F-1. $X, Y, Z$ are topological spaces. Let $f : X \to Y$ be continuous, surjective, and closed (i.e. the $f$-image of closed sets in $X$ are closed in $Y$). Prove that a map $g : Y \to Z$ is continuous if and only if $g \circ f$ is continuous.
F-2. Prove that a topological space $X$ is connected if and only if every map from $X$ to the two-point discrete space is constant.
F-3. The surface $\mathbb{R}P^2$ is the connected sum of the compact, connected surfaces $A$ and $B$. What can $A$ and $B$ be?
F-4. Let $M_g$ be the $g$-fold torus $\#^gT^2$. (a) Find a covering map $M_9 \to M_3$. (b) Do every covering maps $M_9 \to M_3$ have the same number of sheets?
F-5. Let $p : X \to Y$ be the map between the graphs in the picture (i.e. edges $a_i, b_i, c_i$ map homeomorphically to the edges $a, b, c$ respectively, keeping the orientation of the arrows). Show that this is a covering map. What is $\pi_1(X), \pi_1(Y)$? Find the index of the subgroup $p_*(\pi_1(X))$ in $\pi_1(Y)$. What is the group of covering transformations?
F-6. Glue the boundary of a Mobius strip to the equator of $S^2$. Consider the bouquet (one-point-union) of this space with a circle. Find the fundamental group of the resultant space.
F-7. Let $X$ be $\mathbb{R}^2$ with two points removed. Find $H^{1}_{DR}(X)$ (with any method you want). Describe 1-forms on $X$ whose cohomology classes are a basis of $H^1(X)$. 
F-8. Let $X$ be the torus with an open disc removed. Let $B$ be the boundary circle of $X$ with the embedding $i : B \to X$. Compute $\pi_1(X)$. Compute $i_* : \pi_1(B) \to \pi_1(X)$. Prove that there is no retraction of $X$ to $B$. 