision of LCDM and HIST with respect to feature length L*

Table 6: Number of operations needed to obtain λ^c_{mn}

LCDM	Operations		
	Additions	Multiplications	Division
λ^c_{10}, λ^c_{01}	$MN + 1$	$2MN + 5$	1
λ^c_{11}	$MN + 1$	$3MN + 5$	1
λ^c_{20}, λ^c_{02}	$2MN + 1$	$3MN + 5$	1

Table 7: Retrieval time comparison of LCDM, HIST and CM

	Additions	Absolute value	Typical values
LCDM	$[P(P+3)-1]N_{total}$	$[P(P+3)/2-1]N_{total}$	$9N_{total}$, $4N_{total}$ $(P=2)$
HIST	$[2Q^2-1]N_{total}$	$[Q^2-1]N_{total}$	$199N_{total}$, $99N_{total}$ $(Q=10)$
CM	$[2(P+1)(P+2)-1]N_{total}$	$[(P+1)(P+2)-1]N_{total}$	$23N_{total}$, $11N_{total}$ $(P=2)$

3.7 Computation time

3.7.1 Computation complexity of LCDM: The computation complexity of LCDM is important and directly affects the time needed to index the whole image database. Using Horner's rule, the number of operations for obtaining the value of a polynomial of degree n can be reduced to n additions and n multiplications. For Legendre polynomials the number of operations can be further reduced by noting that some of the coefficients of Legendre polynomials are zeros. Taking this into consideration, the number of additions can be reduced to $\mathtt{floor}(n/2)$ and the number of multiplication remains n. Hence by considering (12), to compute λ^c_{mn} for a chromaticity space with MN points, we need $MN[\mathtt{floor}(m/2)+\mathtt{floor}(n/2)]$ additions and $MN(m+n)$ multiplications for obtaining the values for the Legendre polynomials, MN multiplications to obtain the products of the two polynomial values, $MN-1$ additions for summing up these values. The summed value are then multiplied (one multiplication) with A_{mn}, which by itself costs two additions, four multiplications and one division. To sum up, the total number of additions is $MN[\mathtt{floor}(m/2)+\mathtt{floor}(n/2)+1]+1$, multiplications $MN(m+n+1)+5$ and division 1. Table 6 lists the number of operations needed to obtain the individual λ^c_{mn} up to $m+n \le D=2$.

3.7.2 Retrieval time: The retrieval time is generally proportional to the length of the feature vector, L used to index the image database. The larger the dimension of the feature vector, the longer the time is needed to compare the feature between the query image and the images in the database. According to (13) the comparison of two images, the query image and one image from the database, L additions are needed to compute the difference of the feature vectors, L operations are needed to obtain the absolute values, and another $L-1$ operations are needed to sum up all the absolute values. This sums up to $2L-1$ additions and $L-1$ absolute-value operations. The total numbers of operations needed to query the whole database are then $(2L-1)N_{total}$ additions and $(L-1)N_{total}$ absolute-value operations where N_{total} is the total number of images shown in the database. The retrieval time in terms of numbers of operations for LCDM, HIST and CM are tabulated in Table 7. It can be seen that, LCDM needs the least number of operations among the methods considered.

4 Conclusions

We propose an effective scheme for content-based image retrieval based on Legendre chromaticity distribution moments (LCDM). LCDM serve as a set of low-level features which can be easily incorporated into more complex schemes to work together with other features such as texture and shape descriptors. LCDM can be obtained directly from the chromaticity space without first constructing the chromaticity histogram. Moreover, LCDM is compact in the sense that only a few terms of LCDM are needed to obtain retrieval accuracy comparable to that of a full chromaticity histogram. We have shown that for as little as five LCDM terms, a retrieval precision of 0.82 can be obtained. For the same order of length of feature vector, the chromaticity histogram gives a retrieval precision of about 0.5–0.6 and Paschos *et al.*'s chromaticity moments show a result of 0.5–0.75 for different numbers of quantisation bins used. The short-feature length of LCDM also means that less number of computational operations are needed to retrieve images from the database. This is especially important when considering the fact that image databases nowadays are growing rapidly in size. This also implies that the incorporation of LCDM with other descriptors such as texture and shape descriptors will less likely result in a lengthy feature vector, which will in turn result in a long retrieval time.

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6 Appendix

6.1 Relation of Legendre polynomials and monomials

Gram–Schmidt orthogonalisation is a procedure which takes a nonorthogonal set of linearly independent functions and constructs an orthogonal basis over an arbitrary interval $x_1 \le x \le x_2$ with respect to an arbitrary weighting function $w(x), x_1 \le x \le x_2$. Given an original set of linearly independent functions $\{u_n(x)\}_{n=0}^{\infty}$, let $\{\psi_n(x)\}_{n=0}^{\infty}$ denote the orthogonalised (but not necessarily normalised) functions, $\{\phi_n(x)\}_{n=0}^{\infty}$ denote the orthonormalised functions, then it can obtained that

$$\psi_i(x) = u_i(x) - a_{i0}\phi_0(x) - a_{i1}\phi_1(x) - \cdots - a_{i,i-1}\phi_{i-1}(x) = u_i(x) - \sum_{k=0}^{i-1} a_{ik}\phi_k(x) \qquad (20)$$

where

$$a_{ij} = \int_{x_1}^{x_2} u_i(x)\phi_j(x)w(x)dx, \qquad \phi_i(x) = \frac{\psi_i(x)}{\sqrt{\int_{x_1}^{x_2}\psi_i^2(x)w(x)dx}} \qquad (21)$$

Equations (20) can be interpreted as follows. Each function $u_i(x)$ is projected on a set of orthonormal functions $\{\phi_k(x)\}_{k=0}^{i-1}$, which are the results of the orthonormalisation of functions $\{u_k(x)\}_{k=0}^{i-1}$. The projections obtained are $\{a_{ik}\}_{k=0}^{i-1}$. The value $\sum_{k=0}^{i-1} a_{ik}\phi_k$ is then subtracted from u_i itself, forming the new orthogonal component $\psi_i(x)$. Note that this is essentially a process to remove redundant information from $u_i(x)$, where information already stored in $\{\phi_k(x)\}_{k=0}^{i-1}$ are removed.

Legendre polynomials $P_n(x)$ are essentially the orthogonalisation of the monomials $\{x^k\}$, $k = 0, 1, \ldots, n$, in the interval $[x_1, x_2] = [-1, 1]$ with weighting function $w(x) = 1$.

$$P_0(x) = 1$$

$$P_1(x) = x - \left[\frac{\int_{-1}^{1} xdx}{\int_{-1}^{1} dx}\right](1) = x$$

$$P_2(x) = x^2 - \left[\frac{\int_{-1}^{1} x^3dx}{\int_{-1}^{1} x^2dx}\right](x) - \left[\frac{\int_{-1}^{1} x^2dx}{\int_{-1}^{1} dx}\right](1) = x^2 - \frac{1}{3}$$

$$P_3(x) = x^3 - \left[\frac{\int_{-1}^{1} x^3(x^2-\frac{1}{3})dx}{\int_{-1}^{1}(x^2-\frac{1}{3})^2dx}\right]\left(x^2 - \frac{1}{3}\right) - \left[\frac{x\int_{-1}^{1} x^4dx}{\int_{-1}^{1} x^2dx}\right](x) - \left[\frac{\int_{-1}^{1} x^3dx}{\int_{-1}^{1} dx}\right](1) = x^3 - \frac{3}{5}x$$

Normalising so that $P_n(1)=1$ gives the expected Legendre polynomials.

6.2 Discrete form of Legendre moments

Considering a function $f(x_i, y_j)$, $i = 0, 1, \ldots, M-1, j = 0, 1, \ldots, N-1$ and $(x_i, y_j) \in [-1, 1] \times [-1, 1]$. Hence from (1), we have

$$\lambda_{mn} = \frac{(2m+1)(2n+1)}{4}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} f(x_i, y_j) \times \int_{x_i-(\Delta x_i/2)}^{x_i+(\Delta x_i/2)}\int_{y_j-(\Delta y_j/2)}^{y_j+(\Delta y_j/2)} P_m(x)P_n(y)dxdy \qquad (22)$$

where $\Delta x_i = x_{i+1} - x_i$ and $\Delta y_j = y_{j+1} - y_j$. The integral terms in (22) are often evaluated by zeroth-order approximation, that is, the values of Legendre polynomials are assumed to be constant over the intervals $[x_i - (\Delta x_i/2), x_i + (\Delta x_i/2)]$ and $[y_j - (\Delta y_j/2), y_j + (\Delta y_j/2)]$, and the values for each interval are obtained by sampling the Legendre polynomials at the central points of these intervals. In this case, the set of approximated Legendre moments is defined as

$$\hat{\lambda}_{mn} = \frac{(2m+1)(2n+1)}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} f(x_i, y_j)P_m(x_i)P_n(y_j) \qquad (23)$$