Exploratory Factor Analysis

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Table of Contents

CHAPTER I
GENERAL CONCEPTS AND OBJECTIVES OF FACTOR ANALYSIS

1.0. General Introduction

The field of factor analysis involves the study of order and structure in multivariate data. The field includes both theory about the underlying constructs and dynamics which give rise to observed phenomena, and methodology for attempting to reveal those constructs and dynamics from observed data. The purpose of this chapter is to provide an explanation of the central concepts of factor analytic theory and the objectives of factor analytic methodology. This will be done in an entirely non-mathematical fashion.

The material discussed in this chapter, as well as in the remainder of the book, will be presented in the context of the field of psychology. Often the more specific context of the study of mental abilities will be used for demonstrations, examples, and explanations of concepts. This context represents the field in which the primary development and use of factor analysis has occurred. Thus, we will generally consider problems where the entities under study are people, and the variables of interest are attributes of those people. It should be kept in mind, however, that factor analysis is valuable and applicable in a wide range of other settings. It has been used widely in other social sciences, education, business fields, biological sciences, etc. Thus, though much of the theory and methodology has developed within psychology, factor analysis can be applied beneficially to subject matter from many other disciplines.

1.1. Introduction to Factor Analytic Theory

To begin to develop an understanding of the type of problem to which factor analysis can be applied, and the objectives of this approach, let us first consider a process which characterizes much research in psychology. Such research often includes a number of steps which, though they may not be explicitly stated, can be recognized clearly and which represent integral parts of the research process. The first step is the identification of the domain and population of interest. The domain can be defined as the range of phenomena of interest in the research. The domain may be very broad, such as mental abilities, or more narrow, such as verbal abilities. Other examples of domains could include attitude, personality, physical abilities, etc. In addition to defining the domain, the researcher also defines the population, which is the entire set of entities of interest. In psychology the entities are very often people, but may be other things such as animals, groups, etc. For discussion purposes, we will assume that the entities of interest are people. Populations of entities are usually very large (e.g., 12-year-old children), almost always too large to allow for each entity to be measured.

Given a domain and population of interest, the researcher selects from the domain variables which are to be measured. We will refer to these variables as surface attributes; a surface attribute is any one of the many attributes of people that may be observed and measured. For instance, in the domain of mental abilities, tests could be constructed to measure distinct kinds of attributes. One test could contain addition problems; another could test spelling; a third could contain memory problems; etc. Each of these tests represents a surface attribute. Clearly, in any given domain, it would be possible to identify and measure a vast array of surface attributes. A set of surface attributes measured in a given study will be referred to as a battery of surface attributes.

In addition to selecting surface attributes to be measured, a researcher also must select a sample of individuals from the population. Measurements then are obtained for each individual in the sample on each surface attribute. When the researcher views such data, several things usually are quite apparent. First, there normally will be substantial variation among individuals in terms of their measures on the surface attributes. That is, individual differences will occur. In addition, there normally will be substantial covariation (i.e., correlation) between different surface attributes. For instance, an addition test may show a high correlation with a multiplication test. Given more than a few surface attributes, there likely will be a wide range of correlations among the attributes, some correlating highly and others quite low. This observed variation and covariation on the surface attributes may seem chaotic. Viewed at this level, it generally will be difficult to understand and account for in any simple manner. The theory and methods of factor analysis are designed explicitly for this purpose. Factor analysis involves a set of techniques designed to identify order and structure in such data by providing a parsimonious and meaningful explanation for the observed variation and covariation in surface attributes.

The cornerstone of factor analytic theory is the postulate that there exist internal attributes. An internal attribute is taken to be an unobservable characteristic of people on which people differ in extent or degree. Internal attributes are more fundamental than surface attributes. For instance, in the domain of mental abilities, internal attributes might be such things as numerical ability, verbal ability, etc. These attributes cannot be directly measured, but their effects are reflected when one obtains measures on surface attributes. The internal attributes commonly are referred to as factors, or latent variables. These terms will be used interchangeably.

Internal attributes also can be thought of as hypothetical constructs. They are not necessarily taken to be completely real and concrete. Rather they are constructs which, as will be seen, can be used to understand and account for observed phenomena. The central role of this concept of unobserved variables in factor analysis should not be viewed as a basis for criticism of factor analytic theory. The use of hypothetical constructs is routine in many fields of science. For instance, physicists make use of a multitude of constructs (e.g., gravity, magnetism, etc.) to account for observed phenomena. These forces actually represent unobserved constructs whose existence has been hypothesized based on observed events. These constructs, along with theories which make use of them, then serve to account for a vast array of observed phenomena. In much the same way, factor analytic theory postulates that there exist unobservable internal attributes which account for observed variation and covariation across a wide range of surface attributes.

A central theoretical concept in factor analysis involves the relation of surface attributes to internal attributes. The basic principle is that internal attributes influence surface attributes in a systematic fashion. This implies that when a researcher obtains a measurement for an individual on a surface attribute, that measurement is, at least in part, the result of the influence of underlying internal attributes. For instance, an individual's score on an addition test (surface) is influenced by numerical ability (internal). According to the traditional representations of factor analytic theory, these influences of internal attributes on surface attributes are taken to be linear.

At this point it is useful to distinguish between two types of internal attributes, or factors. One type is called a common factor, which is defined as an internal attribute which affects more than one of the surface attributes in the selected set, or battery. For example, if the selected battery of surface attributes includes more than one which is influenced by numerical ability (e.g., both an addition test and a multiplication test), then numerical ability is a common factor. The second type of internal attributes is specific factors, each of which influence only one of the surface attributes in any given battery. There may be a number of specific factors for any given surface attribute; however, their influences can be viewed as being combined into a single specific factor. With a change of the battery of measured surface attributes one of the common factors may become a specific factor for one of the surface attributes, or one of the specific factors for one surface attribute may become a common factor. For example, consider that all surface attributes that are influenced by numerical ability are removed from a battery except for a test of addition; then numerical ability becomes a specific factor for the test of addition. The reverse can be true when a battery is enlarged; putting back into the battery the other tests which are influenced by numerical ability will make this ability a common factor again. For now, suffice it to say that there exist common factors, which are those that affect more than one attribute in the battery, and specific factors, each of which affects only one surface attribute in the battery.

In addition to the two types of internal attributes represented by common and specific factors, there exists a third influence on the surface attributes. This third influence is errors of measurement in observing each surface attribute. These errors of measurement are represented in factor theory as additional factors, though they do not correspond to internal attributes as defined above; i.e., they do not represent unobserved characteristics of individuals. Rather, errors of measurement factors arise from transient, unsystematic events which influence the measurement of the surface attributes. Note the direct relation between errors of measurement factors and the reliability of the measures of the surface attributes. Higher reliability implies lower errors of measurement. Thus, the use of more or less reliable tests in a battery would influence the errors of measurement factors, but would not necessarily affect the common or specific factors. Note also that altering the test battery by removing or adding tests may affect the common and specific factors, as described above, but would not influence the errors of measurement factors since reliability is not affected by such changes in the battery.

For any particular battery the specific factors and errors of measurement factors may be combined into unique factors. There will be one unique factor for each surface attribute, with each unique factor defined as the combination of the specific and errors of measurement factors for the corresponding surface attribute. Note that unique factors will change as a result of any changes in specific or errors of measurement factors. Thus, changes in the battery which affect specific factors will in turn affect the corresponding unique factors, and the use of more or less reliable tests would affect errors of measurement factors and, in turn, the unique factors.

Given these basic definitions of factors, together with the principle that the factors influence the surface attributes, it can be understood that the factors combine to account for an individual's degree or level on a surface attribute. That is, any individual's level on a surface attribute can be viewed as arising from that individual's level on the relevant factors. For instance, an individual's score on an addition test can be viewed as being attributable to the individual's level on the internal attribute of numerical ability, along with his or her level on the specific factor associated with the addition test. Also, errors of measurement contribute to the measure of an individual's level on the addition test. By simple extension of this notion, it can be stated furthermore that individual differences, or variance, on surface attributes can be attributed to the underlying factors. That is, individuals differ on surface attributes because they differ on the internal attributes which influence those surface attributes, as well as due to the presence of errors of measurement. For instance, individual scores on a spelling test vary, in part, because those individuals vary with respect to their level on an underlying internal attribute, verbal ability. Further variation in the test scores can be attributed to individual differences on the specific factor associated with the spelling test, as well as the presence of errors of measurement.

The distinction drawn earlier between common, specific, and errors of measurement factors is very relevant to this view of how factors account for variation on surface attributes. It is very important to note that the observed variance on a given surface attribute in a battery can, according to factor analytic theory, be recognized as arising from three sources. Some of the variance is due to the influence of the common factors. This portion is referred to as the common variance, or communality. A second portion of the variance on the surface attributes arises from the specific factor. This portion is termed specific variance, or specificity. A third portion of the variance on the attribute arises from errors of measurements, and is termed error of measurement variance. The specific variance frequently is combined with the error of measurement variance to form the unique variance, or uniqueness. To state this another way, the observed variation on a given surface attribute in a given battery is due in part to factors which influence other surface attributes in the battery, and in part to factors which influence only the given surface attribute. In this sense, the common, specific, and errors of measurement factors are taken to account for the observed variation on each surface attribute in the battery. Equivalently, it can be said that the observed variation on each surface attribute in the battery can be attributed to common and unique factors, since the unique factors represent the combined influences of the specific and errors of measurement factors.

By a further extension of these concepts, factor analytic theory also can be viewed as accounting for covariation between surface attributes. It is commonly observed that surface attributes within any given domain tend to be correlated with each other to varying degrees. According to factor analytic theory, such correlation is due to the influence of the common factors. That is, a correlation between two surface attributes is due to the dependence of those attributes on one or more of the same common factors. For instance, an addition test and a multiplication test will exhibit a fairly high positive correlation because both depend on the numerical ability factor. By contrast, an addition test and a spelling test will exhibit a much lower correlation because they are influenced by different common factors. Thus, the degree of intercorrelation between surface attributes is taken to be the result of the degree to which those attributes are influenced by the same internal attributes.

It is of utmost importance to understand that it is only the common factors which account for correlations of surface attributes. That is, unique factors do not give rise to such correlations. Equivalently, specific factors and errors of measurement factors do not give rise to correlations among surface attributes. By definition, they cannot because they each influence only a single surface attribute.

To summarize these basic principles of factor analytic theory, it is hypothesized that, in a given domain, there will exist a small number of common factors which influence the potentially vast array of surface attributes. Variation on the surface attributes is attributable, in part, to variation on the common factors. The remainder is due to unique factors, or, alternatively, to specific factors and errors of measurement factors. Covariation of surface attributes is attributable to the dependence of surface attributes on some of the same common factors. Thus, the basic tenets of this theory allow for the explanation of covariation on a potentially large number of surface attributes in terms of a much smaller number of internal attributes, or common factors.

We will now present a series of figures designed to provide a schematic representation of the theoretical concepts developed to this point. These figures are called path diagrams and were devised by Wright (1921). They were first used to represent factor analytic theory by Tucker (1940). Within these diagrams squares represent surface attributes and circles represent internal attributes and other types of factors. Uni-directional arrows represent directional linear influences, indicating a linear effect of one variable on another, via some type of causation or process. Bi-directional arrows represent correlation, where two variables are correlated, but where no directional influence of one on the other is hypothesized.

Figure 1.1 illustrates the relationships between surface attributes and three types of factors defined above. This figure shows four surface attributes on the right. These could be thought of as four mental tests, for example. On the left are three types of factors. The two internal attributes at the top each are represented as influencing more than one of the surface attributes. Thus, these satisfy the definition of common factors. According to the pattern of effects represented in the diagram, the first two surface attributes would be correlated with each other because both are influenced by the first factor. The last three surface attributes would also be intercorrelated because they are influenced by the second factor. Note that bi-directional correlational arrows among the surface attributes are not included in Figure 1.1. This is because the correlations among the surface attributes are seen as arising from the effects represented in the figure. That is, the effects of the common factors on the surface attributes imply that the surface attributes will be intercorrelated, so those intercorrelations are not explicitly represented in the figure.

It is important to note that one of the surface attributes in Figure 1.1 is represented as being influenced by both common factors. This is emphasized here in an effort to dispel a common misconception in applied factor analysis; i.e., that each surface attribute should be associated with only one factor. In fact it is entirely permissible and, indeed, quite common for a surface attribute to be influenced by more than one common factor. For example, if the two common factors in Figure 1.1 were numerical ability and verbal ability, a test composed of mathematical "story problems," requiring an ability to read and understand the problem as well as to solve it mathematically, would be dependent on both factors.

Another important aspect of Figure 1.1 is that the two common factors are represented as being intercorrelated; note that there is a bi-directional path connecting these two internal attributes. This illustrates the point that common factors may be related to each other, and, in fact, are often found to be so in practice. For instance, factors such as numerical ability and spatial ability may be correlated rather than uncorrelated constructs. Just as with surface attributes, correlations between internal attributes indicate a degree of linear relationship between measures for individuals on these attributes.

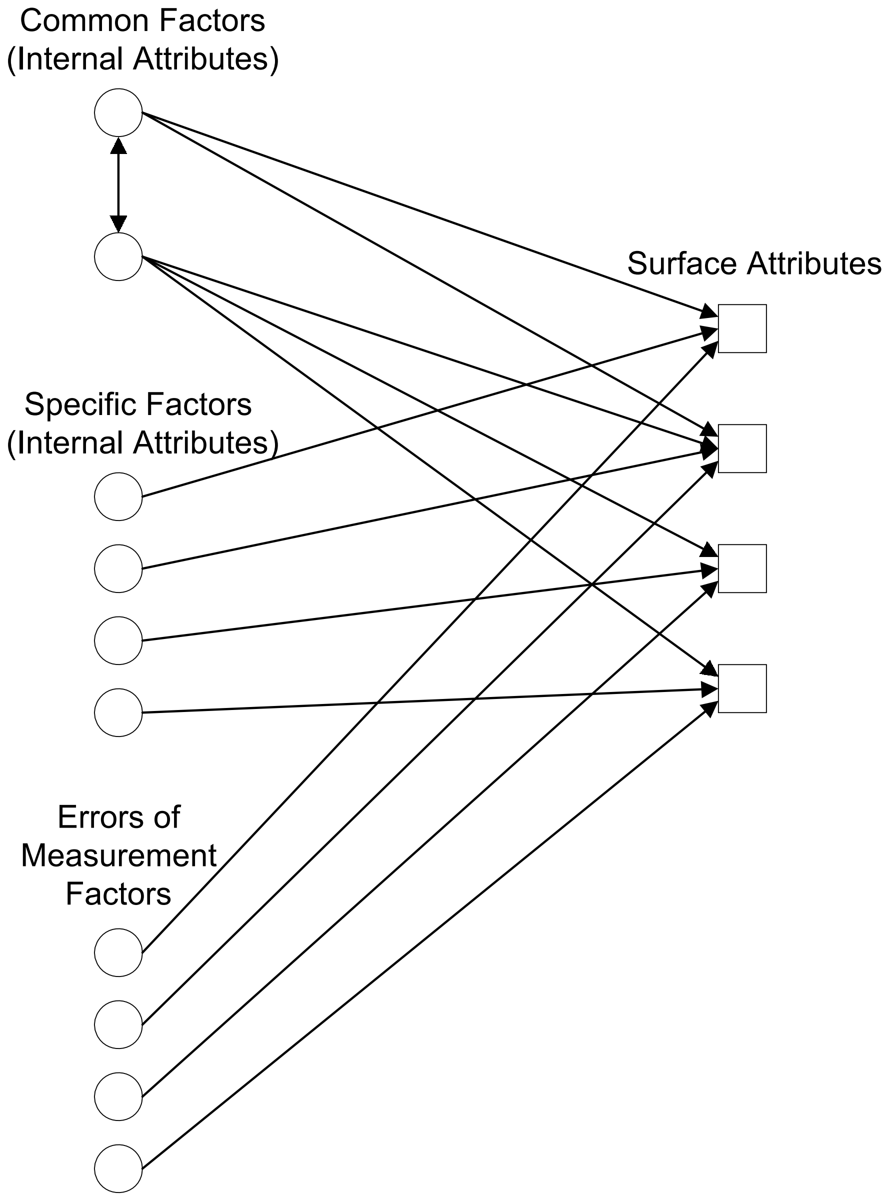

Figure 1.1: Basic Schemata for Common Factor Theory

Below the common factors in Figure 1.1 are shown four specific factors, which also represent internal attributes. There is one specific factor for each surface attribute. Note that each specific factor is shown as influencing only a single surface attribute. Below the surface attributes in Figure 1.1 are shown four errors of measurement factors. These factors are not designated as internal attributes since they do not correspond to unobserved characteristics of the individuals, but rather arise from unsystematic, transient events. As shown in Figure 1.1, there is one errors of measurement factor for each surface attribute, with each such factor influencing only a single surface attribute.

Figure 1.1 then illustrates a number of fundamental points. Considering each surface attribute separately, the variation in a given surface attribute is seen as arising from the effects of common, specific, and errors of measurement factors. As described earlier, the variance arising from the common factors is termed the communality of a surface attribute; the part of the variance on a surface attribute arising from the specific factor is termed the specificity of the surface attribute; and the part of the variance due to errors of measurement is termed the error of measurement variance of the surface attribute. Thus, Figure 1.1 indicates how the variance on a surface attribute may be partitioned into three sources: communality, specificity, and error of measurement variance. Inclusion of errors of measurement within factor analytic theory is a very important point which is overlooked by many practitioners in factor analysis. With regard to the separate issue of relations between surface attributes, Figure 1.1 illustrates how the relation between any two surface attributes is a result of the degree to which those attributes are influenced by the same common factors.

The representation of common factor theory in Figure 1.1 is expanded in Figure 1.2 to show how the specific and errors of measurement factors can be viewed as combining to form unique factors. The common factor portion of Figure 1.2 is the same as that shown in Figure 1.1. In Figure 1.2 four unique factor are shown. Each unique factor is represented as the combination of the specific and errors of measurement factor for a given surface attribute. In this view of common factor theory, each surface attribute is taken to be affected by some number of common factors and a single unique factor. According to this framework, the covariation of surface attributes is still attributed to the influences of common factors, and the variation in a given surface attribute is seen as arising from the effects of common and unique factors. In terms of variance, it was stated earlier that specificity and error of measurement variance are combined to form the uniqueness of the surface attribute. Thus, Figure 1.2 indicates how the variance on each surface attribute may be partitioned into communality and uniqueness.

The presentation of basic factor analytic theory is often limited to the view represented by Figures 1.1 and 1.2. However, it is important to develop an expanded view which considers the internal attribute in more depth. This expanded view will provide a further understanding of the

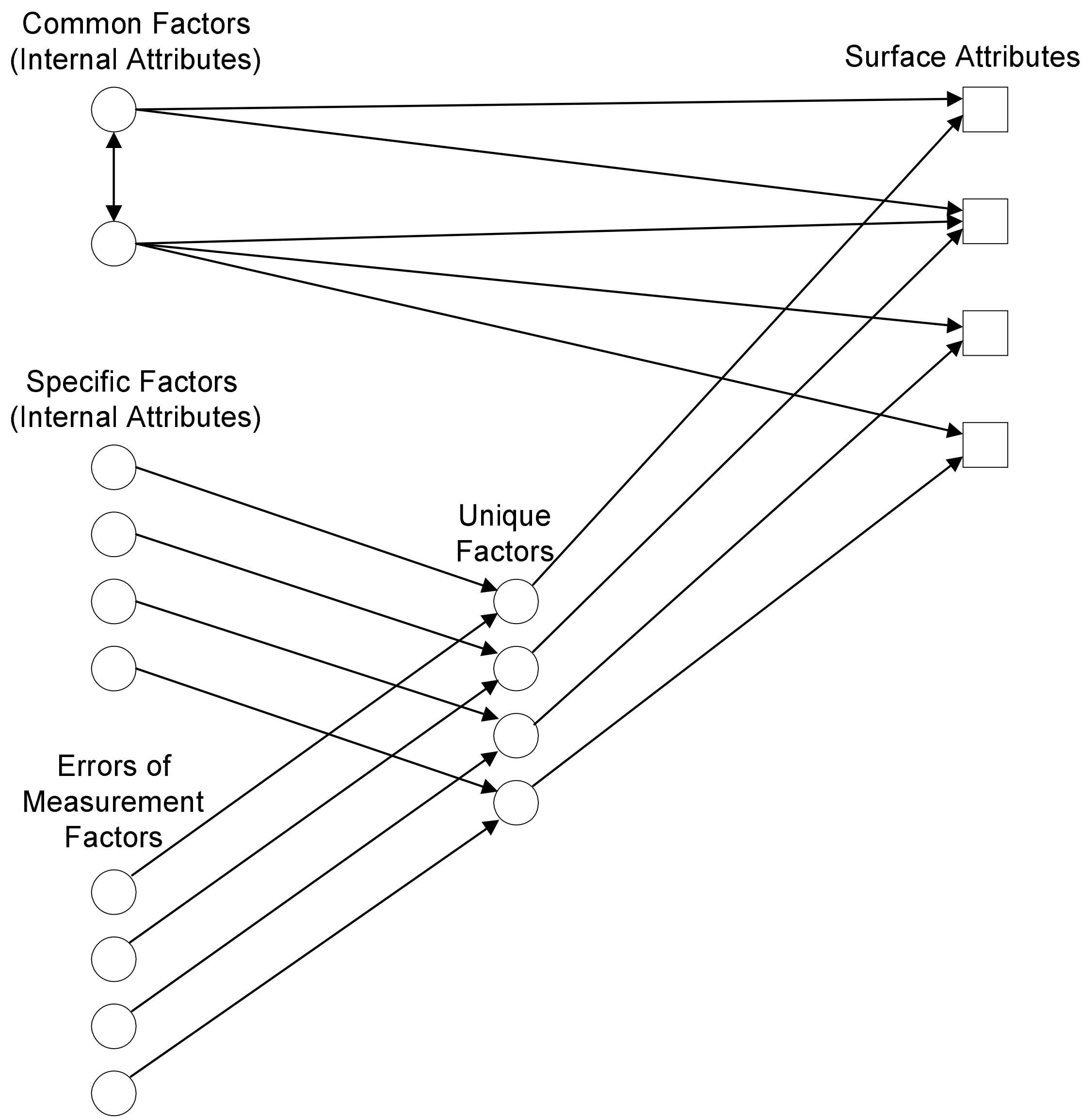

Figure 1.2: Expanded Schemata for Common Factor Theory
Showing Unique Factors

nature of these attributes and the influences which give rise to them, and thus yield a fuller conceptualization of the theory. Figure 1.3 provides an expanded path diagram which illustrates a more complete representation of factor analytic theory. The right half of Figure 1.3 is identical to Figure 1.2, with one exception: the intercorrelation of the two common factors no longer is represented explicitly by a bi-directional path. The reason for this change is given in the next paragraph.

The far left side of Figure 1.3 represents the sources and dynamics which give rise to common, specific, and errors of measurement factors. This portion of Figure 1.3 contains two different types of influences. At the top is shown a group of effects designated "Basic Characteristics and External Influences." These are intended to represent (a) basic characteristics of individuals, such as innate abilities, age, etc., and (b) external influences, such as education, experience, cultural effects, etc. These characteristics are shown as influencing both the common and specific factors. The common factors can thus be thought of as constructs which are made up of combinations of basic characteristics and experiential elements of individuals. The common factors are reflections of the coalescing of such elements as they influence surface attributes. The broad arrows representing the effects of the basic characteristics and external influences on the common factors are intended to represent this general coalescing, which might include nonlinear relations. As implied by Figure 1.3, different common factors may be influenced by some of the same basic characteristics and external influences. Such a phenomenon would give rise to correlated common factors. Thus, correlations among common factors are a result of the influences represented in Figure 1.3, and so need not be represented explicitly by bi-directional paths. This is the same reasoning which eliminates the need to represent correlations among surface attributes explicitly, since such correlations are accounted for by other effects represented in the figures. In general in path diagrams, relations which are accounted for by effects in the model are not represented explicitly.

The basic characteristics and external influences also are shown as affecting specific factors. This simply means that, just as those effects can coalesce to yield a common factor (which affects more than one surface attribute), they can also combine to yield a factor which has a systematic effect on only one surface attribute.

In the lower left section of Figure 1.3 are shown "Transient Effects." These are effects which influence the performance of the individuals on the surface attributes, but which are unstable, or transient. The most common example of such effects would be errors of measurement, which are unsystematic and transient, yet influence observed measures. Each surface attribute will be influenced by some transient effects; thus, Figure 1.3 contains a transient effect for each surface attribute. These effects can be conceived of as giving rise to the errors of measurement factors, as shown in Figure 1.3.

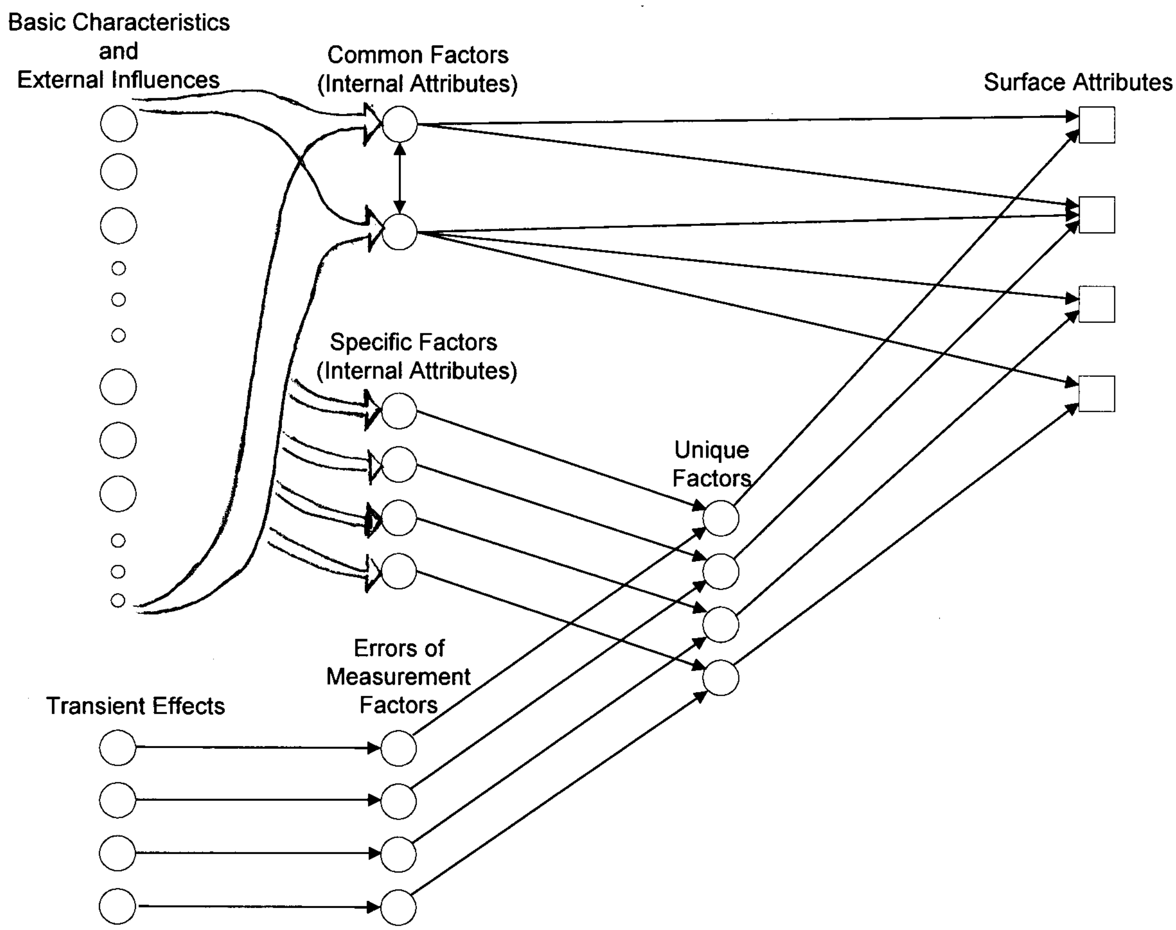

Figure 1.3: Full Schemata for Common Factor Theory

It is important to keep in mind that the factor analytic theory as described here and represented in Figure 1.3 is most assuredly a theory. It is a theory about the influences and dynamics which give rise to observed variation and covariation on surface attributes. Whenever a theory of any kind is used in science, a central issue becomes the degree of correspondence between the theory and the real world. That in, how well does the theory account for the observed phenomena under study? The degree of success of factor analytic theory will vary from one application to another, and it should never be assumed that the theory is a correct representation of the real world in any particular study. The issue of goodness of fit of the theory to the observed data will have to be considered as a routine part of any application of factor analysis.

In terms of the conceptualization of the theory developed here, an interesting way to view the issue of correspondence to the real world is to conceive of each surface attribute as being composed of two parts. One part arises from formal factor analytic theory, as portrayed in Figure 1.3. The other part is the remainder, which cannot be represented in the given framework. In terms of the theory, the former part is systematic and can be accounted for, while the latter part is unsystematic and cannot be accounted for. The unsystematic portion may arise from any number of sources, such as a multitude of minor common factors (i.e., common factors which have real effects on the surface attributes, but whose effects are very minor compared to the major common factors represented in Figure 1.3), and nonlinear effects in places where the model postulates linear effects. Regardless of the source, this lack of perfect correspondence between the theory and the real world must be acknowledged and accepted as an inherent aspect of the use of factor analysis. Furthermore, as with any scientific theory, the degree of fit of the theory to observed data must be evaluated in practice.

1.2. Objectives of Factor Analysis

Of primary interest in factor analytic theory is the nature of the common factors. Therefore, the primary objective in factor analysis methodology is to determine the number and nature of those factors, and the pattern of their influences on the surface attributes.

In very general terms, this is accomplished by making use of the implications of factor analytic theory as discussed earlier. Since the influence of common factors on surface attributes gives rise to correlations among those attributes, the observation of correlated surface attributes can be taken as an indication that common factors are operating. Given correlations among a number of surface attributes drawn from a particular domain, it may be possible to determine, from the pattern of correlations, the number of common factors operating and something about their nature. The methodology of factor analysis is designed to achieve this goal. Based on the intercorrelations among surface attributes, it is possible to estimate the number of common factors and to obtain numerical coefficients representing the degree of effect of each common factor on each surface attribute. Based on this information, it will be seen that it is possible to attempt to interpret the nature of the common factors themselves, as well as to obtain estimates of the amount of common and unique variance in each surface attribute. In practice, when factor analysis is conducted on a single battery in a single sample with no further information, only the common and unique variances can be determined. That is, it will not be possible to partition the uniqueness into portions due to specific and errors of measurement factors without further information about the reliabilities of the measures in the battery. However, this is a tangential issue in factor analysis. The central objective of factor analysis is focused on the common factors -- to gain an understanding of their nature and the dynamics of their relationships to each other and to the surface attributes.

We wish to emphasize that we do not view this as an attempt to determine measures for the individuals on the common factors. The purpose of factor analysis is not to measure individual differences on the common factors, or to obtain such measures for further analysis. Such measures are referred to as <u>common factor scores</u>. Though it is possible to estimate such scores for individuals on the common factors, and though we will cover this issue in detail in Chapters 15 and 16, this endeavor should not be viewed as a primary objective of factor analysis. Rather, the goal is to account for variation and covariation on the surface attributes by identifying relevant common factors. This is the end toward which factor analytic theory and methodology are primarily directed.

An important final point is that it generally is not possible to achieve these objectives in a single study. Rather, a succession of studies, each one building on knowledge gained from preceding studies, is necessary. The purpose of the several studies should be enhancement of the understanding of the internal attributes. Repeated studies with the same battery of measured attributes using new samples of entities may be beneficial but are not sufficient. Rather, the battery of measured attributes should be changed in successive studies by some elimination of uninformative attributes as well as, most importantly, by the addition of new measures constructed from interpretation of the factors. A desirable feature would be the prediction of the factorial composition of these new measures. For instance, attempts might be made to construct new measures which would load in a predictable fashion on two or three factors. These predictions, then, could be checked in a new study. Each success in construction of new measures with predictable characteristics would strengthen the interpretation of the factors. Even when predictions are not borne out, the interpretation of the factors could be revised to a stronger position. Following such a plan of a succession of studies would lead to better understanding of the domain being investigated. Factor analysis research must not be viewed as involving single studies. Further discussion of this view of factor analysis research is provided in Chapter 6.

1.3. <u>Illustration of Factor Analysis</u>

In an effort to clarify the concepts and objectives discussed to this point, a simple illustration of an application of factor analysis will be presented. In presenting this illustration, we will avoid explicit discussion of methodology and focus simply on the potential for factor analysis to achieve the objectives described in the previous section. To accomplish this, it is necessary first to describe (a) the type of data to which factor analysis is applied, and (b) the most important results which are obtained from a factor analysis and how those results are interpreted. This discussion is somewhat oversimplified, but will serve the purpose of introducing the reader to these points and providing a framework for the subsequent illustration.

In most applications of factor analysis, the data analyzed consist of sample correlations or covariances among the surface attributes. Issues differentiating the analysis of correlations vs. covariances will be discussed in subsequent chapters. The present illustration employs correlations. These typically are presented in the form of a correlation matrix, where the rows and the columns represent the surface attributes and each element is a correlation for a given pair of attributes. When such data are subjected to factor analysis a considerable amount of information can be generated. For present purposes, the most important results consist of common factor weights, common factor intercorrelations, and communalities.

The common factor weights also are referred to as common factor loadings. These loadings normally are arranged in the form of a matrix, where the rows represent the surface attributes, the columns represent the common factors, and each element is a factor loading representing the effect of a given factor on a given surface attribute. In Figures 1.1, 1.2, and 1.3, the loadings can be thought of as numerical coefficients corresponding to the directional paths connecting the common factors to the surface attributes. The loadings provide the basis for attempting to interpret the nature of the common factors. In very basic terms, the substantive meaning of each factor is interpreted by examining the coefficients in the corresponding column of the factor loading matrix. High loadings represent surface attributes which are influenced strongly by the factor, and low loadings represent surface attributes which are influenced weakly by the factor. By examining the pattern of loadings for a given factor, the researcher attempts to identify a construct whose effects on the surface attributes correspond to the pattern of loadings. That is, the construct should be one that plays a strong role in the surface attributes which have high loadings and a weak role in those that have low loadings. This process will be illustrated below.

The other fundamental results of a factor analysis mentioned above are common factor intercorrelations and communalities. The former provide measures of the degree of relationship among the constructs which have been identified, and the latter indicate how much of the variance in each surface attribute is accounted for by the common factors.

Our example involves selected data from a study by Thurstone and Thurstone (1941) which consisted of analyses of two large batteries of tests given to seventh and eighth grade students in Chicago. For the current example, we will consider data on nine of these tests from the eighth grade students only. The sample consisted of 710 students. A brief description of the nine tests follows:

(1) Addition -- Items involved adding columns of numbers.

(2) Multiplication -- Items involved multiplying two numbers.

(3) Three-Higher -- Each item consisted of a series of numbers; the task was to identify each number in the series that was exactly three more than the number just before it.

(4) Figures -- Each item consisted of a series of figures, such as letters, rotated or reflected in various ways; the task was to identify each figure in the series which could be rotated to match the first figure.

(5) Cards -- Each item consisted of a series of cards representing geometric shapes, rotated or reflected in various ways; the task was to identify each card in the series which could be rotated to match the first card.

(6) Flags -- Each item consisted of two flags; the task was to determine whether the two flags were the same, within a simple rotation.

(7) Identical Numbers -- Each item consisted of a column of numbers; the task was to mark each number in the column that was identical to the first number.

(8) Faces -- Each item consisted of a set of three faces, where two of the three were identical; the task was to mark the one that was different.

(9) Mirror Reading -- Each item consisted of a typed word, followed by four other words which were printed backward; the task was to identify which of the four was the same as the original word.

The nine tests were administered to the sample of students, and correlations among the tests were obtained. The correlation matrix is shown in Table 1.1. Note that the nine tests correspond to surface attributes, and the primary objective of factor analysis is to identify internal attributes which account for the relationships among these tests. Factor analysis was applied to the correlation matrix in Table 1.1. Discussion of the exact methodology employed is not practical at this point. Suffice it to say that three common factors were identified, and the resulting factor loadings, communalities, and factor intercorrelations are presented in Table 1.2. One can attempt to interpret the substantive nature of the common factors by examining the factor loadings, as described above. Considering the first column of loadings in the factor matrix, it is clear that this factor has a strong influence on the first three tests, but essentially is absent in the remaining tests. Since the first three tests explicitly involve numerical calculations, while none of the remaining tests require such activity, this first factor can be interpreted as a numerical

Table 1.1

Correlations among Nine Mental Tests (Sample: 710 Eighth Grade Students)

Attribute	1	2	3	4	5	6	7	8	9
1. Addition	1.000								
2. Multiplication	.499	1.000							
3. Three-Higher	.394	.436	1.000						
4. Figures	.097	.007	.292	1.000					
5. Cards	.126	.023	.307	.621	1.000				
6. Flags	.085	.086	.328	.510	.623	1.000			
7. Ident. Number	.284	.467	.291	.044	.114	.086	1.000		
8. Faces	.152	.235	.309	.319	.376	.337	.393	1.000	
9.Mirror Reading	.232	.307	.364	.213	.276	.271	.431	.489	1.000

Table 1.2

Factor Analysis Solution for Nine Mental Tests

Factor Weights				
		Factor		Communalities
Attribute	1	2	3	
1	.66	.05	-09	.38
2	.67	-.05	.11	.53
3	.52	.32	.01	.43
4	.00	.72	-.04	.51
5	-.02	.80	.03	.63
6	.02	.71	.02	.51
7	.24	.03	.49	.44
8	-.06	.40	.53	.45
9	.08	.28	.52	.44

Factor Intercorrelations

Factor	1	2	3
1	1.00		
2	.15	1.00	
3	.57	.11	1.00

calculations factor. Considering the second column of factor loadings, it is clear that this factor has a strong influence on tests 4, 5, and 6, and a moderate to weak influence on the remaining tests. Since those three tests all require an ability to recognize relations between shapes which are in different spatial orientations, this factor could be thought of as spatial relations. This interpretation is consistent with the finding that this factor has a mild effect on some of the other tests (e.g., mirror reading). The third factor is characterized by high loadings on the last three tests and very low loadings on the other six. The last three tests all require the subject to rapidly examine simple stimuli and identify those which are the same. This factor could be thought of as a perceptual speed factor.

The factor intercorrelations shown in Table 1.2 reveal that these three factors are not completely independent. Specifically, there is a substantial positive correlation between the numerical calculations and perceptual speed factors. The spatial relations factor is correlated only weakly with the other two. The communalities show how such of the variance in each of the tests is accounted for by the three factors. These values reveal that each of the tests has at least a moderate amount of variance which is not accounted for by the common factors; i.e., which is due to unique factors.

This illustration demonstrates the potential for factor analysis to achieve its objective of identifying internal attributes which account for variation and correlation on surface attributes. In the demonstration, three common factors are found which can be shown to account quite well for the relationships among the nine surface attributes, as well as for a portion of the variance on each surface attribute. Furthermore, these factors were found to be easily interpretable through their relationships with the surface attributes. As a result, the factors can be viewed as substantively meaningful constructs which have been found to be related to surface attributes in a systematic way. These constructs and their relationships to each other and to the surface attributes serve to account for observed phenomena; i.e., the complex pattern of variation and covariation of the surface attributes. However, this illustration should not be viewed as an isolated study which achieves all of the objectives of factor analysis. As noted at the end of the previous section, a study such as this would be one in a series of studies. Preceding studies led to the development of the battery of nine tests which represented the three common factors identified. Subsequent studies could be conducted in which new tests could be added to the battery so as to further verify and refine the understanding of the internal attributes.

Finally, we wish to emphasize that we do not consider factors such as those identified in the illustration just presented to be equivalent to fundamental, unitary mental abilities. Factor analysis is not intended to identify factors which correspond to real, fundamental, underlying attributes of individuals. The factors are constructs which, as described in the previous section, can be viewed as a coalescing of basic characteristics and experiential elements of individuals.

This view of factors as constructs should not, however, be taken to imply that factors are artifacts. They are not. Factor analysis does not create factors, but rather reveals them based on patterns of intercorrelations among surface attributes. Each factor indicates the presence of some systematic influence operating on the surface attributes. The factors vary with respect to the degree with which they can be understood in substantive terms. The purpose of factor analysis methodology is to extract numerical information about these factors from the data, and the responsibility of the researcher is to make use of factor analytic theory and knowledge of the surface attributes to achieve an understanding of the meaning of the factors and the dynamics by which they operate.