

Analyzing variation in continuous reaction norms

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Joint work with

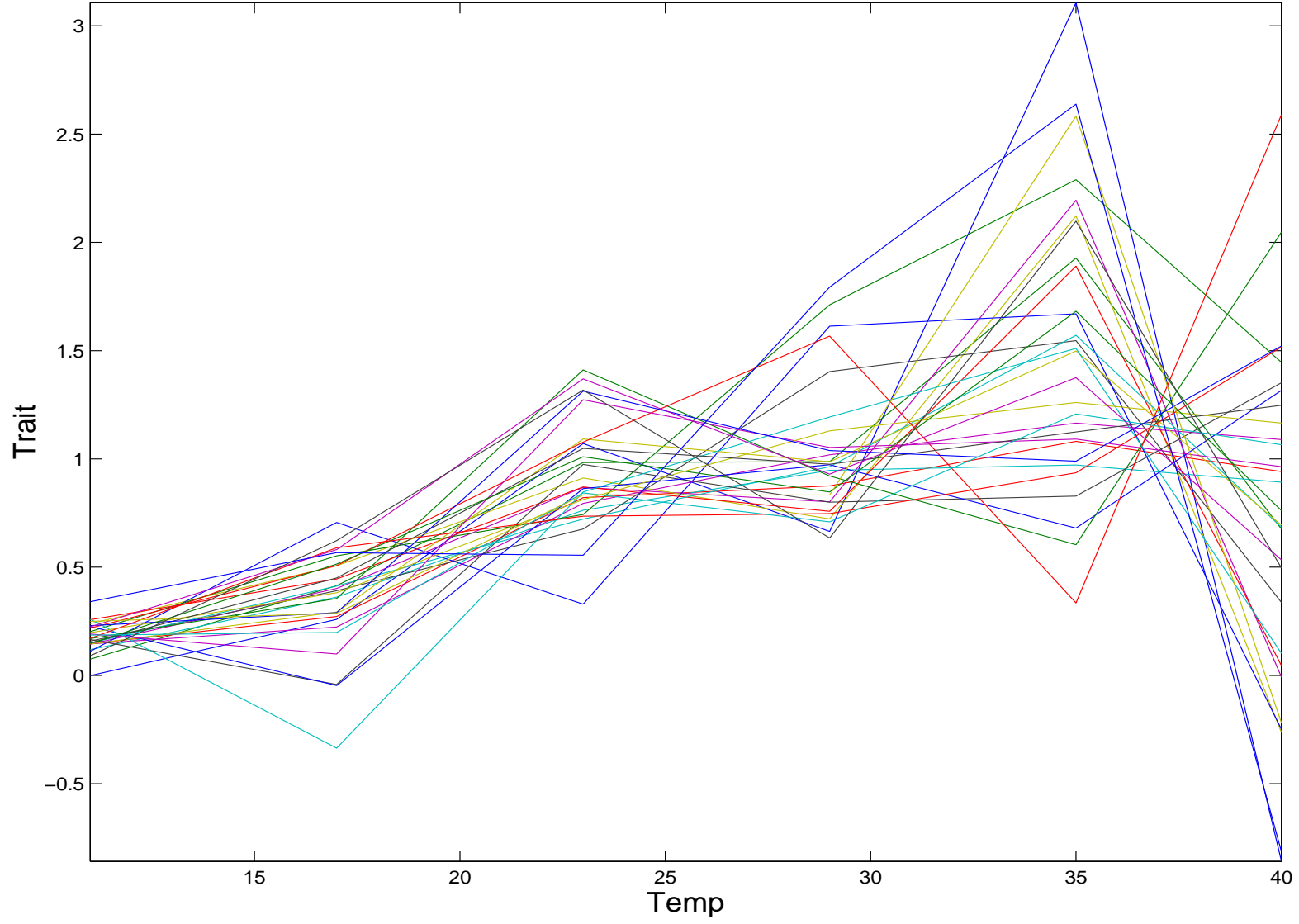
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and

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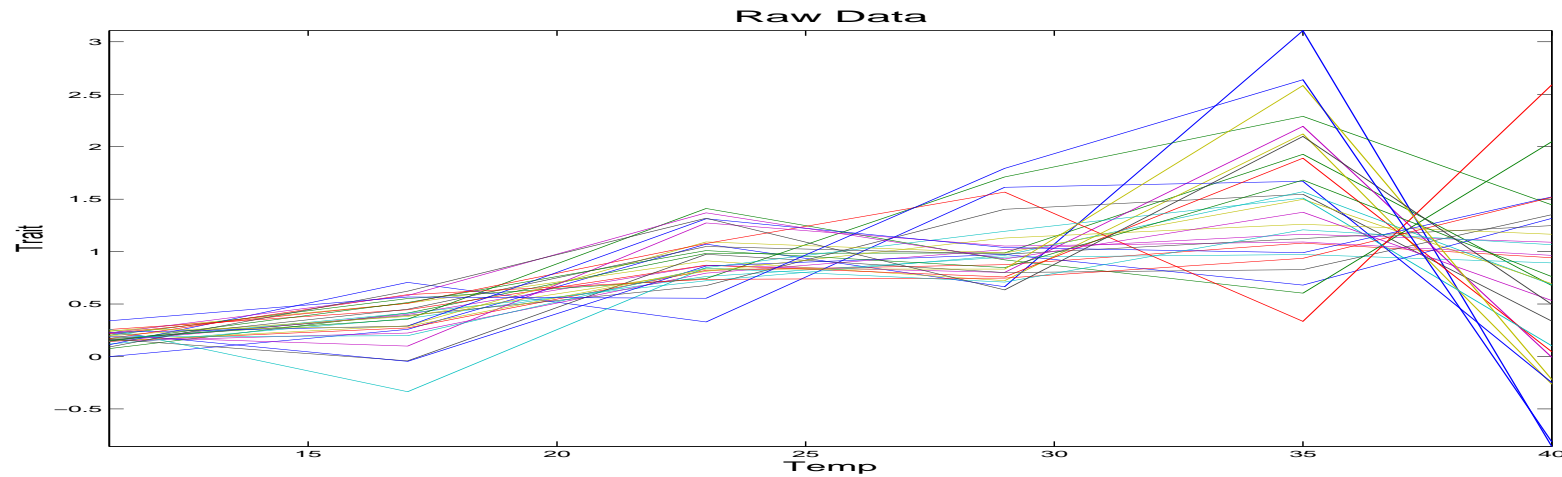
Raw Data



Outline

- Data
- Motivation and model
- Toy example
- Results

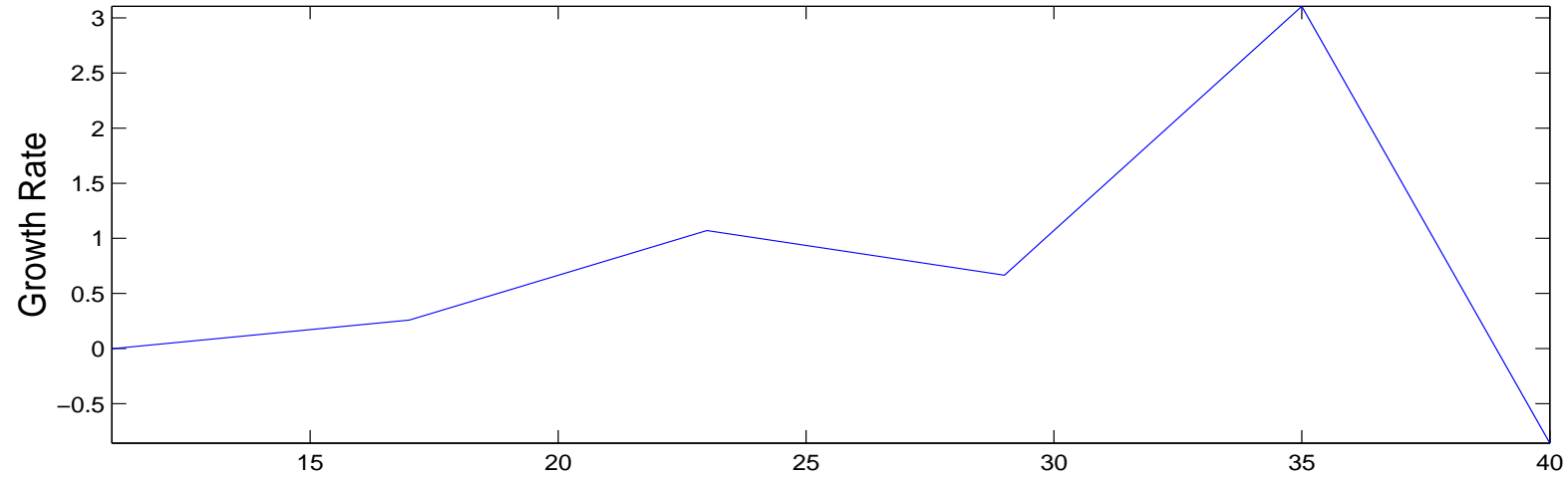
- Performance z . Ex: $z =$ growth rate.
- t is the temperature. Ex: $d = 6$, and $t_1, \dots, t_6 \in [11, 41]$ Celsius.
- Thermal performance curve (TPC): $z = f(t)$.
- Reaction Norm Curve(CRN): $z = f(e)$.



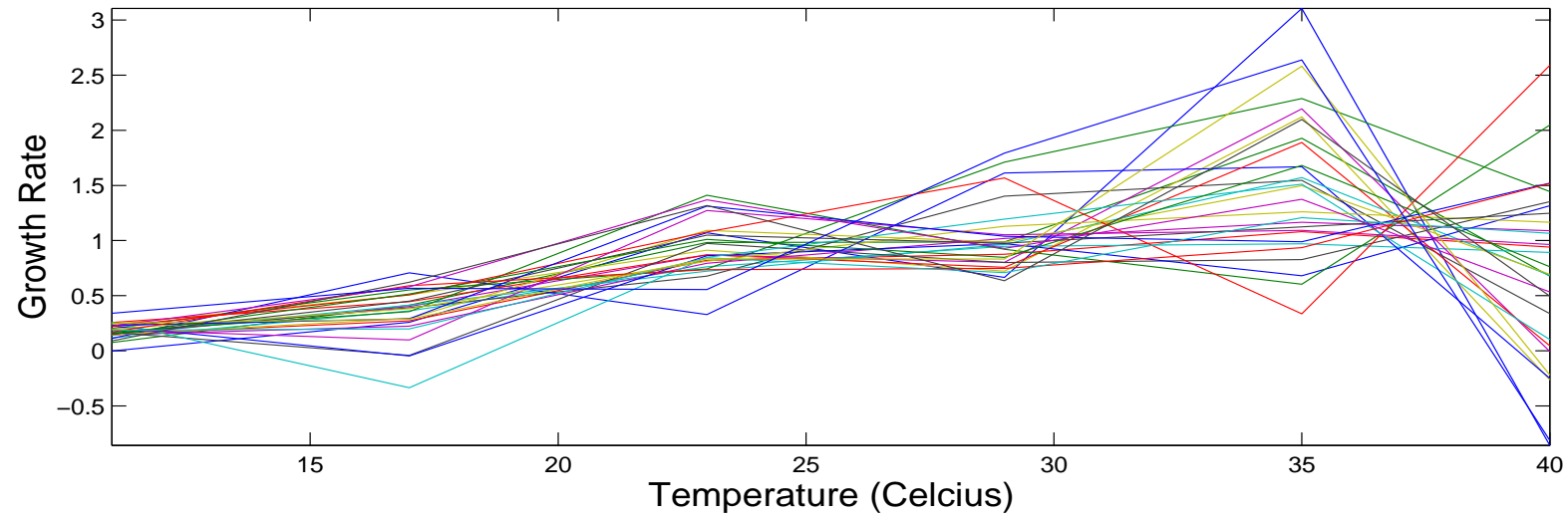
Template shape $f(t)$ of TPC:

- \uparrow , reaches a max and \downarrow .
- Rate of \uparrow slower than rate of \downarrow .
- Tendency toward a unique max.

Thermal Performance Curve for a family



Thermal Performance Curves for the population



- Family = individuals of similar genotype.
Ex: offsprings of same parents, clones,...
- Population = n families, Ex: $n = 32$.

Outline

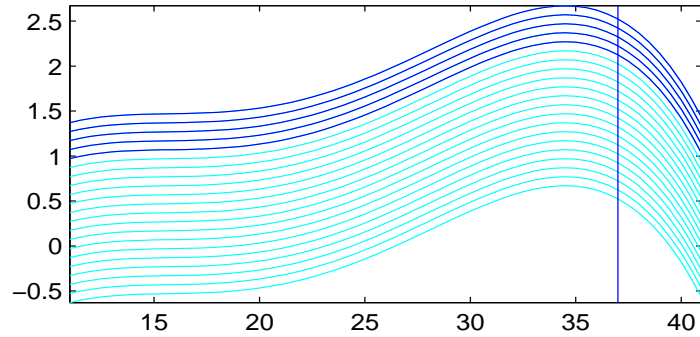
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Variation in TPC = genetic variation in a population

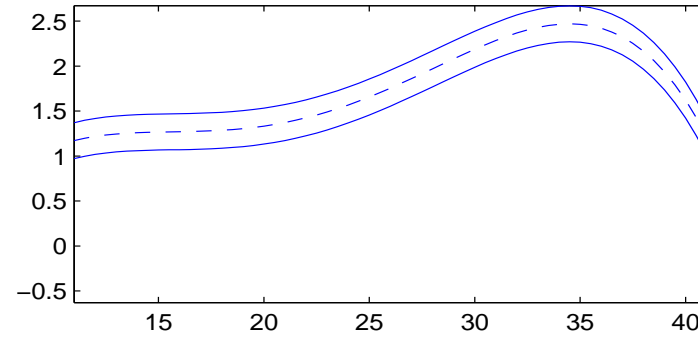
- **Goal:** quantify three TPC modes of variation.
- **Motivation:** **evolution** of genetic variation under **selection**.
- Mode of variation \equiv Constraint to evolution

Mode of variation before selection – Genetic variation after selection

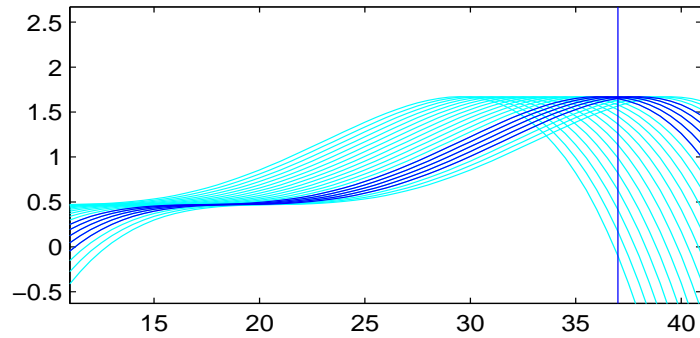
Vertical Shift



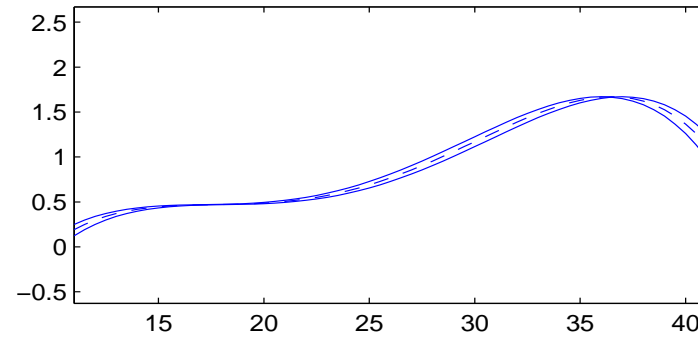
top 25% at t=37



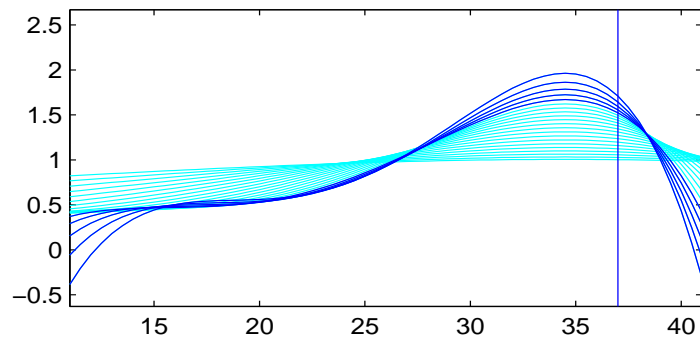
Horizontal Shift



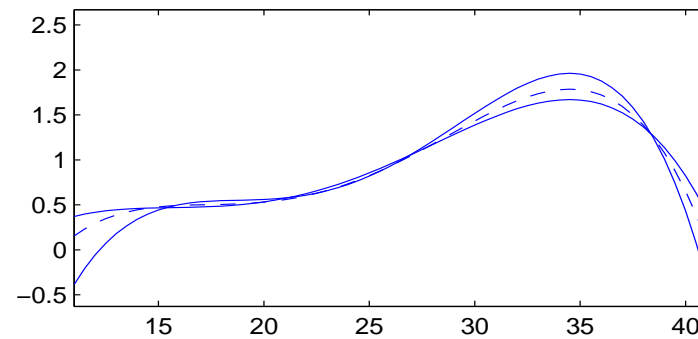
top 25% at t=37



Generalist–Specialist

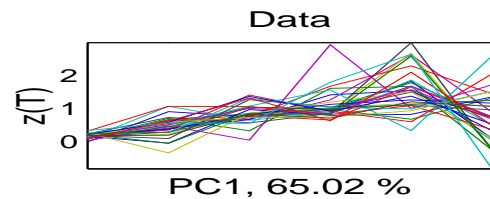


top 25% at t=37

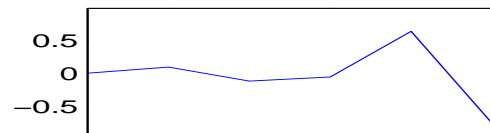


Mode of variation \equiv Constraint to evolution

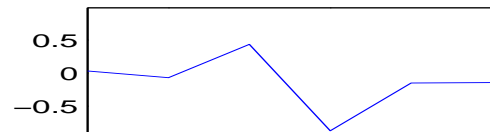
- Vertical Shift, linear.
- Horizontal Shift, nonlinear.
- Generalist-Specialist, nonlinear.



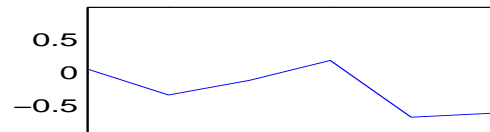
PC1, 65.02 %



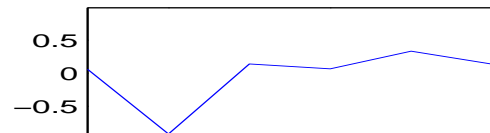
PC2, 17.67 %



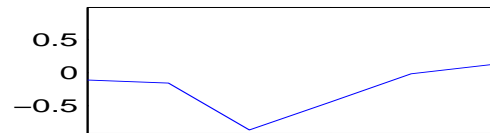
PC3, 8.98 %



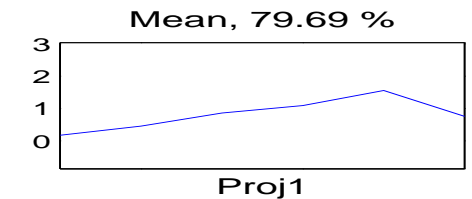
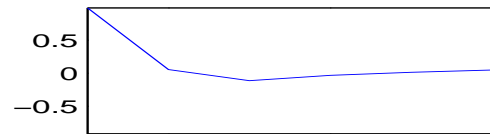
PC4, 5.61 %



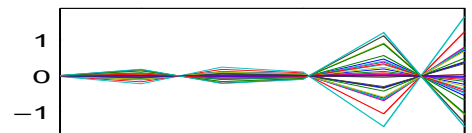
PC5, 2.40 %



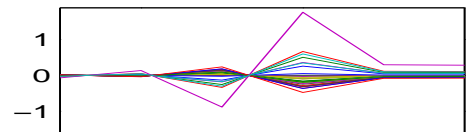
PC6, 0.32 %



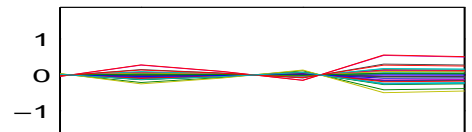
Proj1



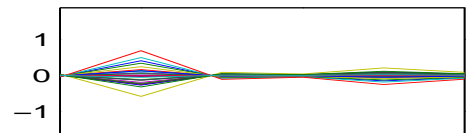
Proj2



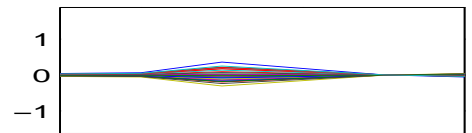
Proj3



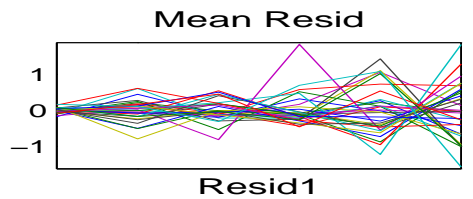
Proj4



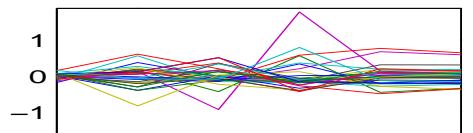
Proj5



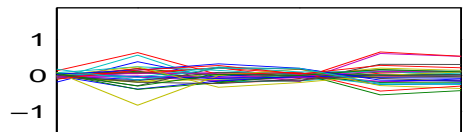
Proj6



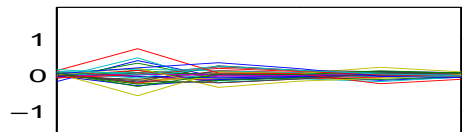
Resid1



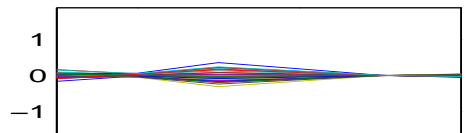
Resid2



Resid3



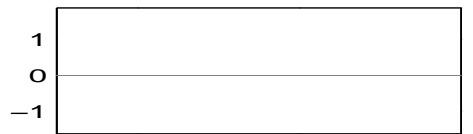
Resid4



Resid5



Resid6



PCA

- 1. PC1 direction of greatest variability
- 2. PC2 direction of greatest variability \perp to PC1
- 3. PC3 direction of greatest variability \perp (PC1,PC2) ...
- Each variation is linear quantified by RSS.

PCA

No assumption on shape

Linear PC of G

Meaning of each PC?

RSS quantify a PC

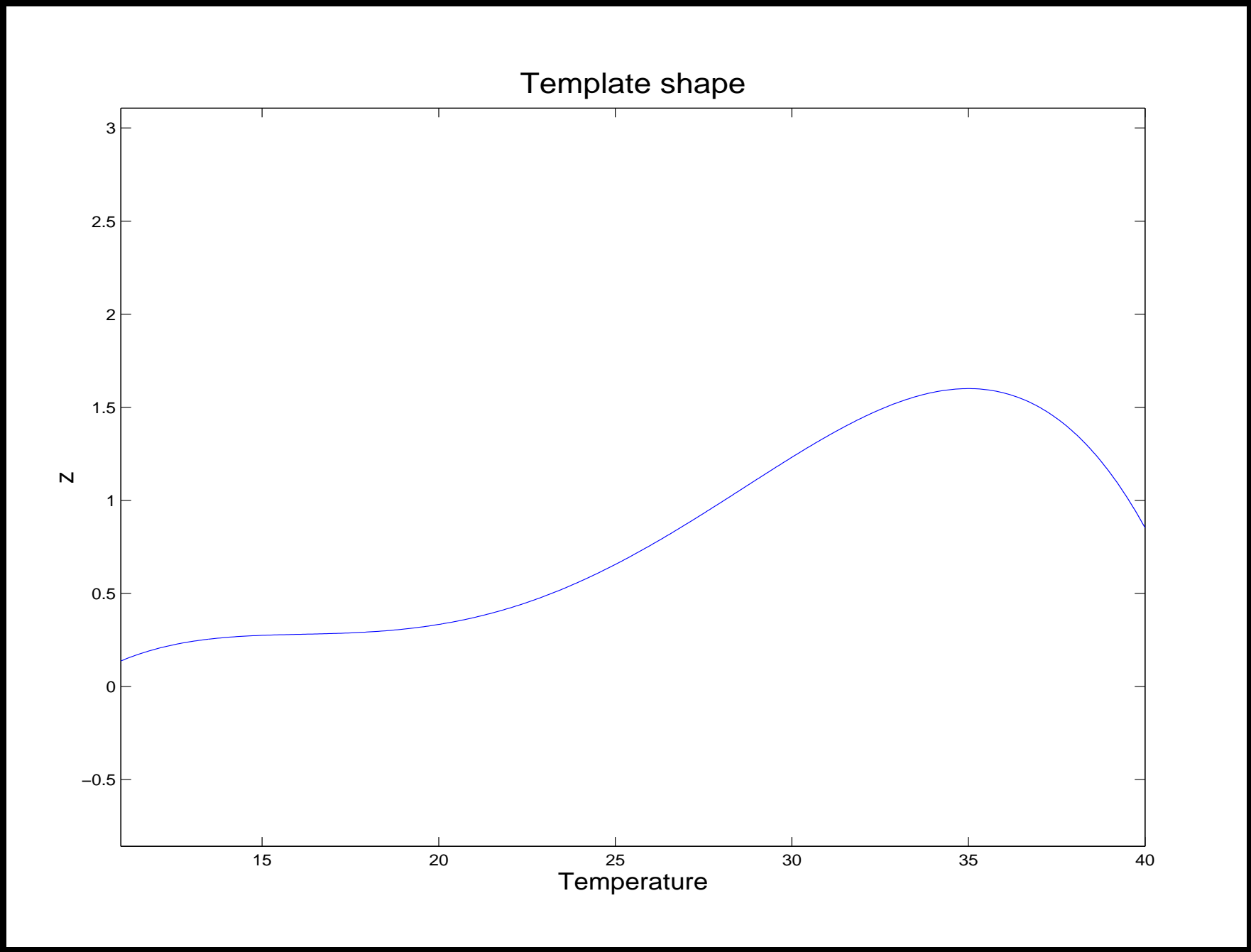
Template Modes of Variations

Template shape of TPCs.

Nonlinear modes of variation of G .

Biological modes.

\widetilde{RSS} quantify a mode.



Model

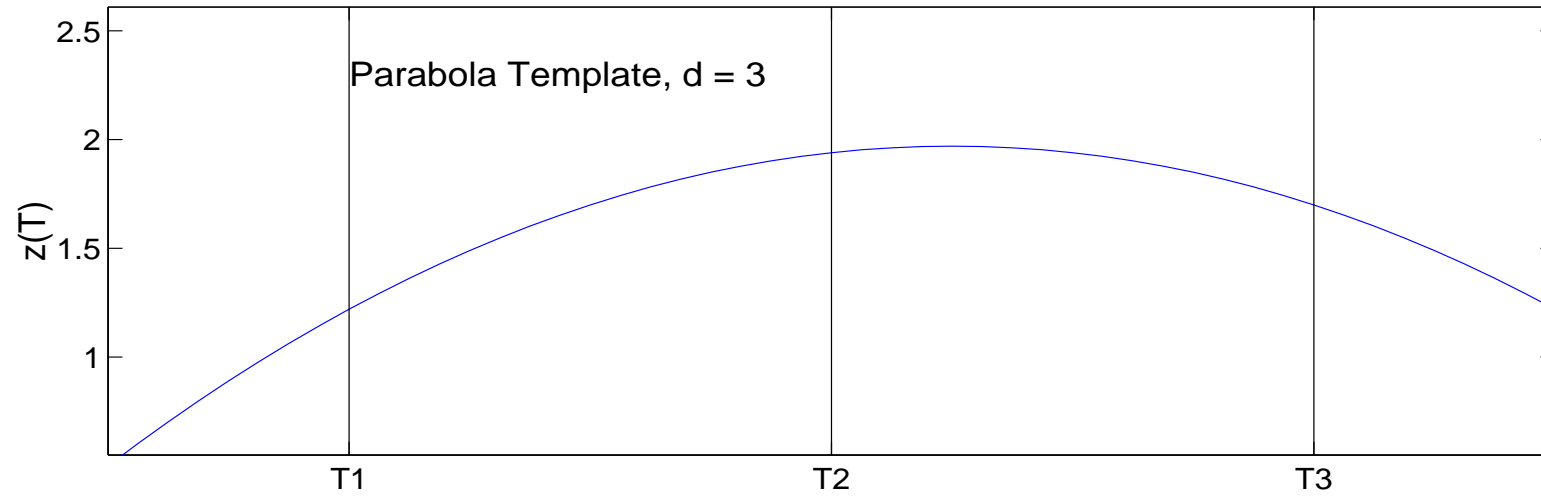
$$Z_i(t_j) = R_i(t_j) + \epsilon_{i,j}, 1 \leq i \leq n \text{ and } 1 \leq j \leq d.$$

	$R_i(t)$	Parameter	Variation
Vertical shift	$f(t) + h_i$	h_i	linear
Horizontal shift	$f(t - m_i)$	m_i	nonlinear
Gen-Spec	$g(w_i)f(w_it),$ $g(w_i)$ such as $\int_{t_1}^{t_d} R_i(t)dt = C.$	w_i	nonlinear

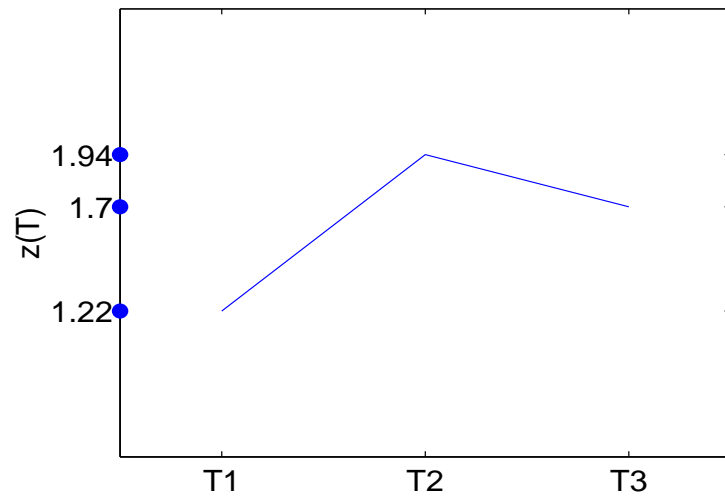
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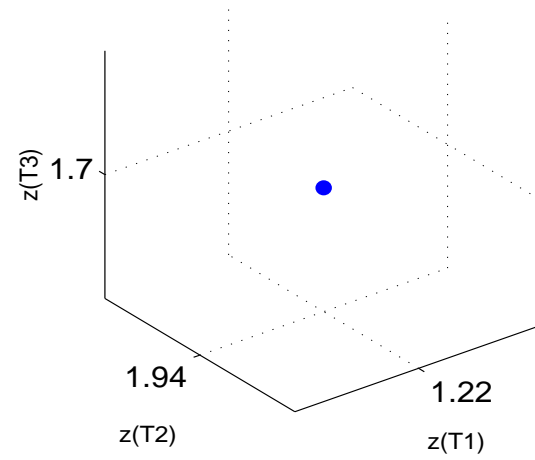
Curve space



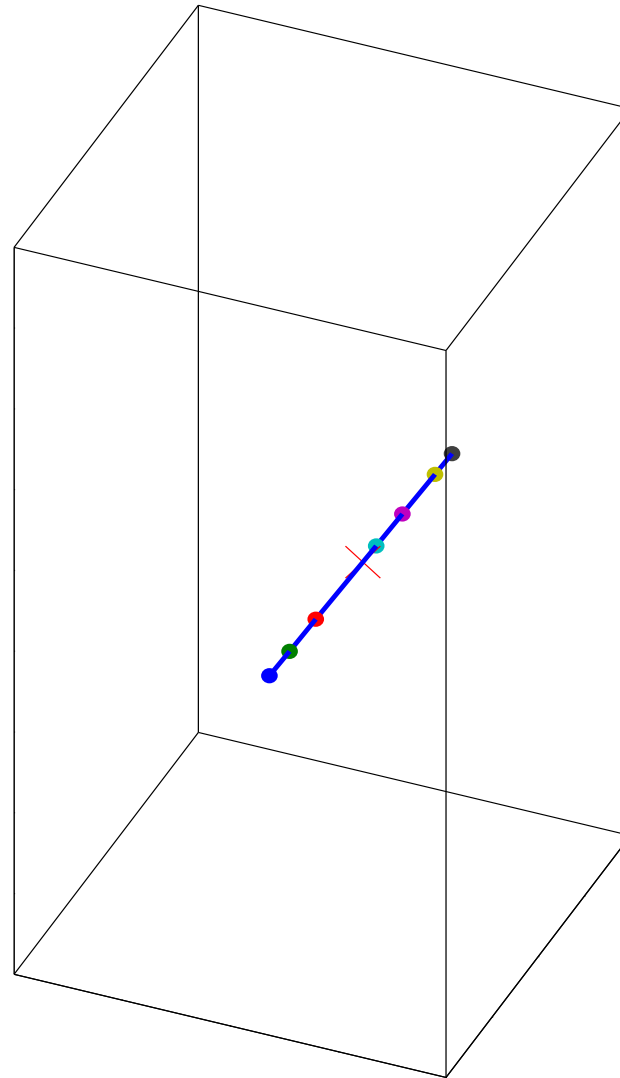
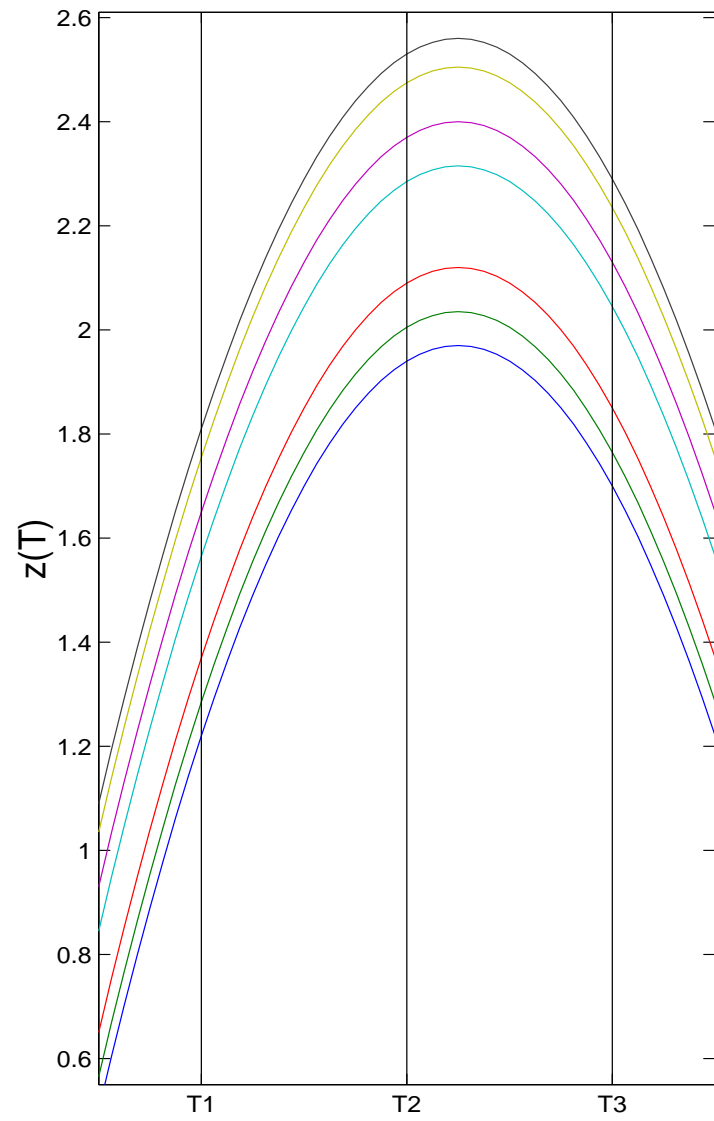
$d = 3$



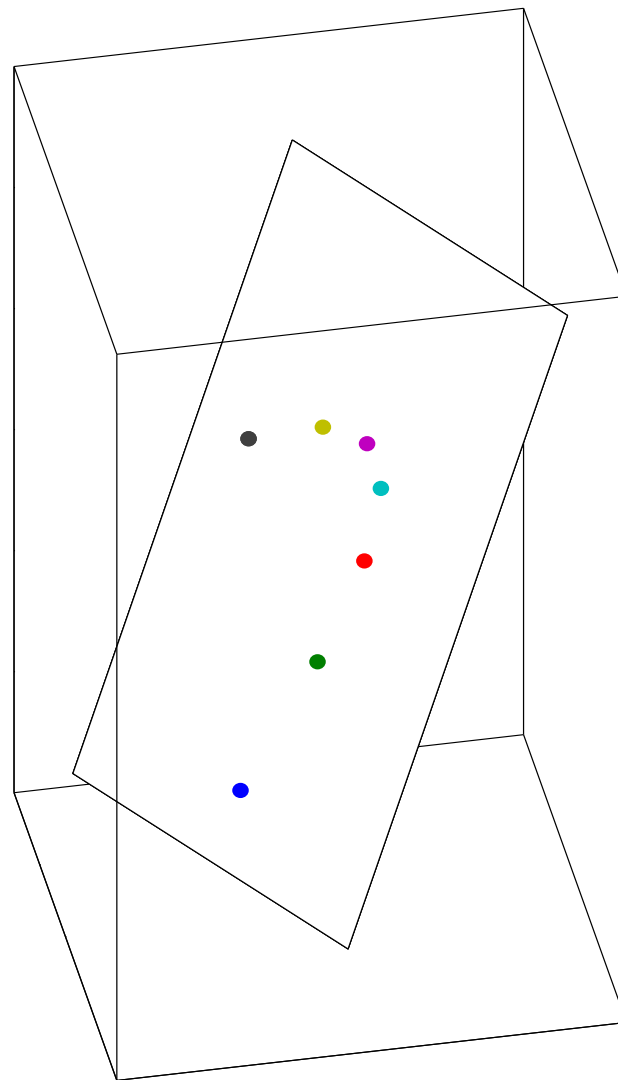
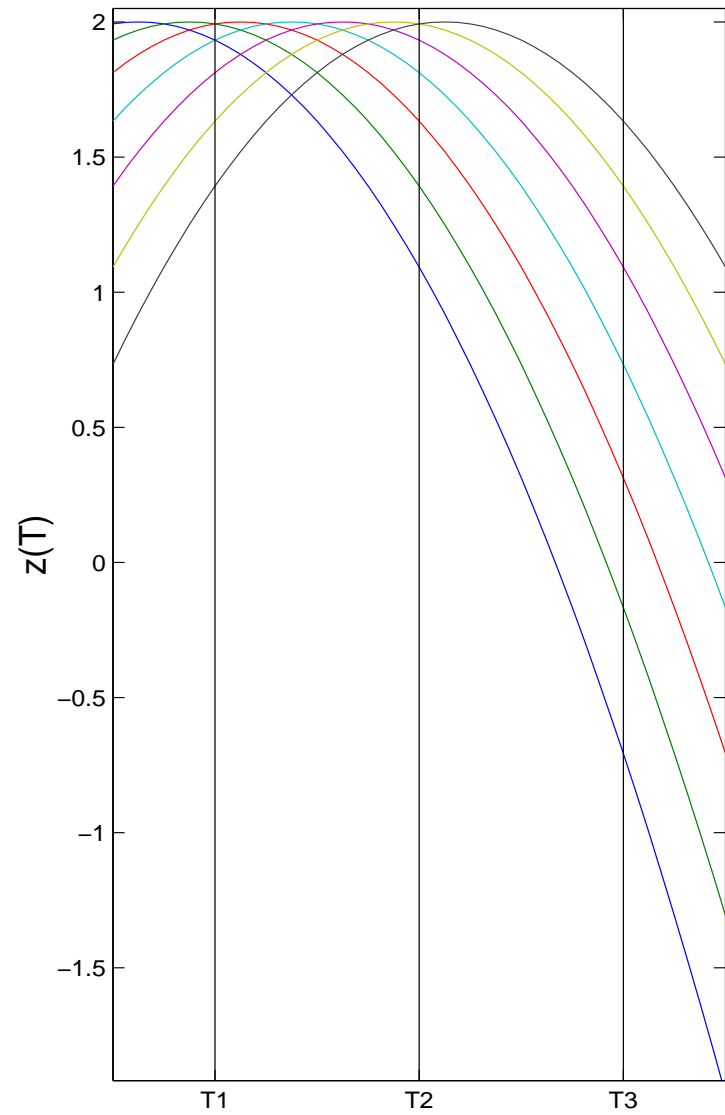
Point cloud space



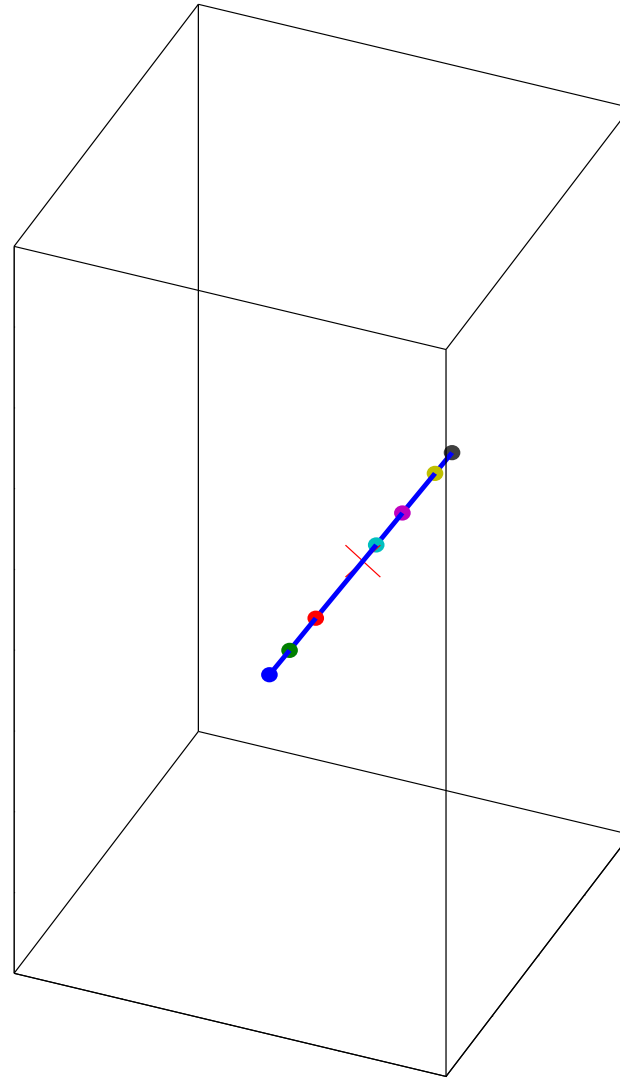
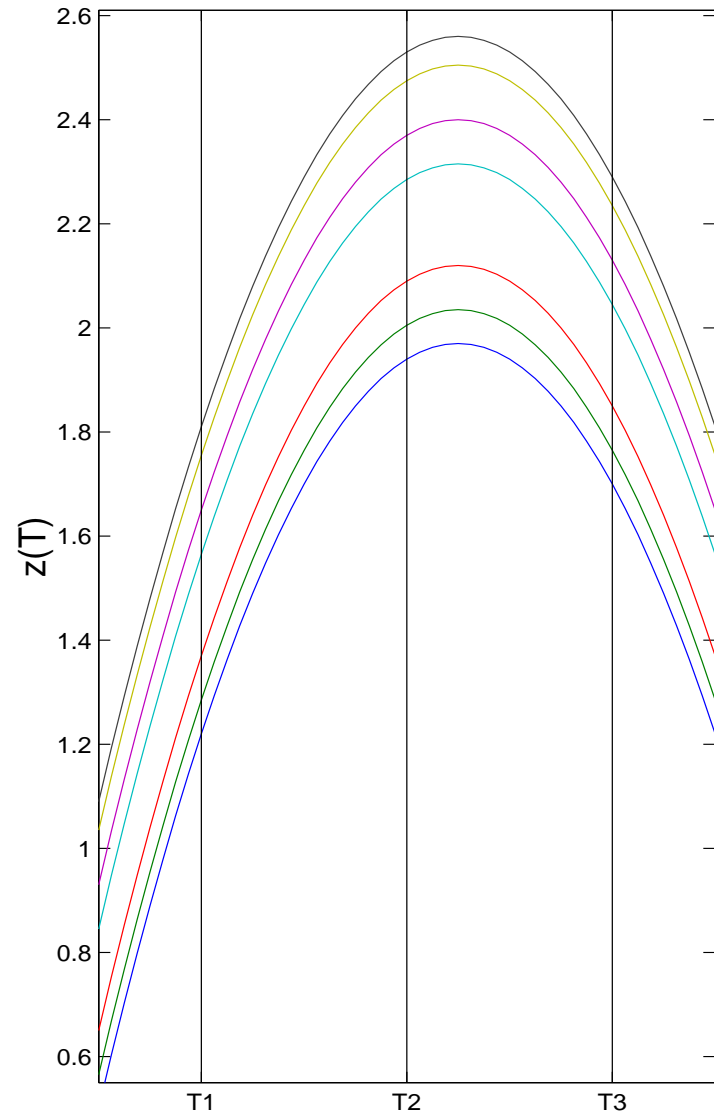
Vertical Shift



Horizontal Shift

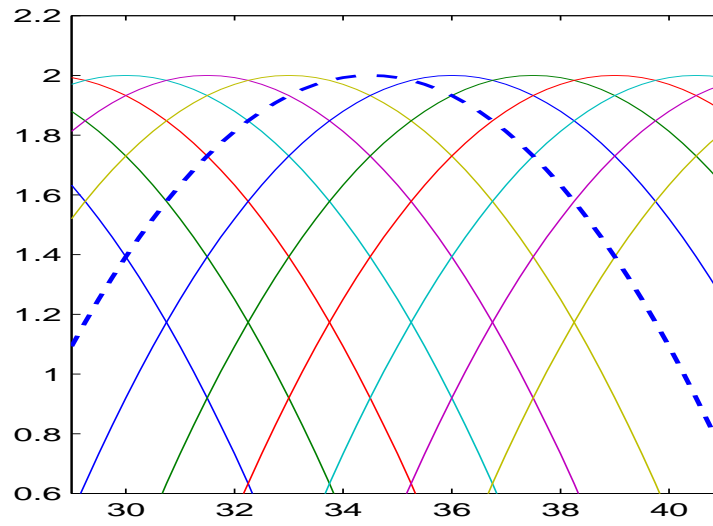
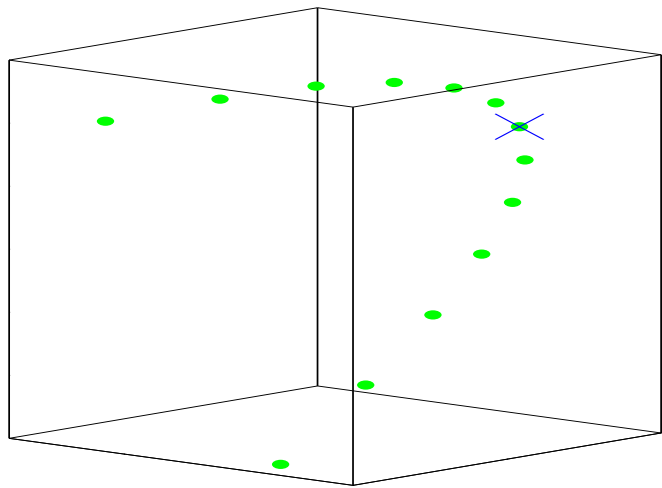
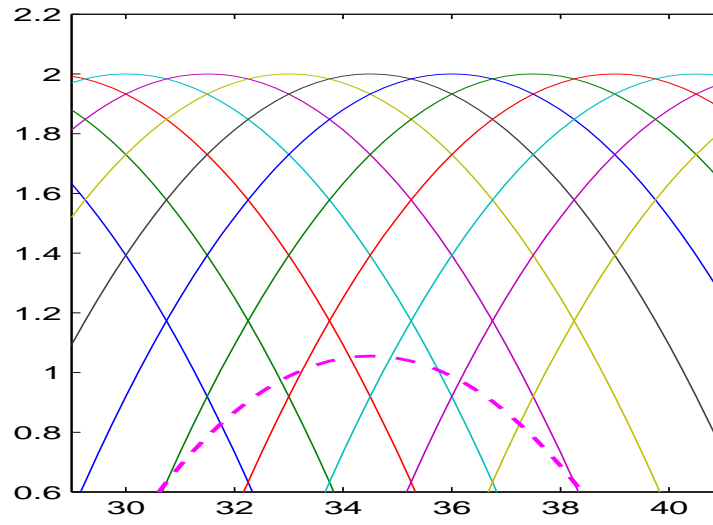
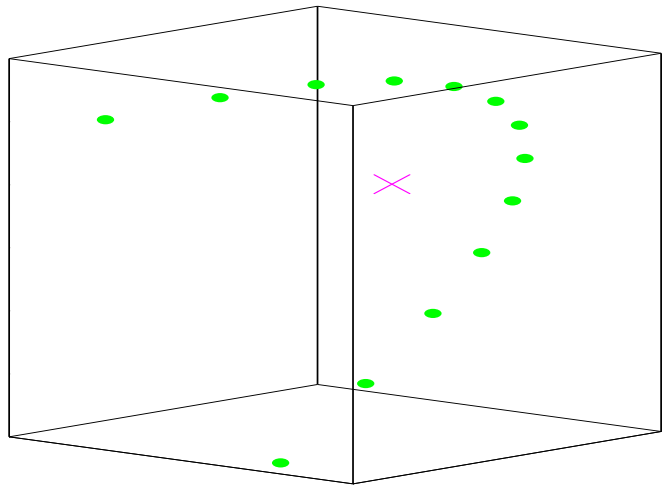


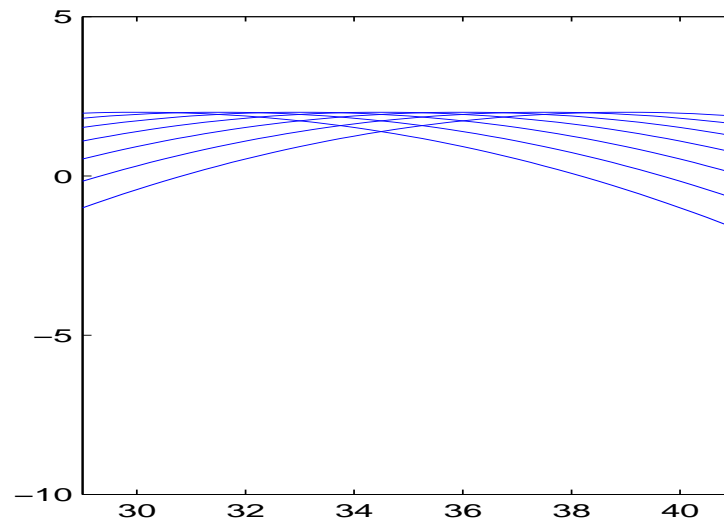
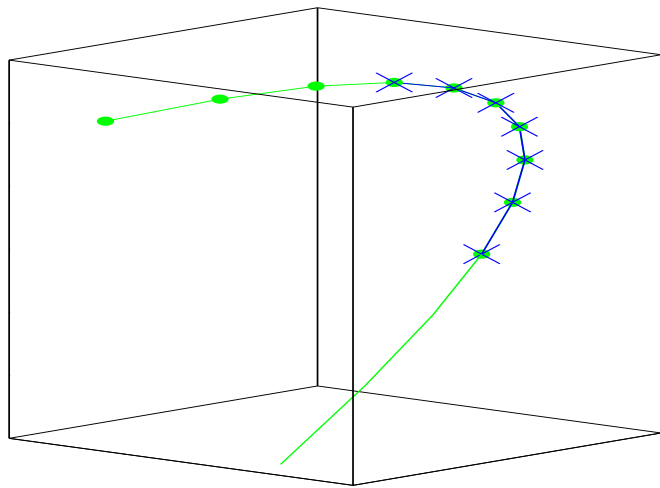
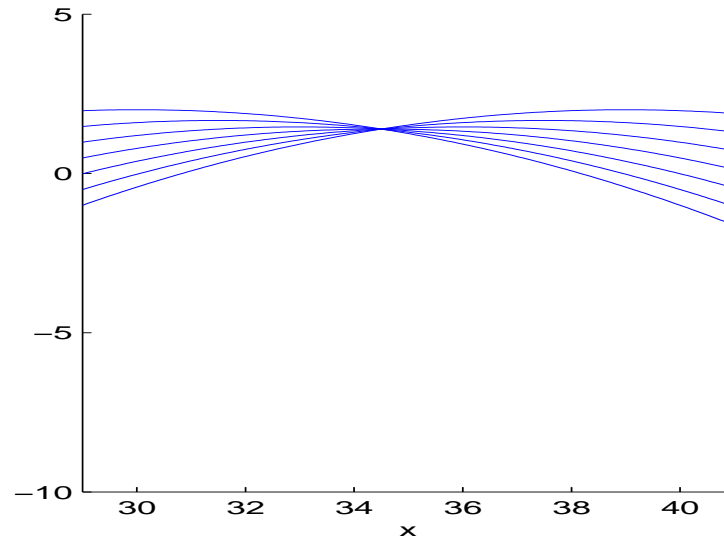
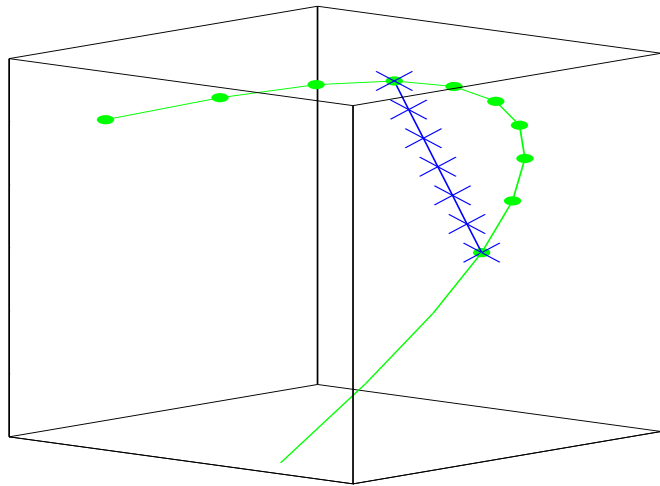
Vertical Shift



Linear case:

- Center of the projections: \bar{R} , and $\bar{R} = \bar{Z}$.
- Spread: $SSM = \sum_{i=1}^n \|R_i - \bar{Z}\|^2$.
- $RSS = \frac{SSM}{SSM+SSE}$ quantifies the variation.





Nonlinear case

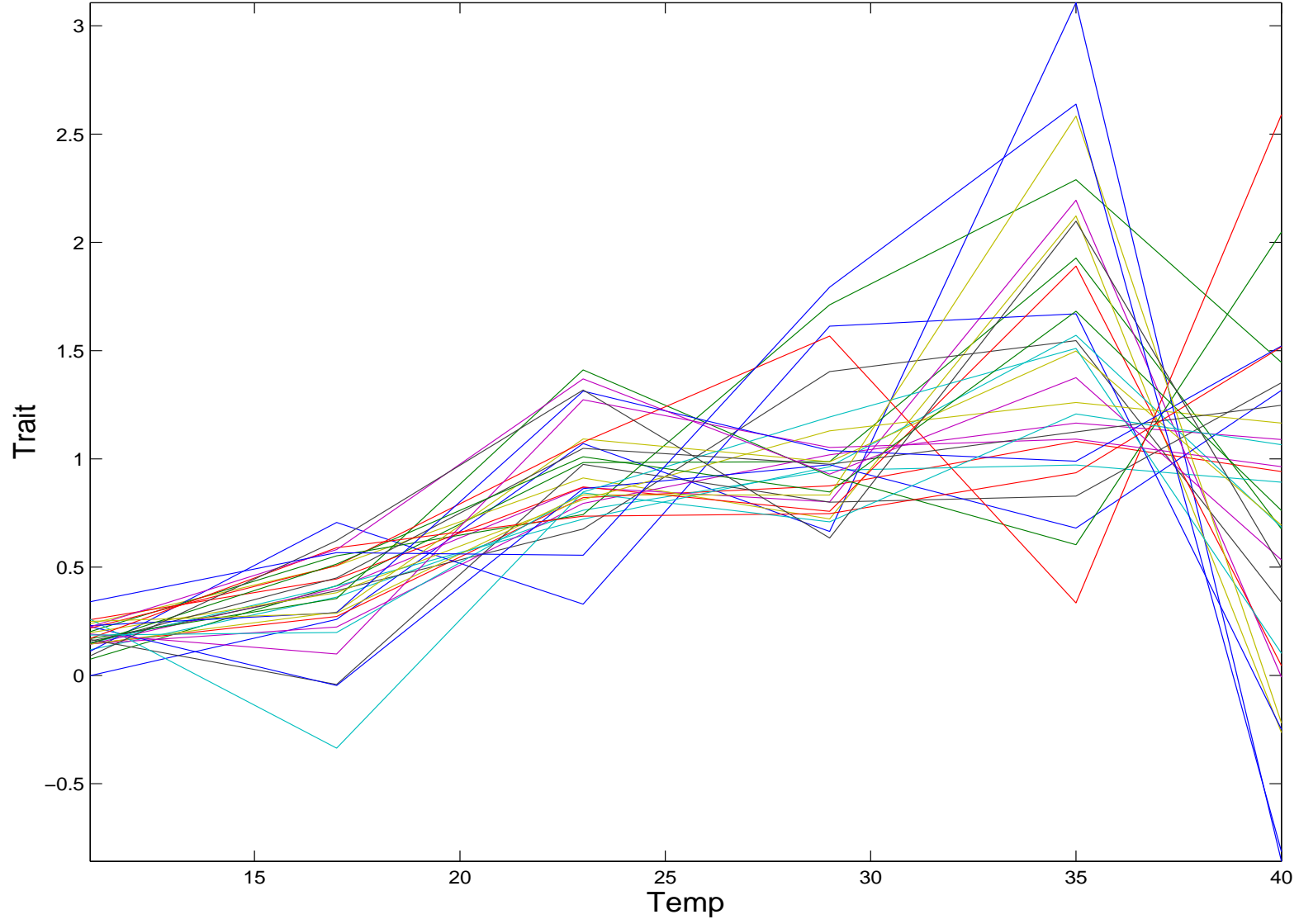
- \tilde{R} is the sample geodesic mean.
- d_g is the geometric distance.
- \widetilde{RSS} quantifies nonlinear variations.

Linear	Nonlinear
$RSS = \frac{SSM}{SSM+SSE}$	$\widetilde{RSS} = \frac{\widetilde{SSM}}{\widetilde{SSM}+SSE}$
$SSM = \sum_{i=1}^n \ R_i - \bar{Z}\ ^2$	$\widetilde{SSM} = \sum_{i=1}^n \left(d_g(R_i, \tilde{R}) \right)^2$
$\bar{Z} = \bar{R}$	$\bar{Z} \neq \tilde{R}$

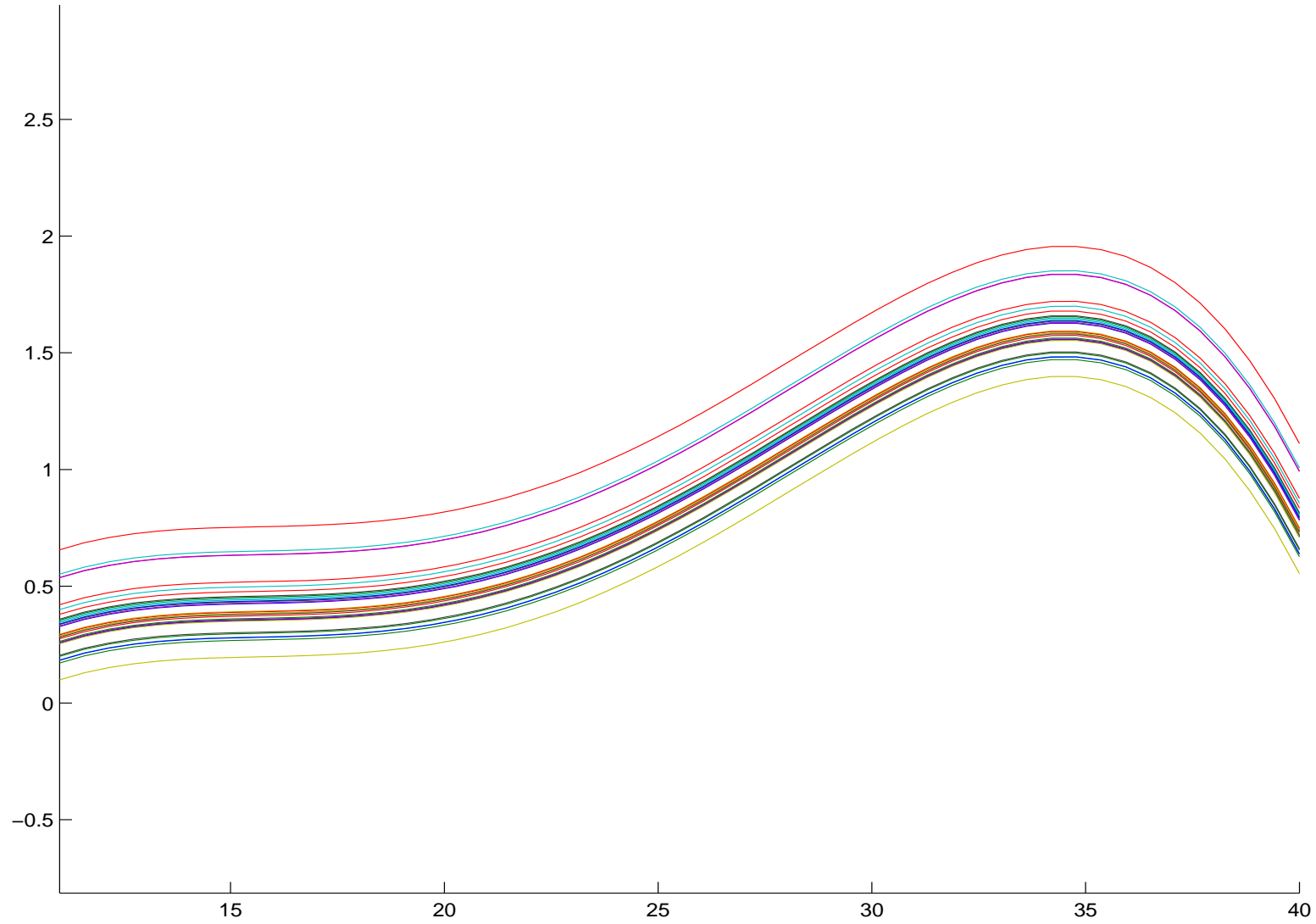
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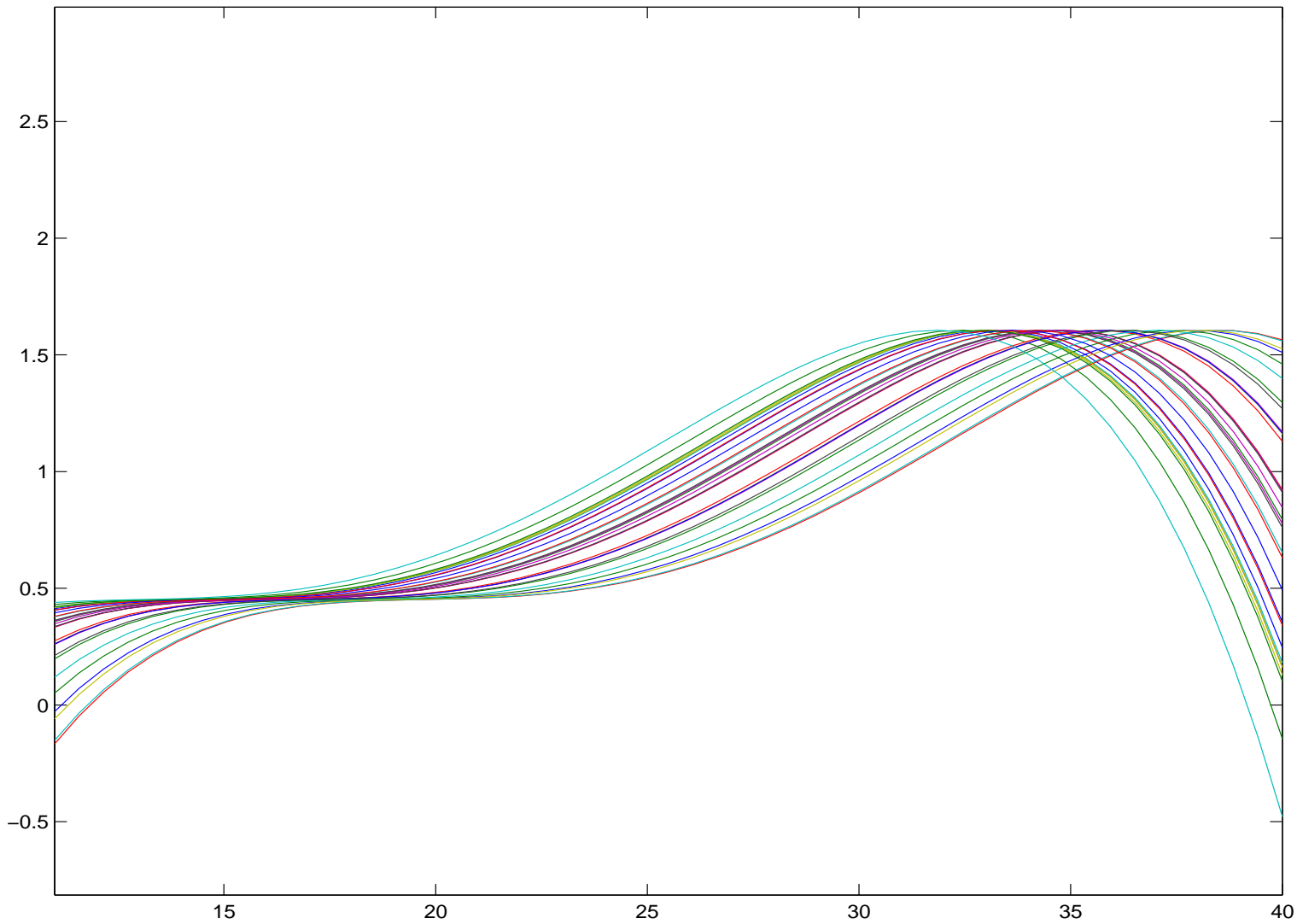
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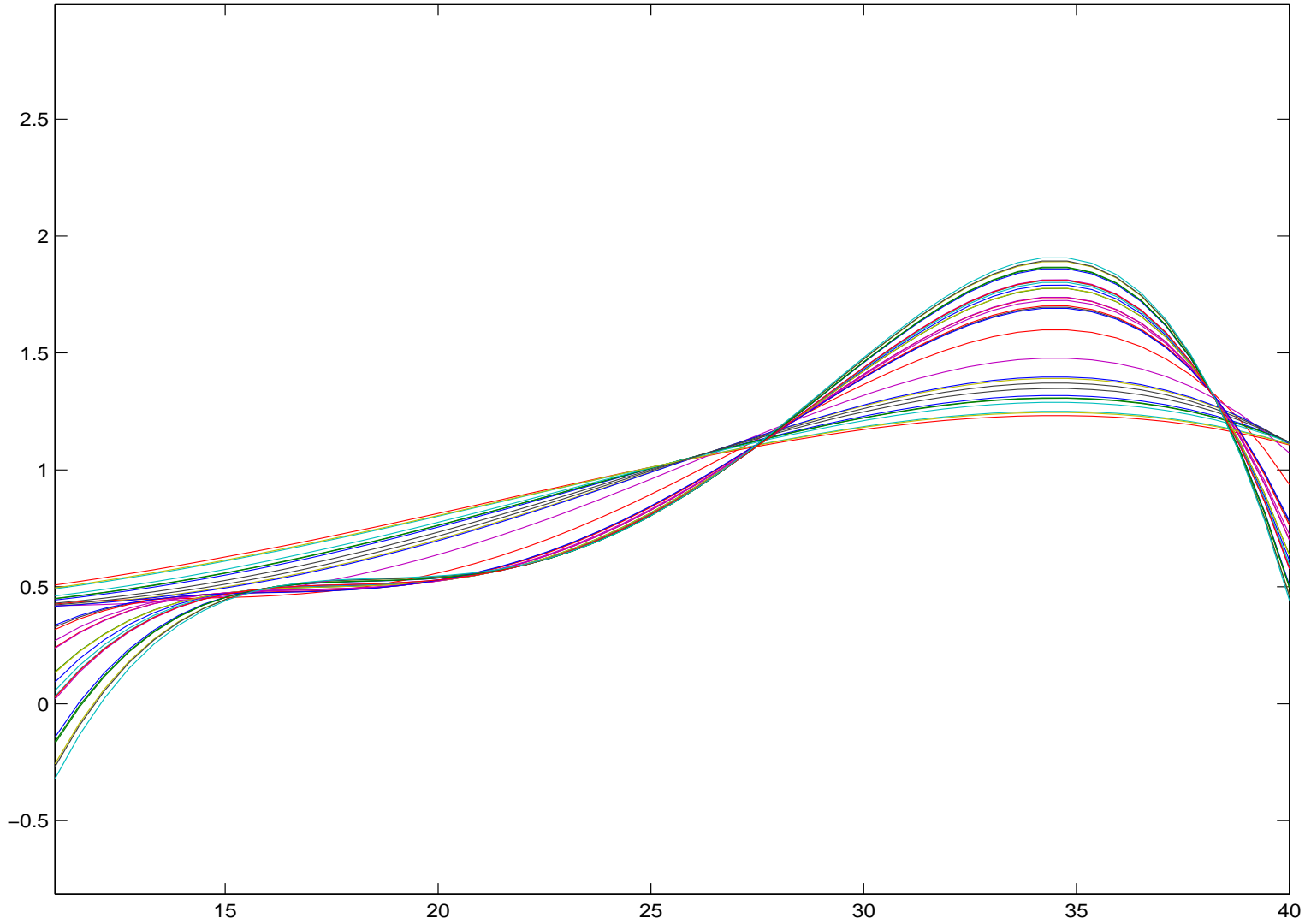
Vertical Shift projections



Horizontal Shift projections



G-S projections



Mode of variation	SSE	$SSM(RSS)$	$\widetilde{SSM}(\widetilde{RSS})$
Vertical Shift	42.67	2.69(6%)	2.69(6%)
Horizontal Shift	32.44	12.11(27%)	13.36(29%)
Generalist Specialist	31.97	6.63(17%)	9.71(23.30%)

Conclusion in evolutionary biology: Selecting for higher growth rate in a range of temperature will not result in higher growth rate at all temperatures.

Family parameters

Mode of variation	Parameter	range (mean,sd)
Vertical Shift	h_i	[1.38, 1.96](1.62, 0.12)
Horizontal Shift	m_i	[31.85, 38.59](34.9, 1.88)
Generalist-Specialist	w_i	[0.49, 1.19](0.92, 0.26)

Summary: One variation at a time

- Parameters optimized to minimize the SSE .
- The modes of variations overlap each other.

Work in progress: Simultaneous variation $g(w_i)f(w_i(t - m_i)) + h_i$

- Projection on each mode.
- Quantification of each variation.
- Decomposition of the total variation in the three modes.