

Functional Data Analysis in Continuous Reaction
Norms: Identifying Nonlinear Variations

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Joint work with

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and

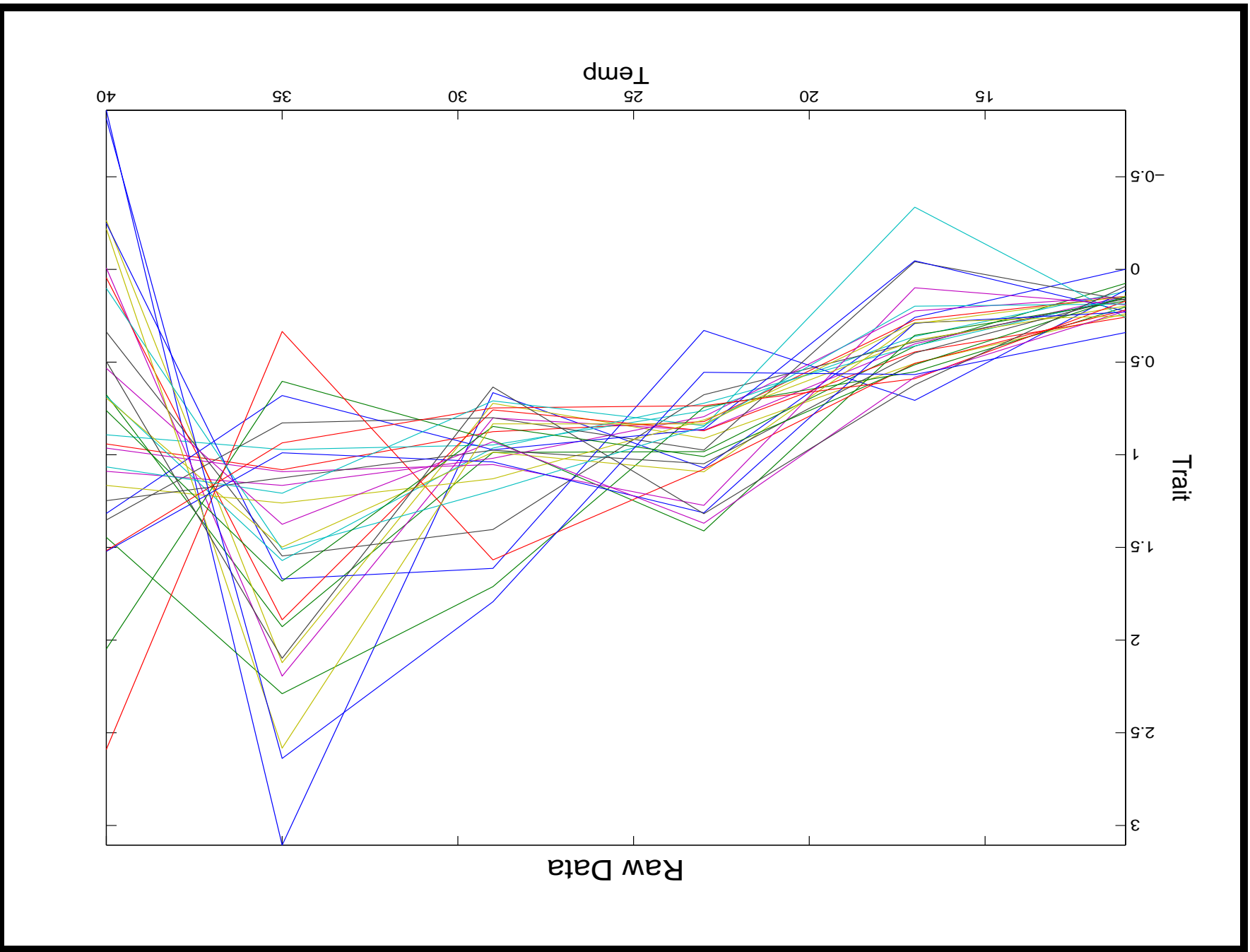
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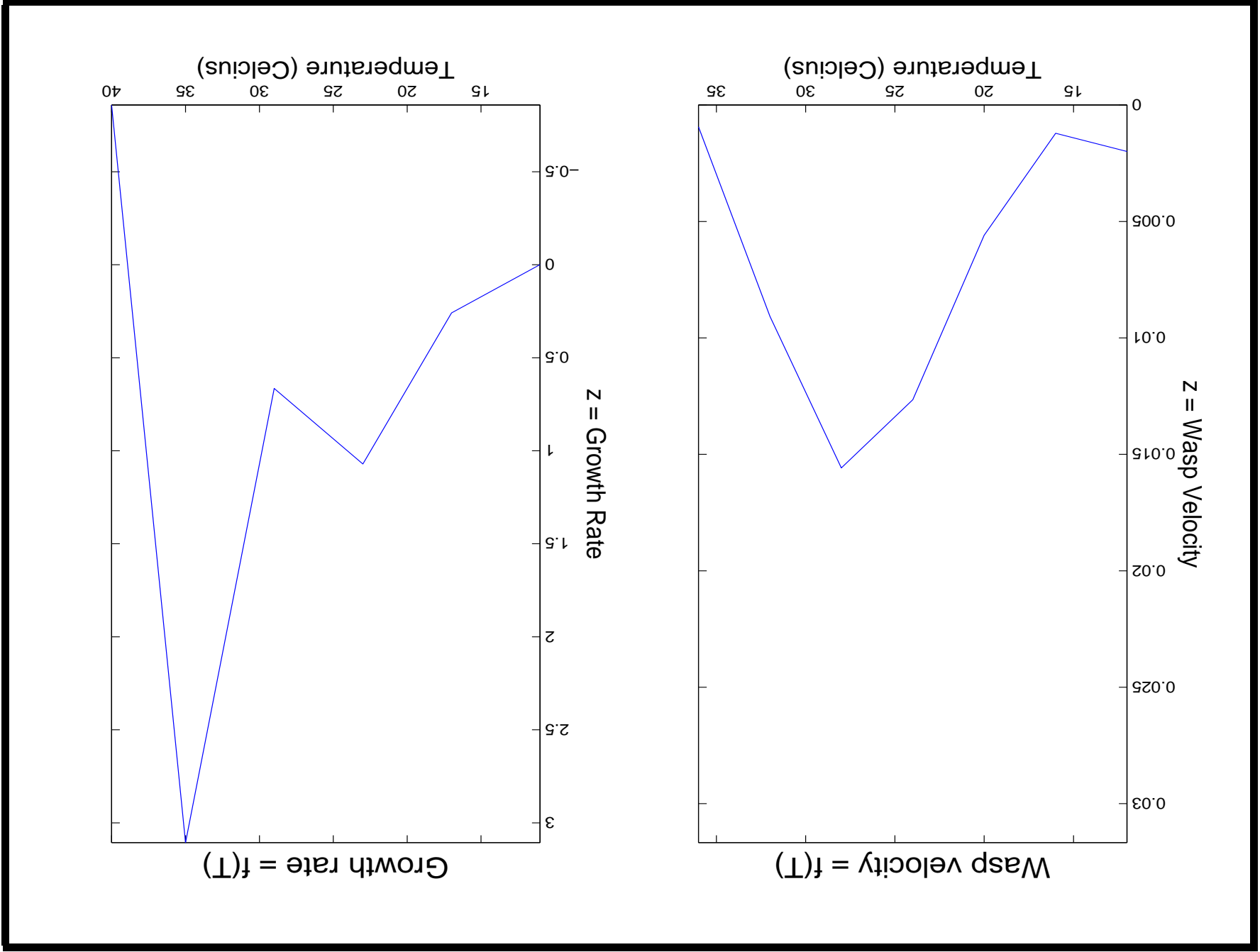
July 16th, 2002

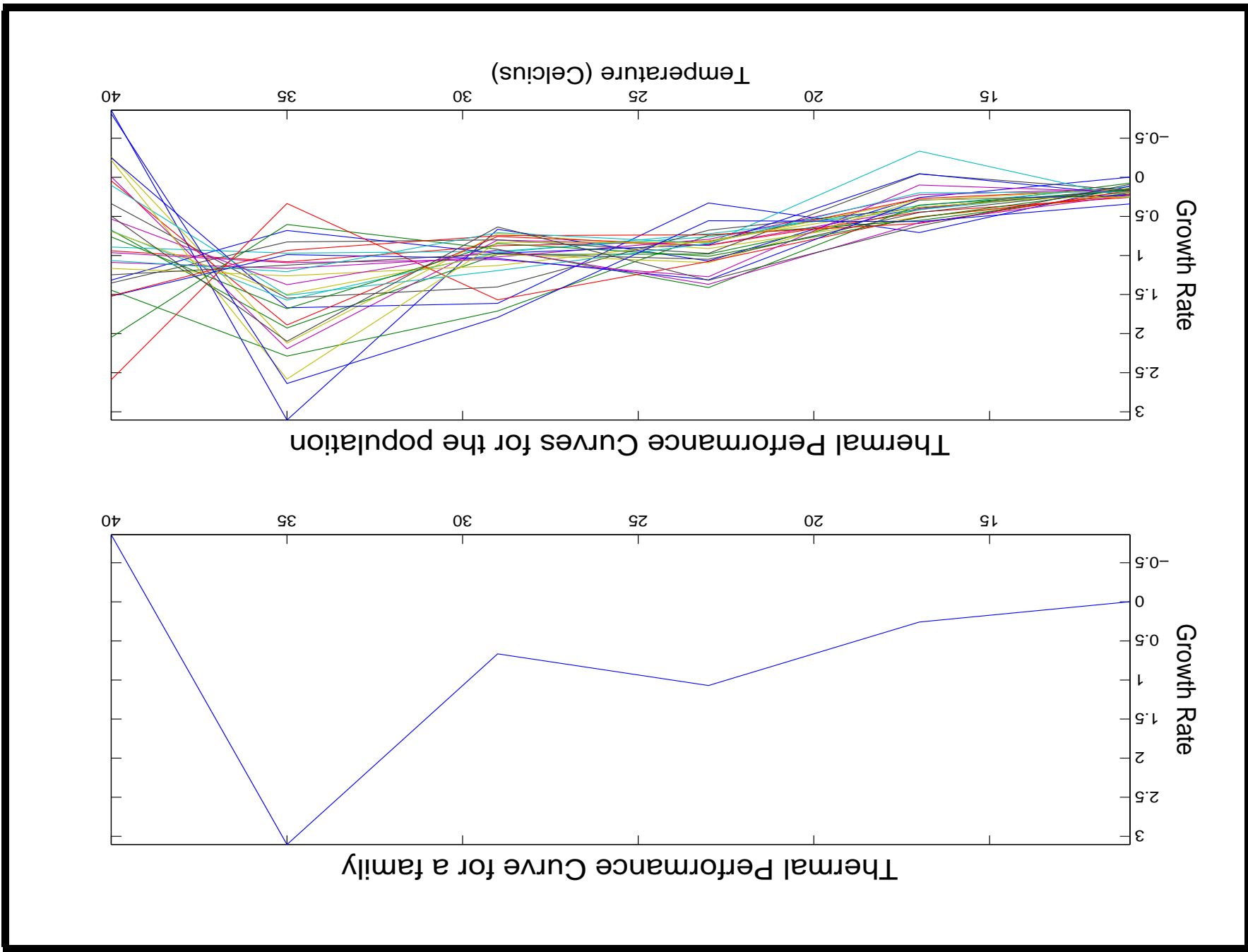
Outline

- I. Thermal performance curves (TPCs).
- II. Modes of variation.
- III. Example: Growth rate curves.



- Continuous Reaction Norm: $z = f(e)$.
- Trait z : phenotype characteristic of a genotype.
- Thermal Performance Curve (TPC): $z = f(T)$,
 T temperature.





- 1 curve: TPC for 1 family.
- n curves: TPCs for the population.
- $d = \text{dimension} = \# \text{ temperatures}$.

PCA	No assumption on shape	Linear PC of G	Meaning of each PC?
Template Modes of Variations	Template shape of TPCs.	Nonlinear modes of variation of G .	Biological modes.

I. Thermal performance curves (TPCs).

II. [Modes of variation](#)

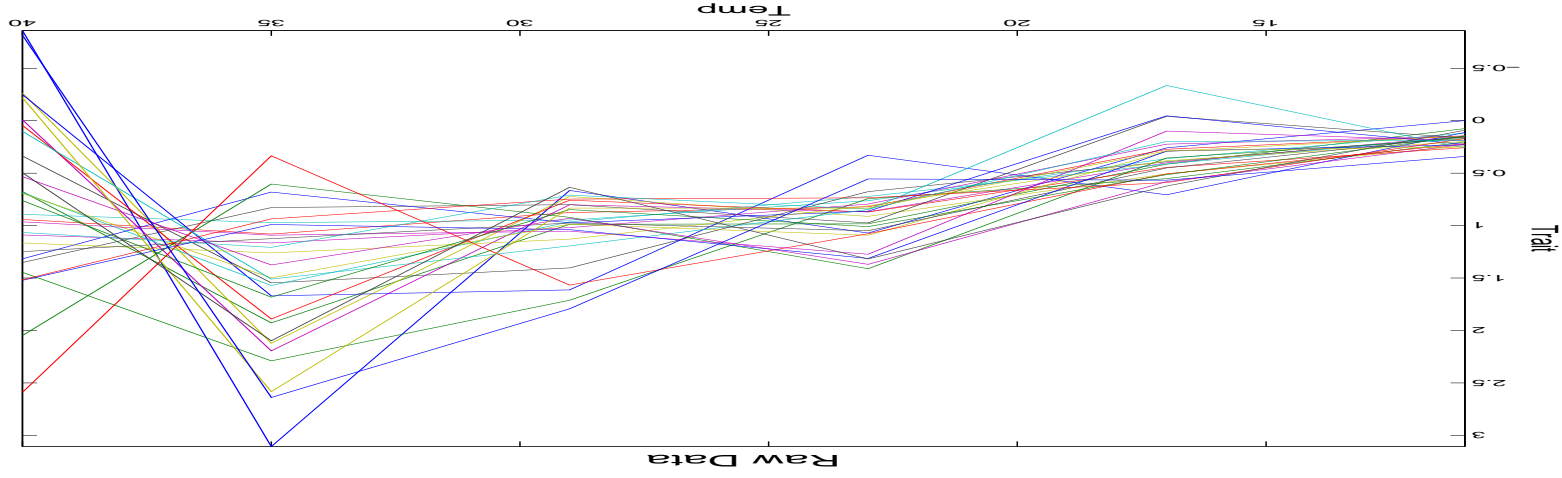
1. Template shape and Modes of variations.

2. Illustration: Parabola.

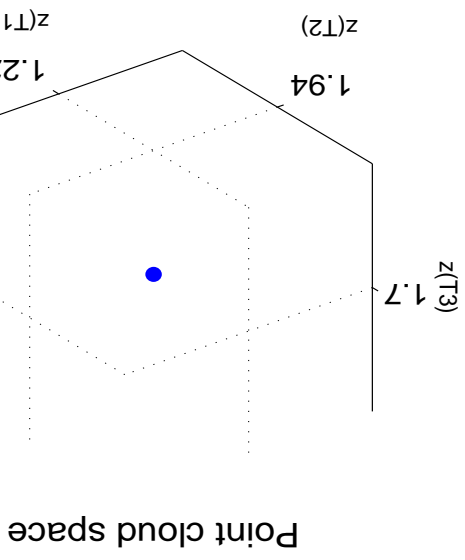
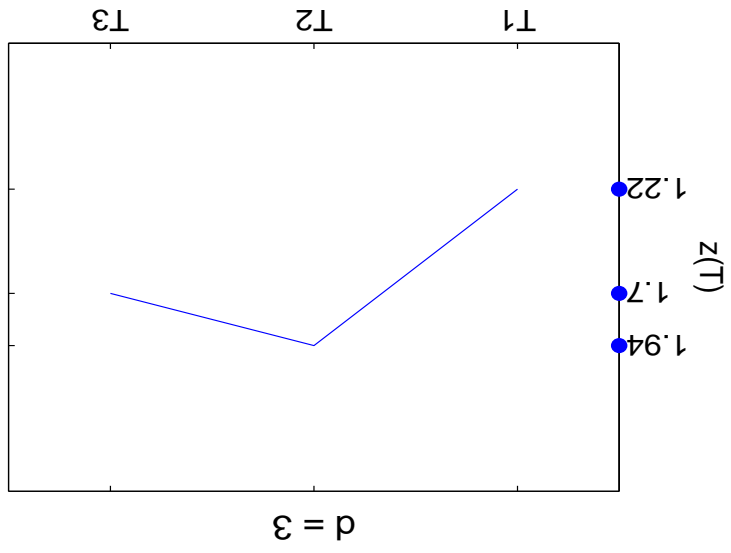
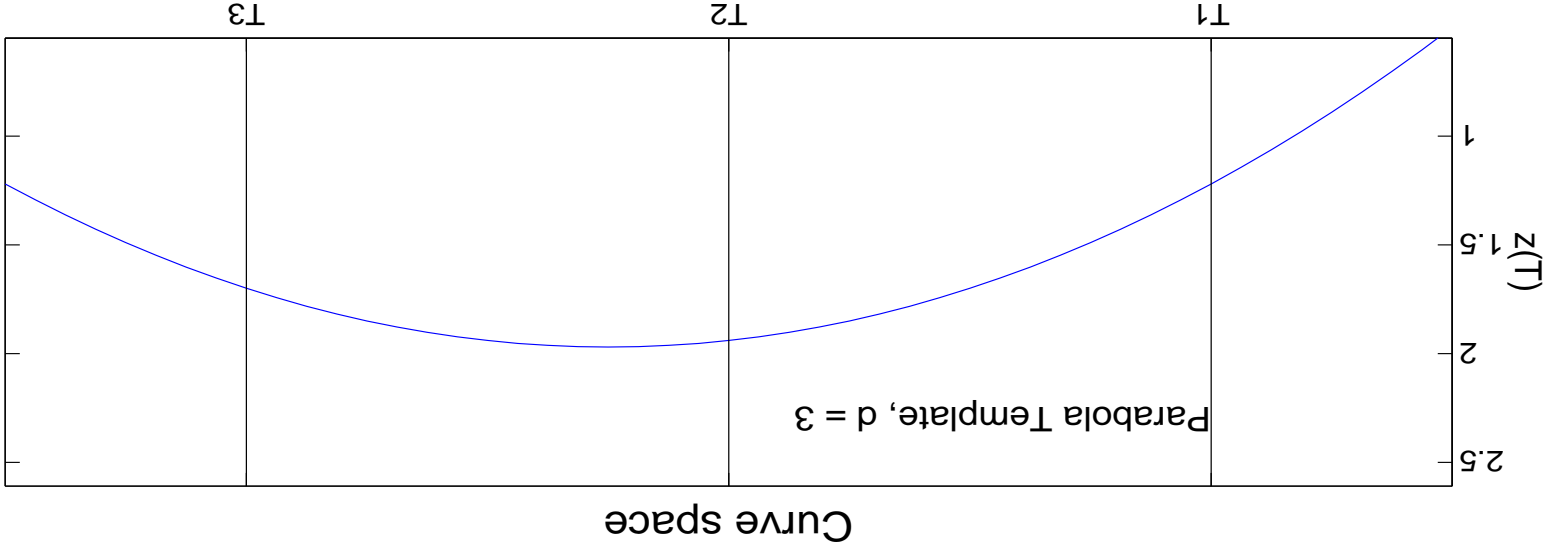
III. Example: Growth rate curves.

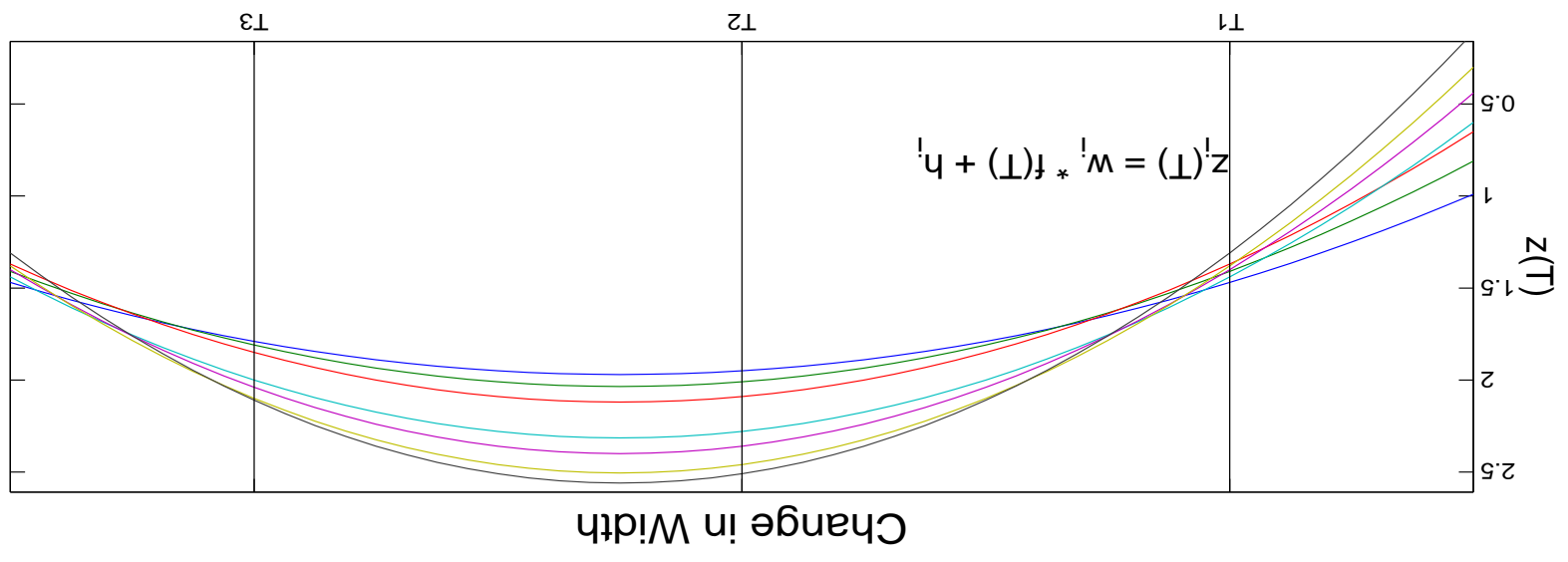
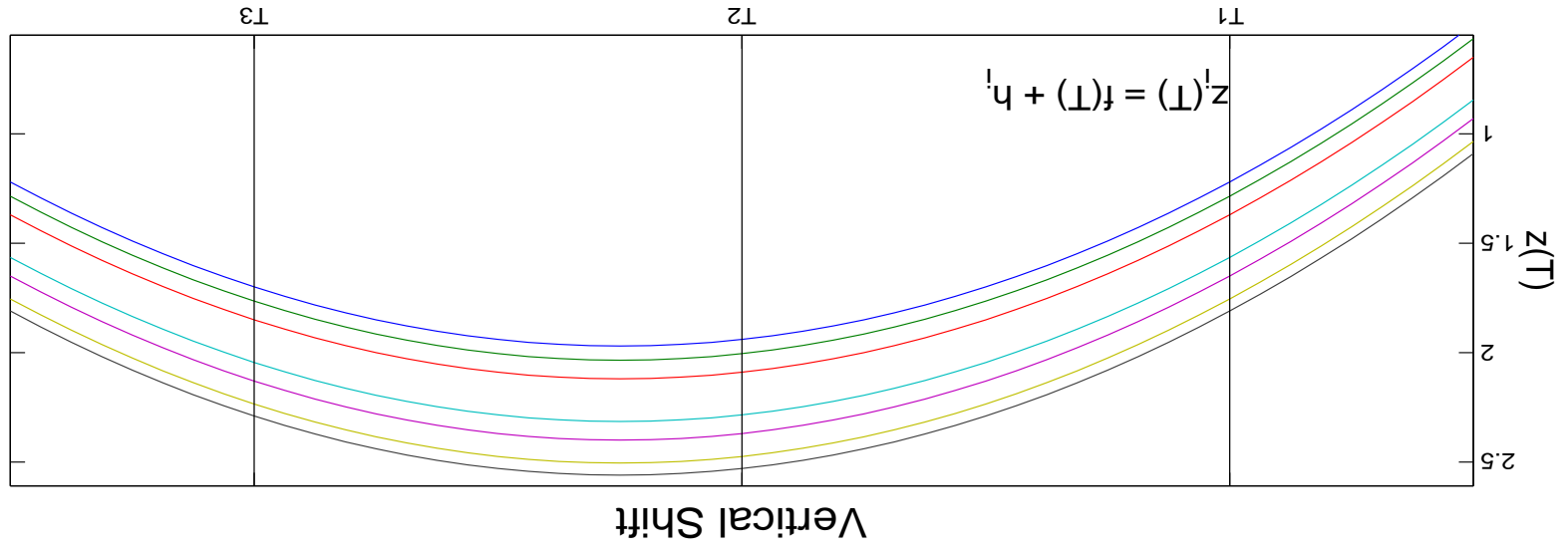
Template shape $f(T)$ of TPC:

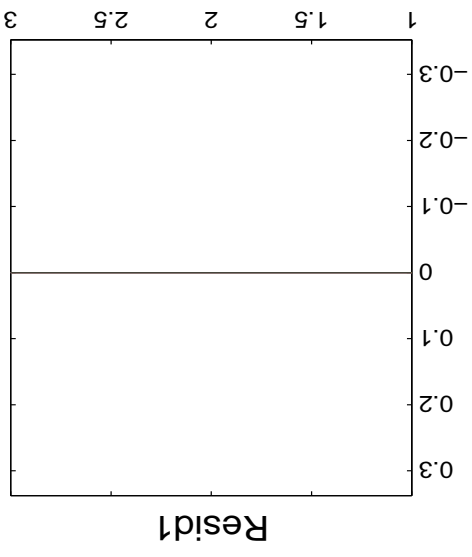
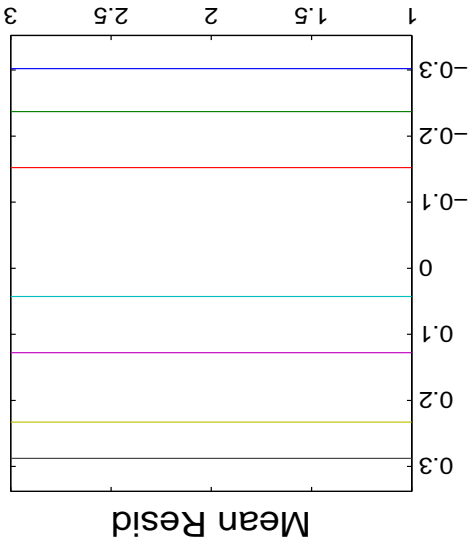
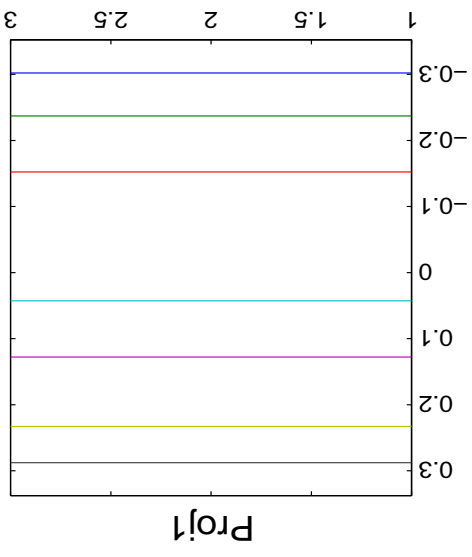
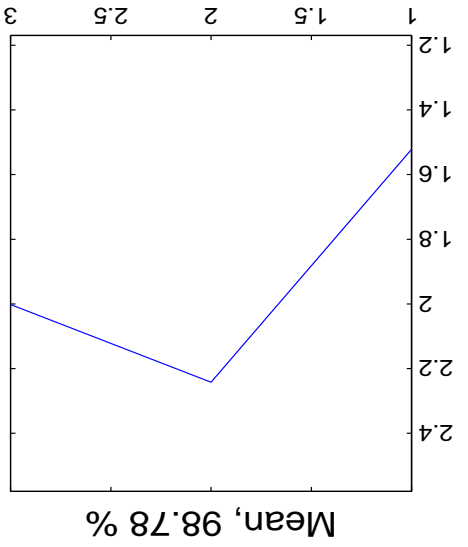
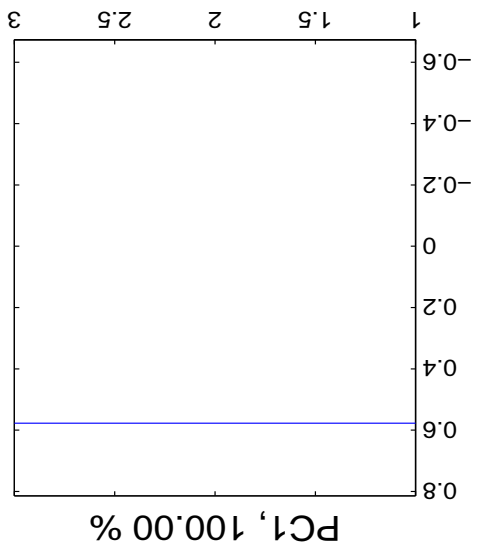
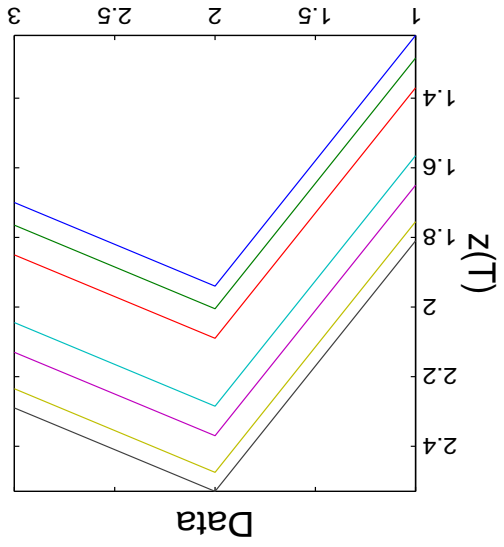
- \downarrow , reaches a max and \uparrow .
- Rate of \downarrow slower than rate of \uparrow .
- Tendency toward a unique max.

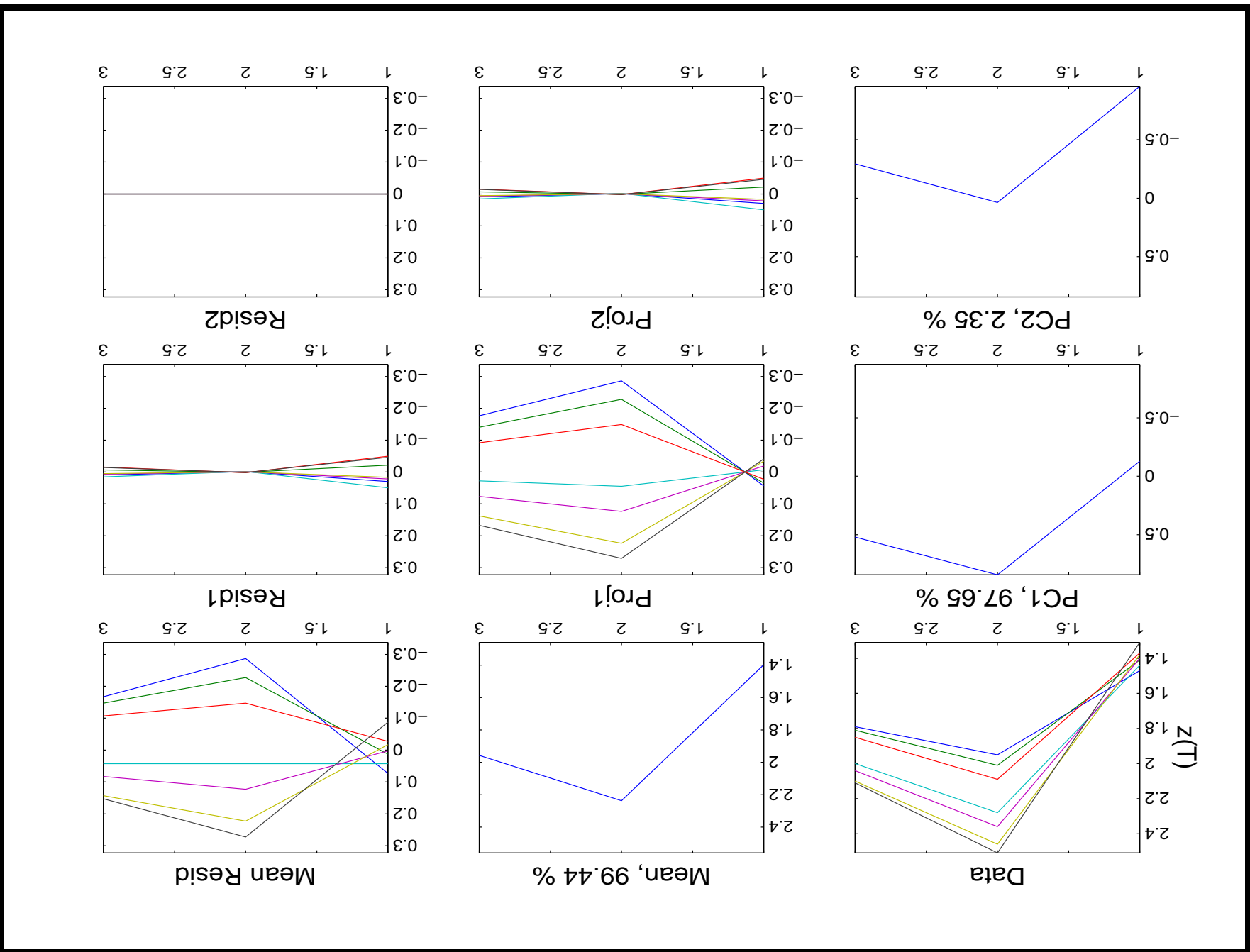


- Modes of variations of Curves \equiv Genetic constraints
- Vertical Shift \equiv Faster-Slower (F-S).
 - Change in Width \approx Generalist-Specialist (G-S).
 - Horizontal Shift \equiv Hotter-Colder (H-C).

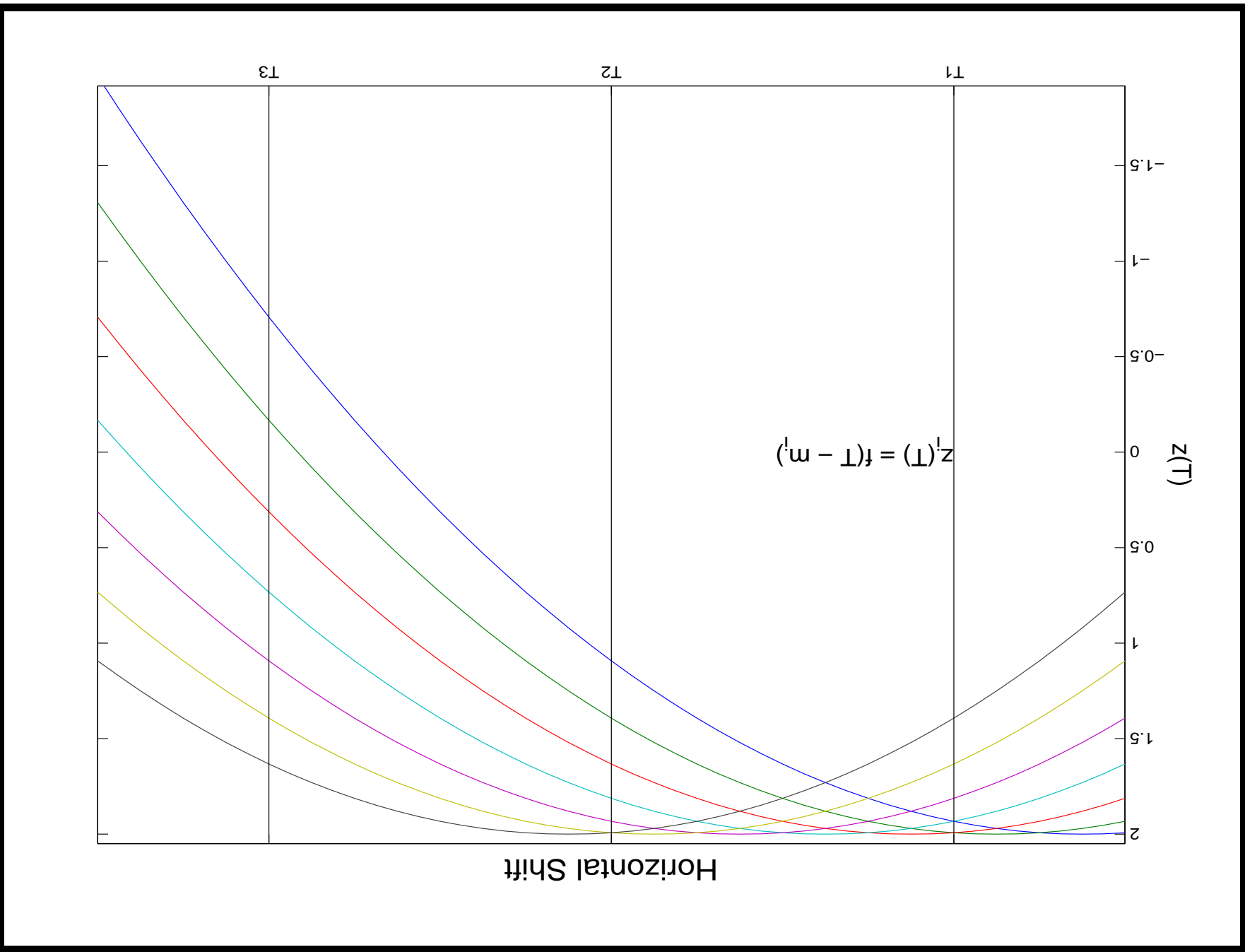








- PCA captures linear modes of variations.
- Some biological modes are non-linear (ex Horizontal Shift).



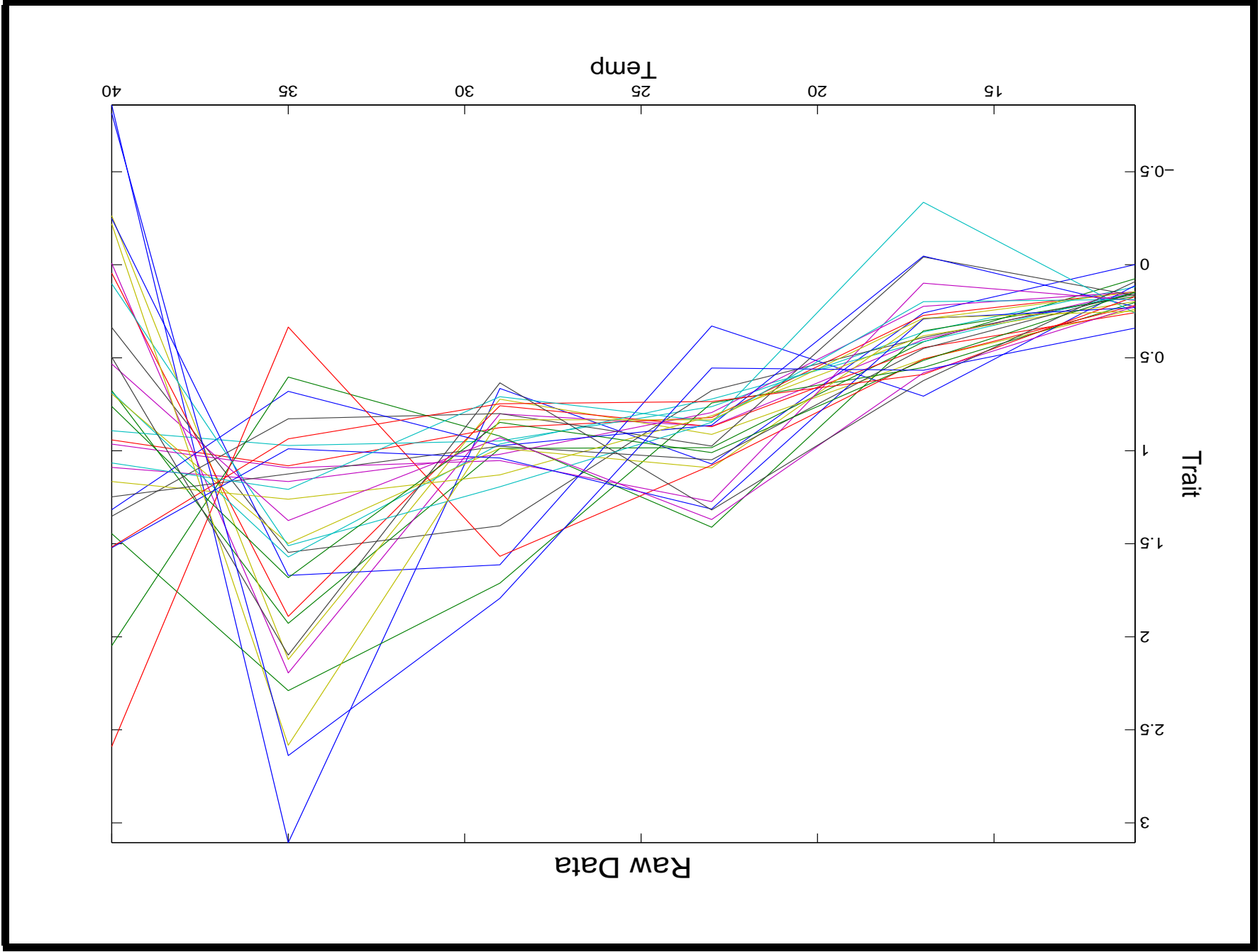
- Summary of Vertical Shift and Change in Width modes:
- Vertical Shift: linear, entirely explained by PC1.
 - Change in Width: entirely explained by a plane.
 - Vertical Shift \subset Change in Width.

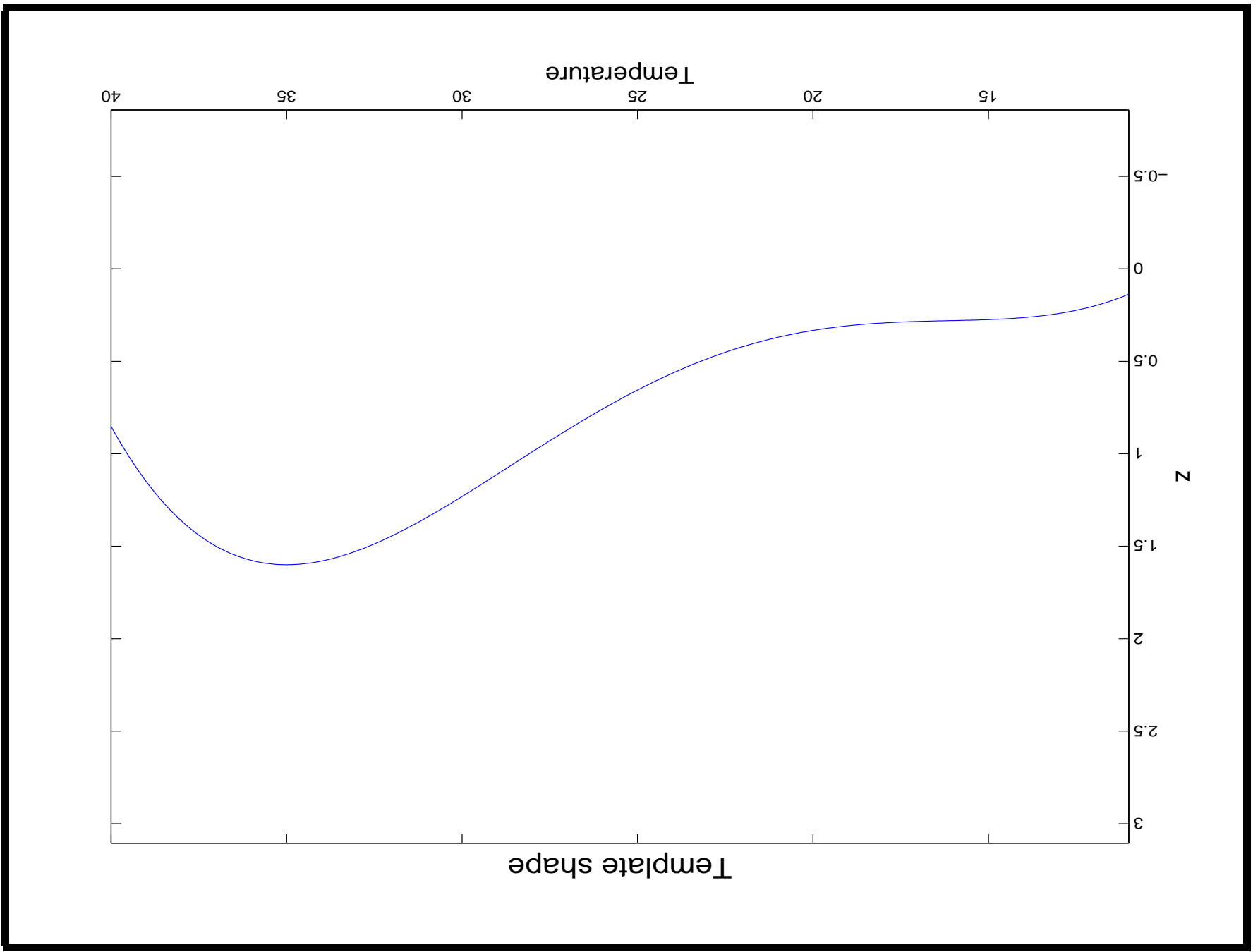
Summary of Horizontal Shift mode:

- Curved.
- Curve at most in a $(d-1)$ dimensional space.
- For ex: for $d = 3$, Parabola curve in a plane.

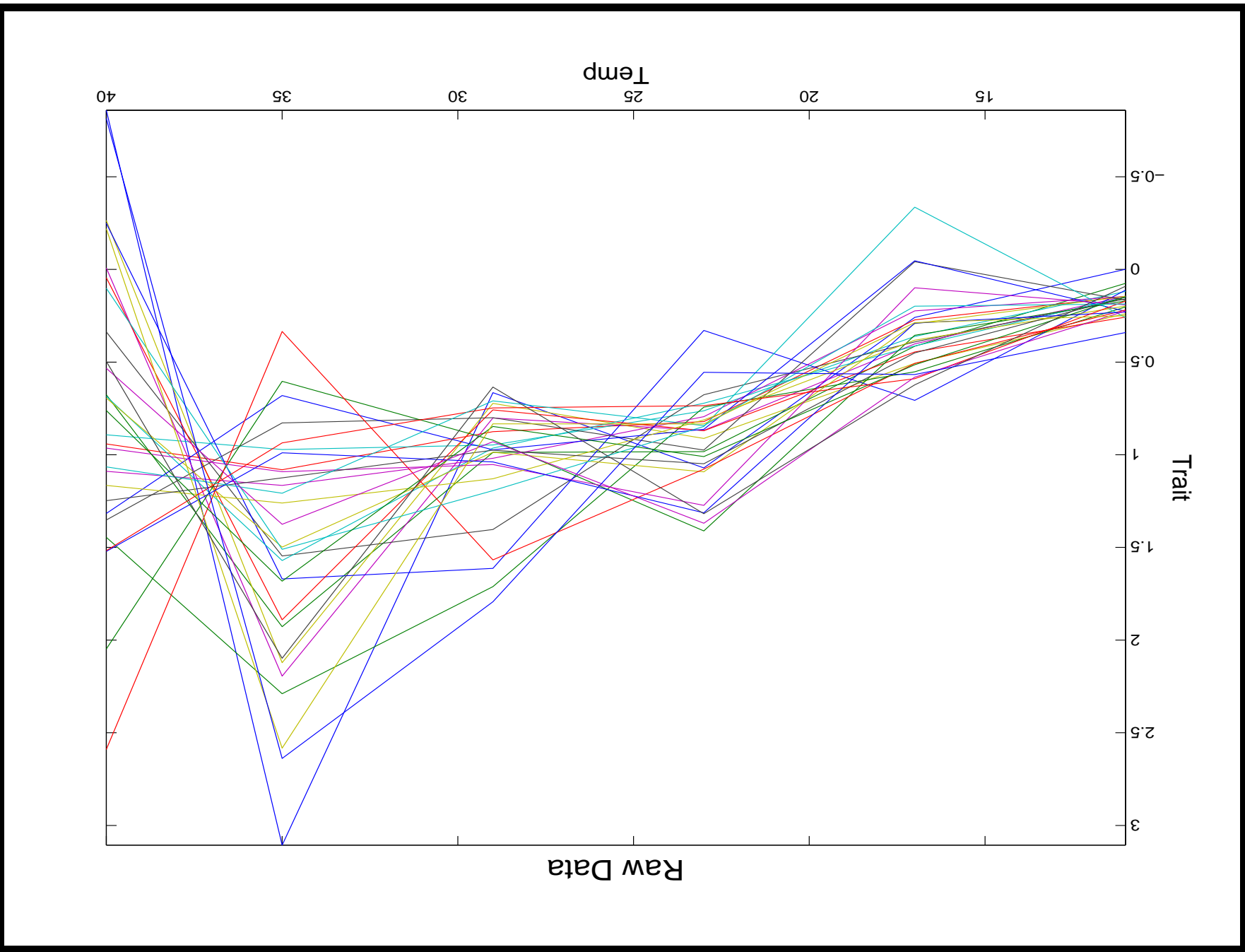
- Goal: For nonlinear modes, project in a curve instead of linear space.
- Curve in a d dimensional space: $x \mapsto (f_1(x), \dots, f_d(x))$

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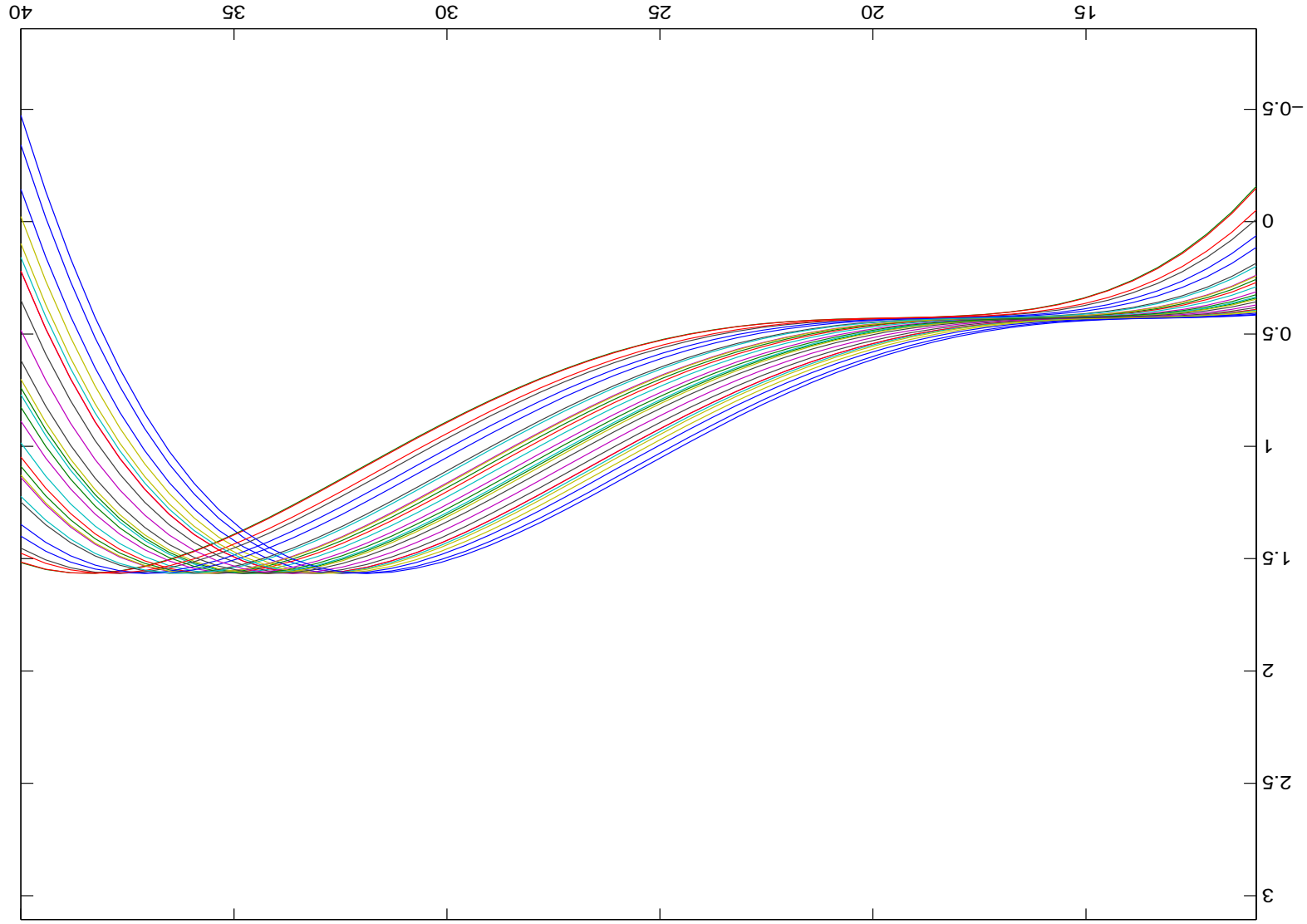


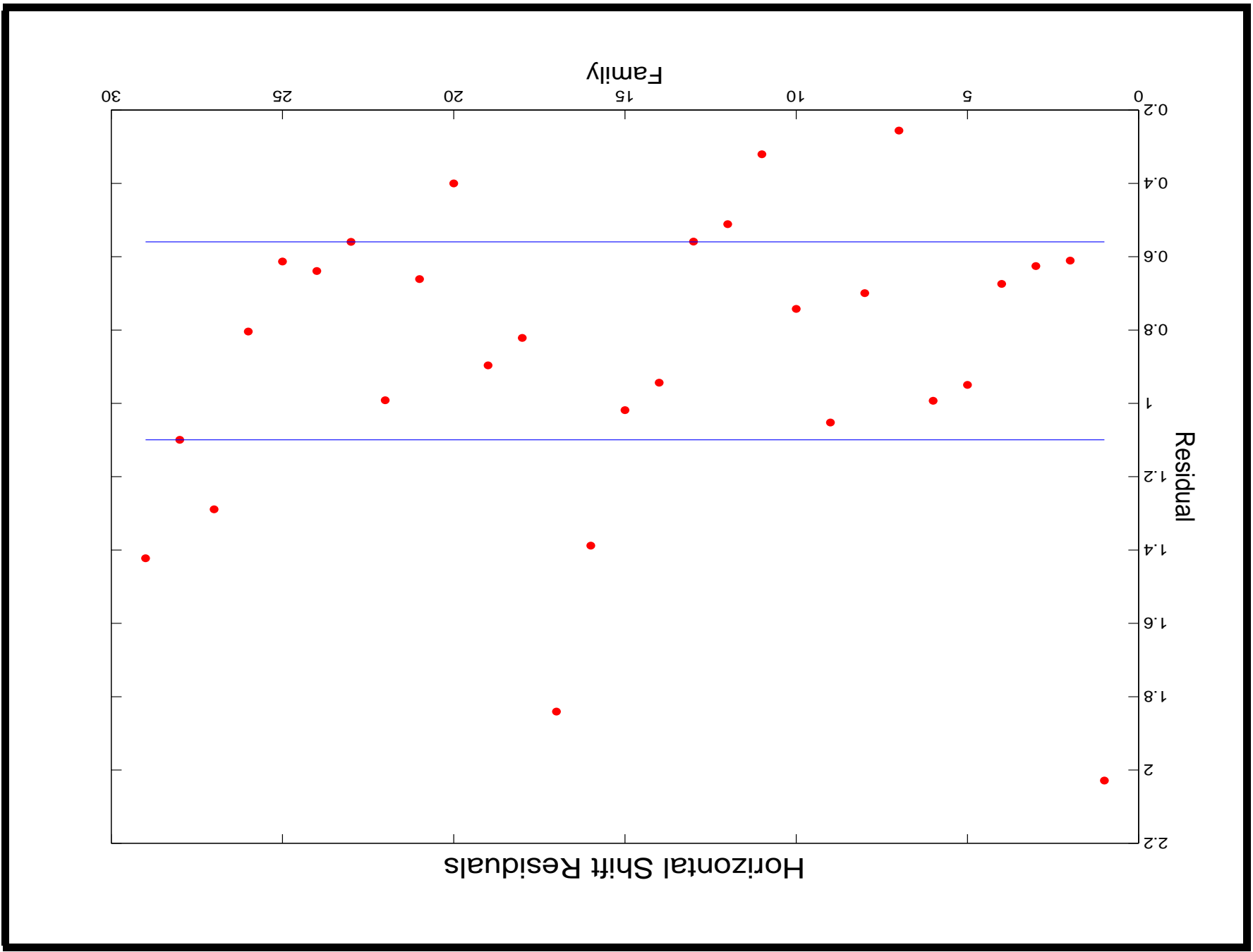


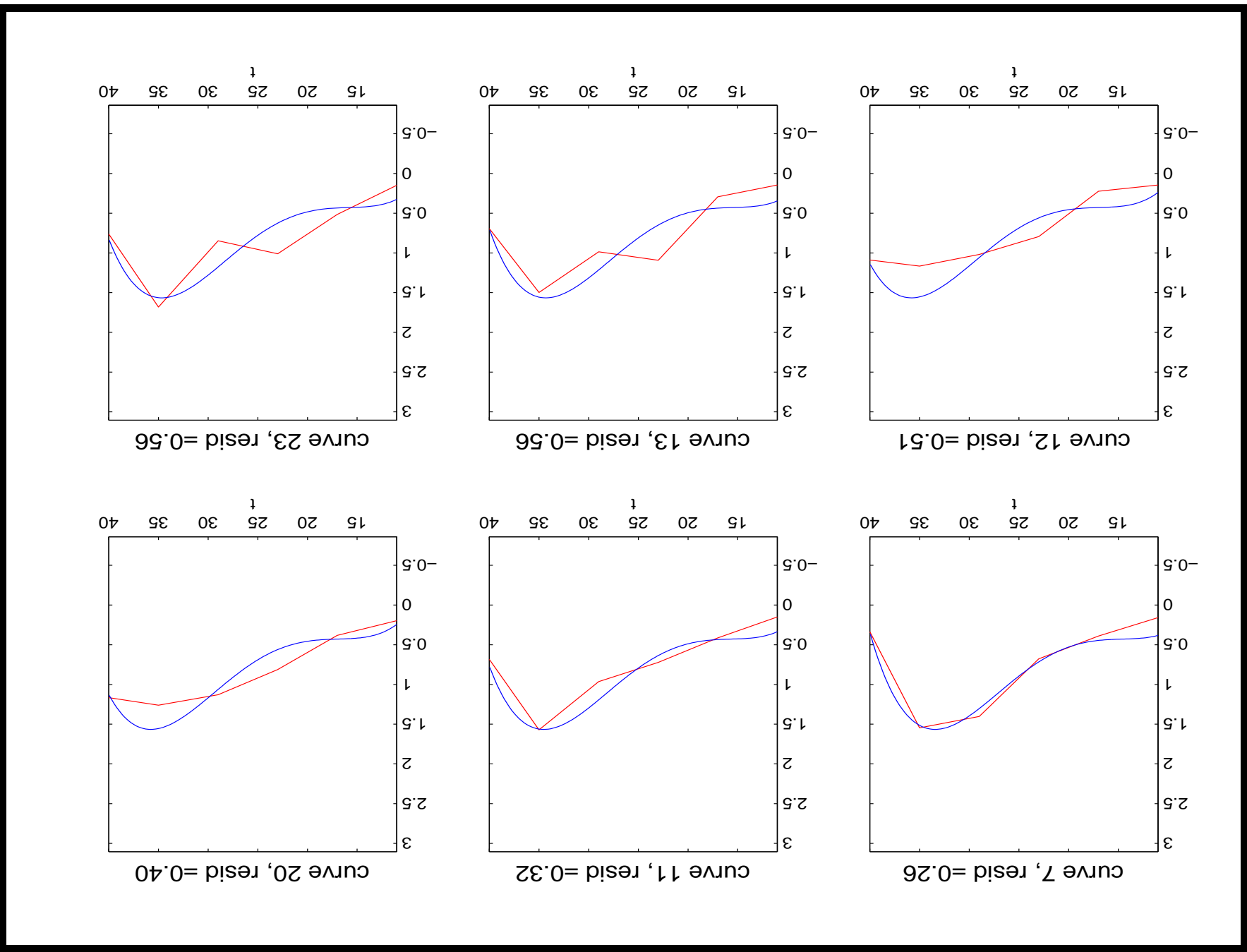
- For $d = 6$, choose smooth template shape:
- polynomial of degree four.
 - has a unique max in \mathbb{R} .

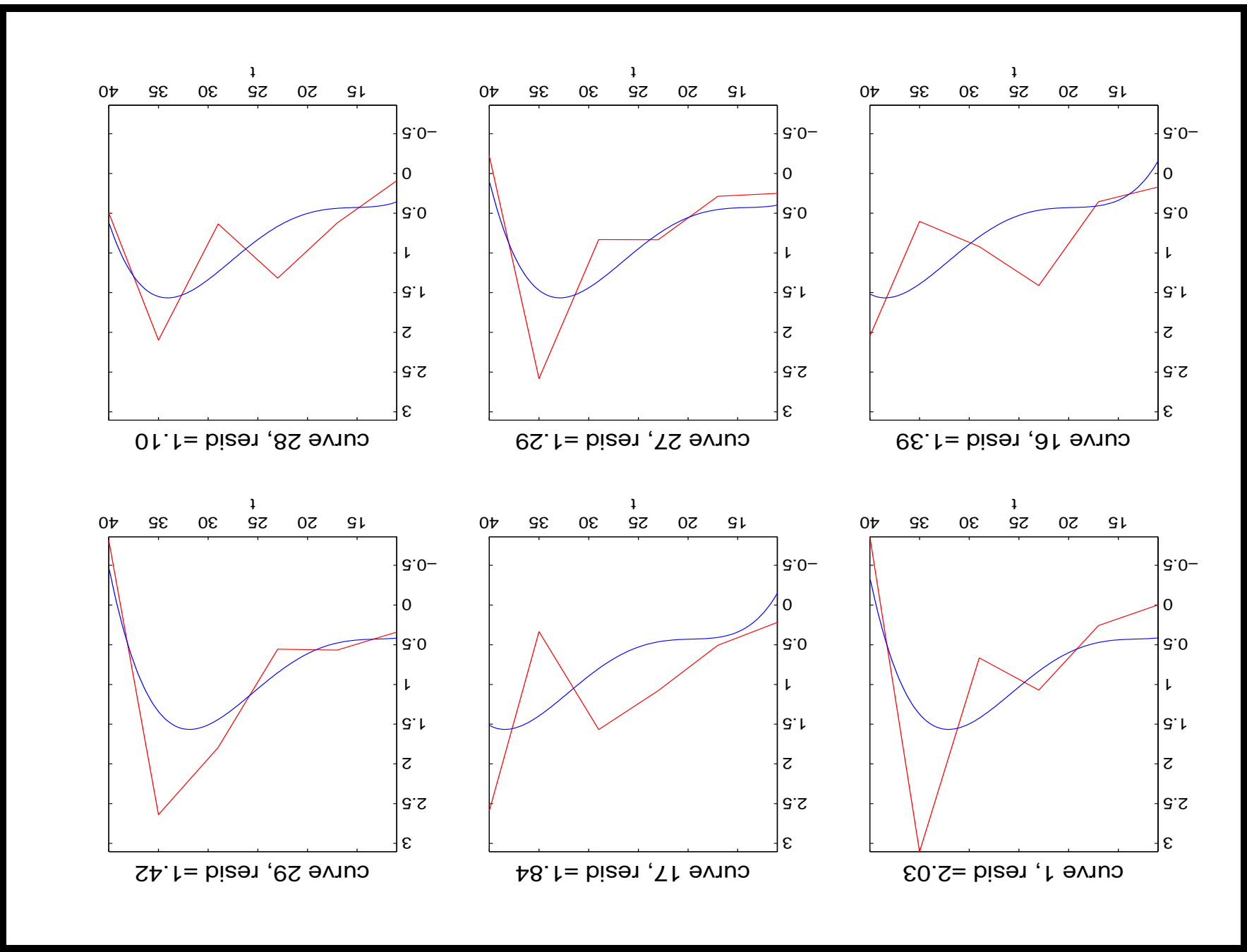


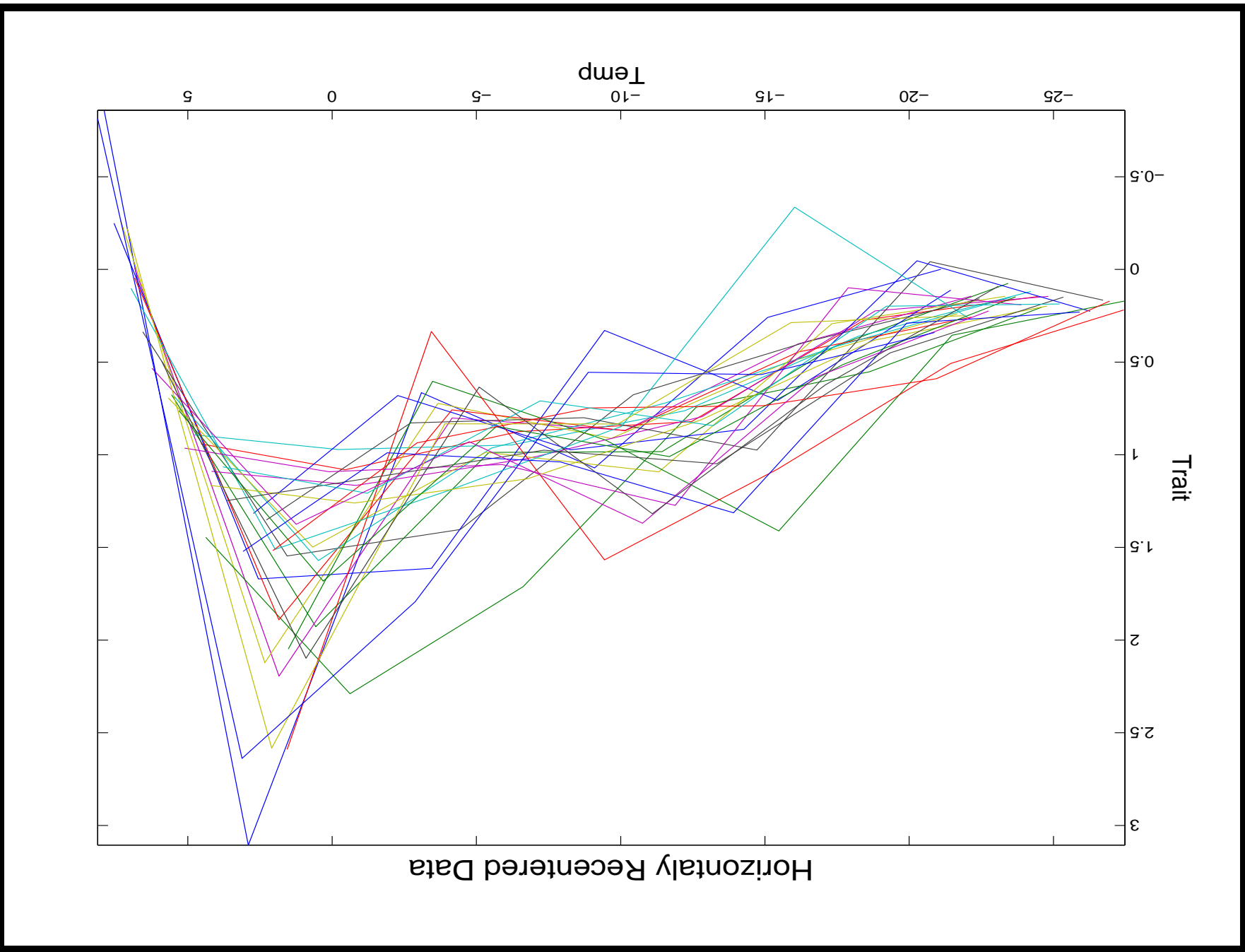
Horizontal Shift projections, MS = 25.11 %, MSE = 74.89 %

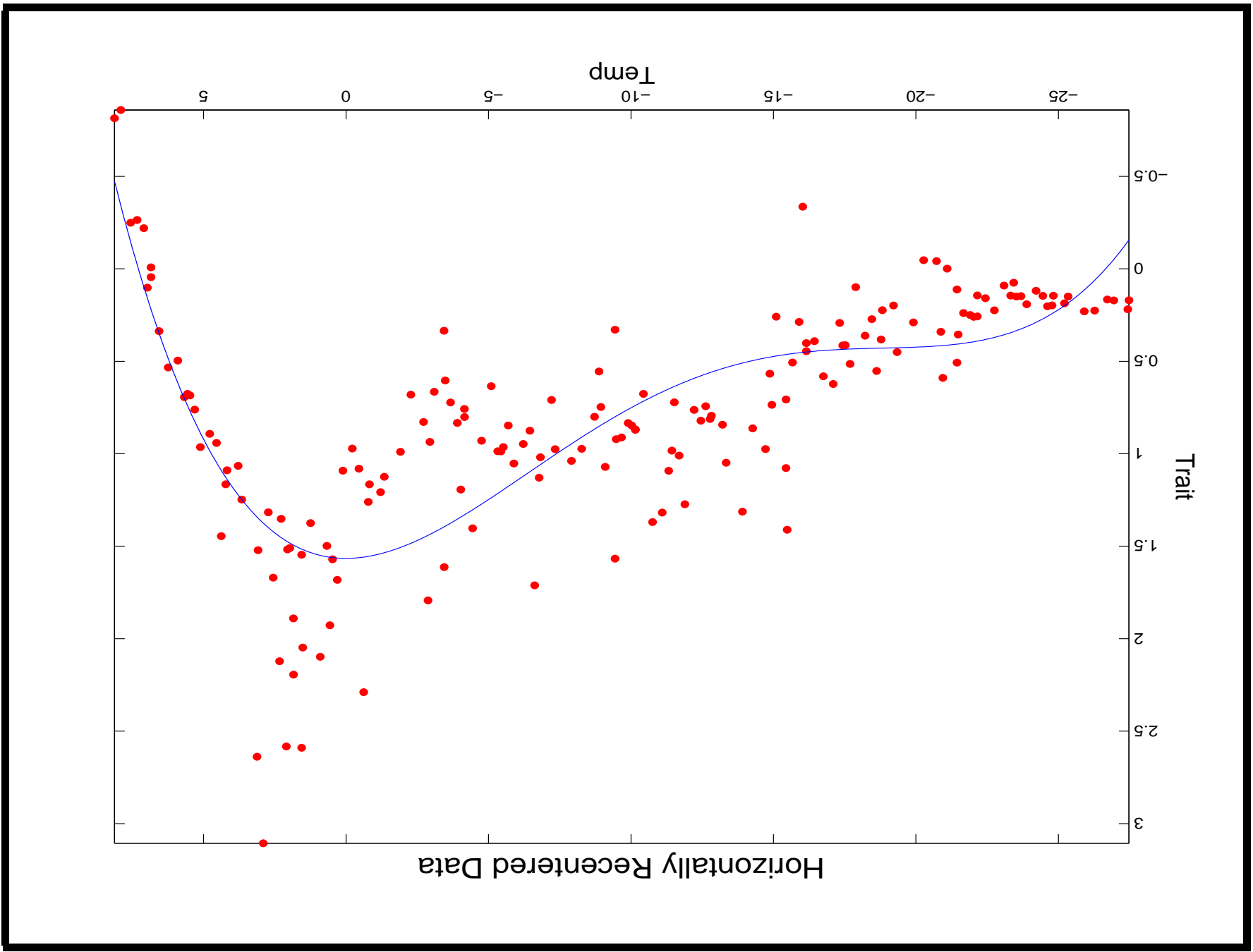












Results for Vertical Shift and Change in Width modes

- Vertical Shift mode explains 6.38% of the variation.
- Change in Width mode explains 23.14% of the variation.

Results for Horizontal Shift mode

- Horizontal Shift mode explains 25% of the variation.
- Outliers
 - $f(T^{max}) = cst$ constraint.
 - Unique global max constraint.

Conclusion

- Vertical Shift accounts for very little of the variation in the data.
- Selecting for faster growth rate in a range of temperature will not result in selecting for faster growth rate at all temperatures.