

Analyzing Nonlinear Variations in Thermal Performance Curves

Rima Izem

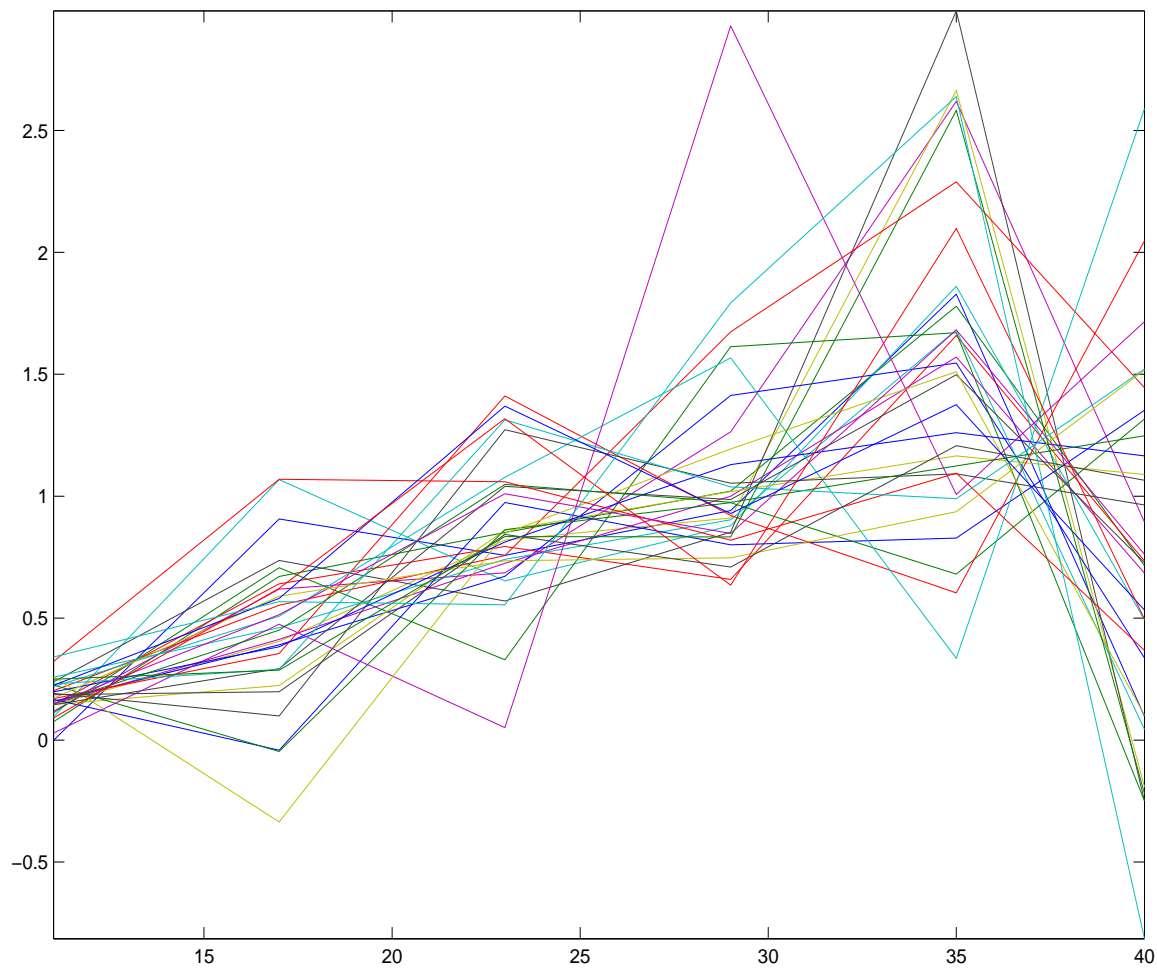
Joint work with

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and

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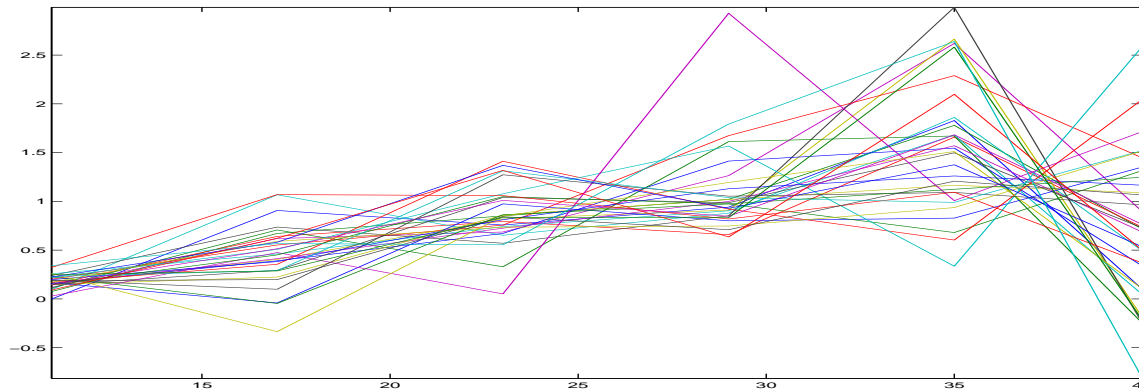
November 18, 2003



Outline

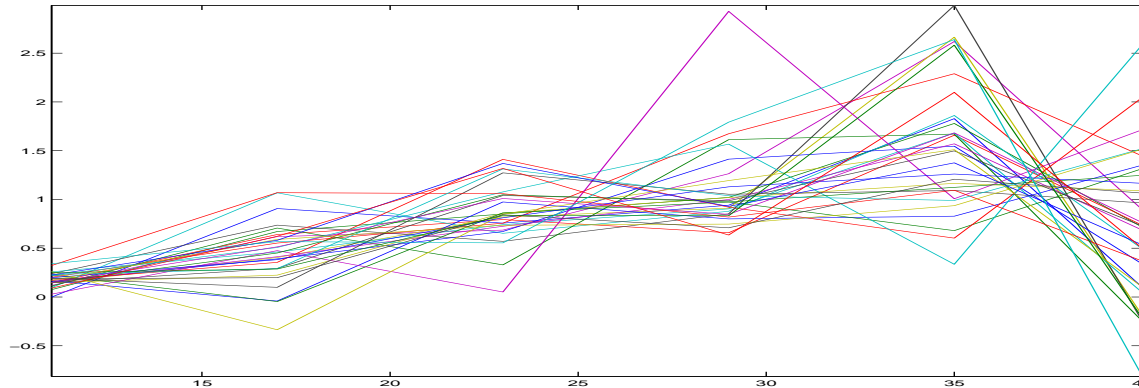
- Data
- Motivation and model
- Toy example
- Results

- Performance z . Ex: $z =$ growth rate.
- Temperature t . Ex: $d = 6$, and $t_1, \dots, t_6 \in [11, 40]$ Celsius.
- Thermal performance curve (**TPC**): $z = f(t)$.
- Continuous Reaction norm: $z = f(e)$



Feature(1): Curves' variation \equiv Genetic variation

- Family = individuals of similar genotype.
Ex: offsprings of same parents, clones, . . .
- Population = n families, Ex: $n = 32$.



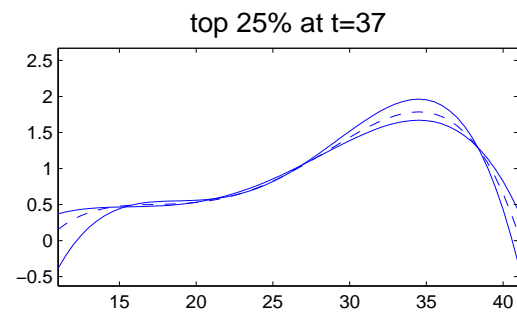
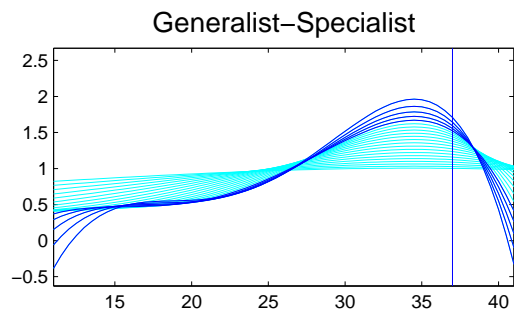
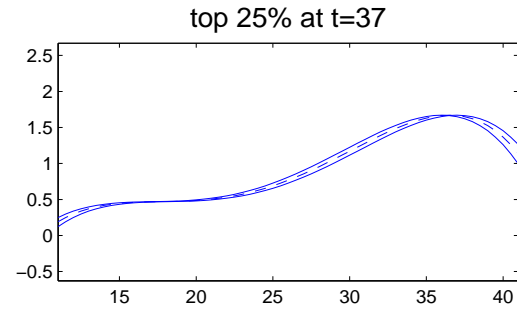
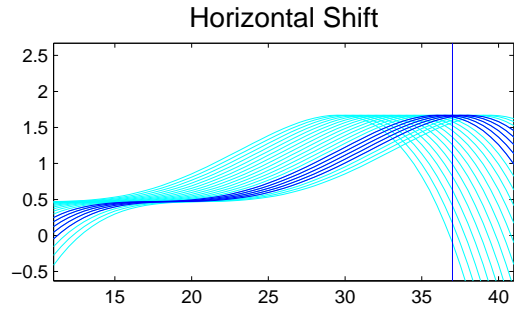
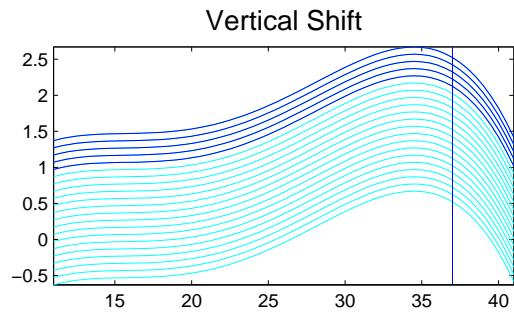
Feature (2) : Template shape $f(t)$ of TPC:

- \uparrow , reaches a max and \downarrow .
- Rate of \uparrow slower than rate of \downarrow .
- Tendency toward a unique max.

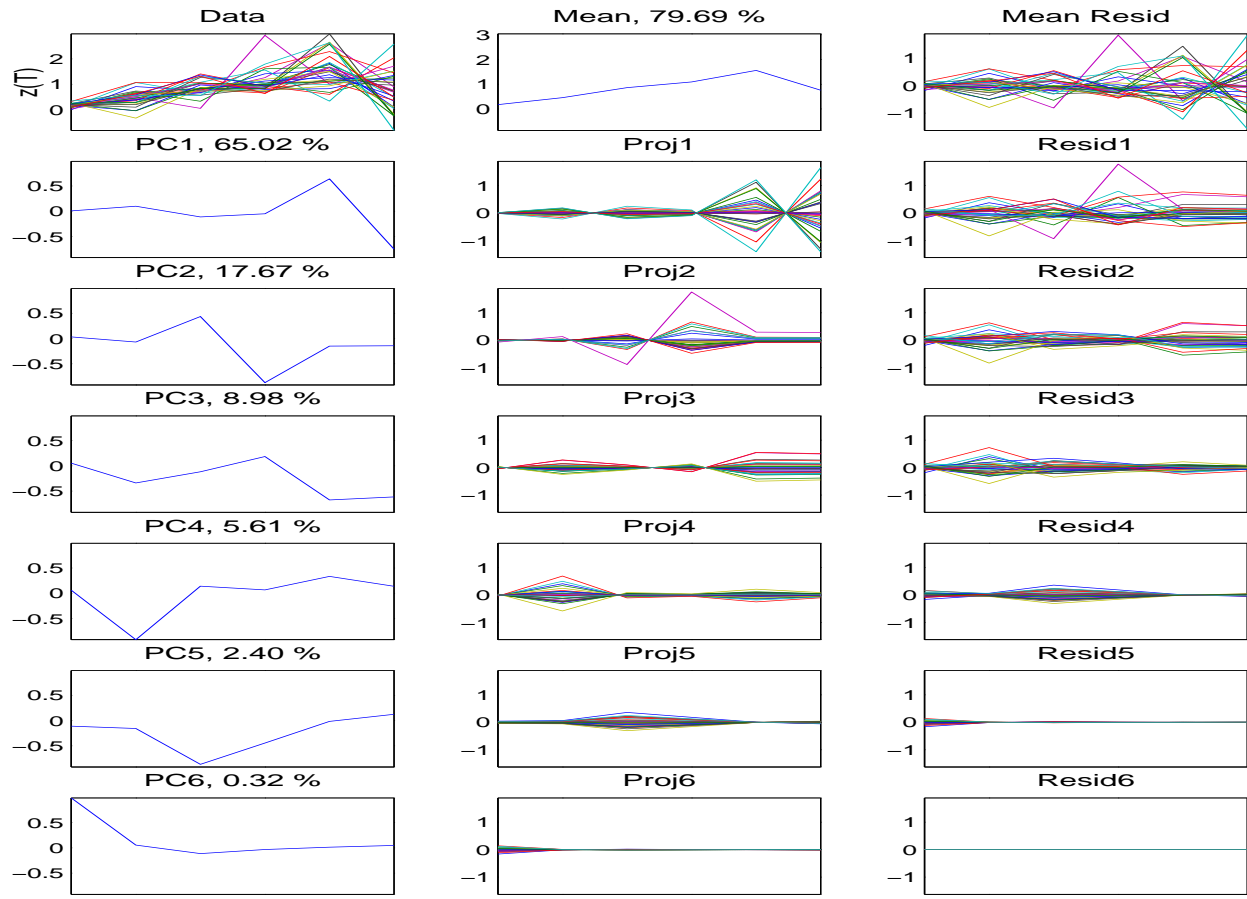
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- **Goal:** Analyze three TPC modes of variation.
- **Motivation:** **evolution** of gen. variation under **selection**.
- Mode of variation \equiv Constraint to evolution



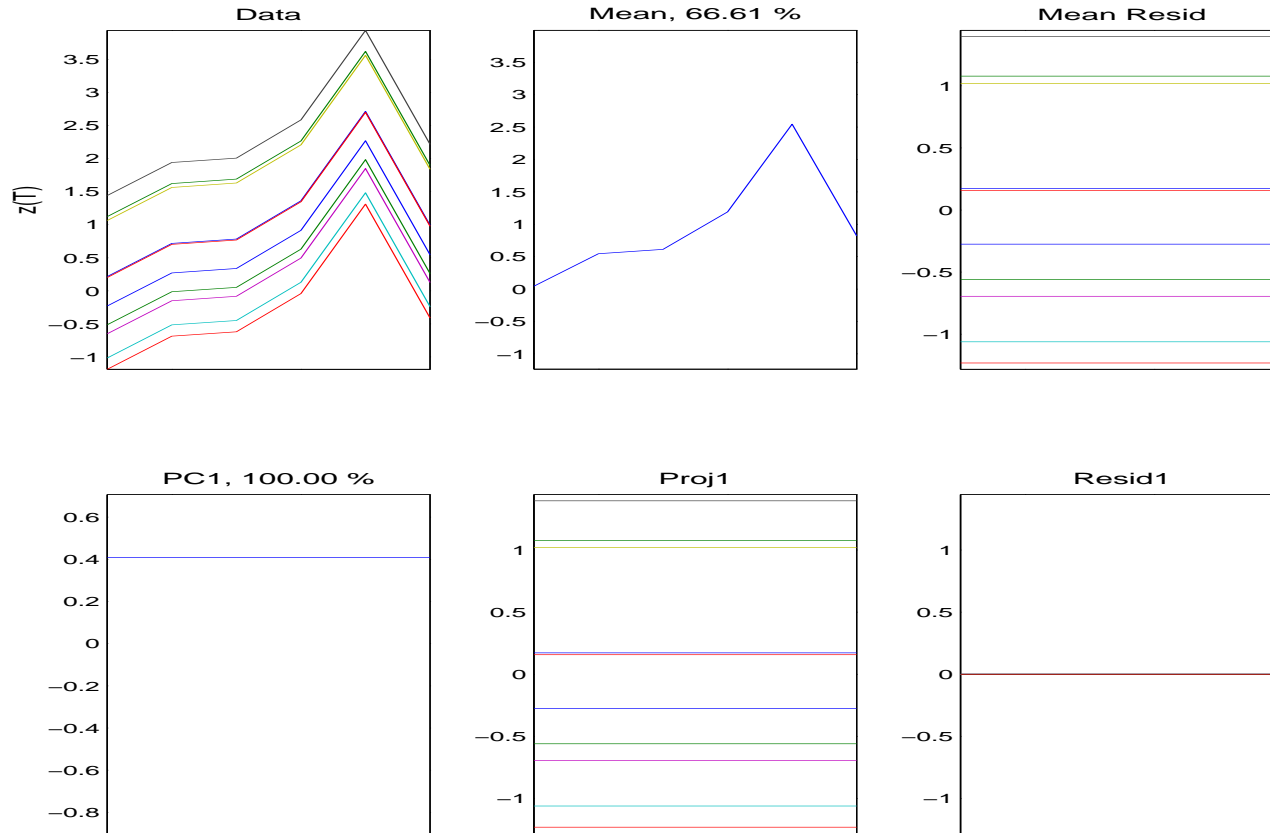
Mode of variation **before** selection – Genetic variation **after** selection



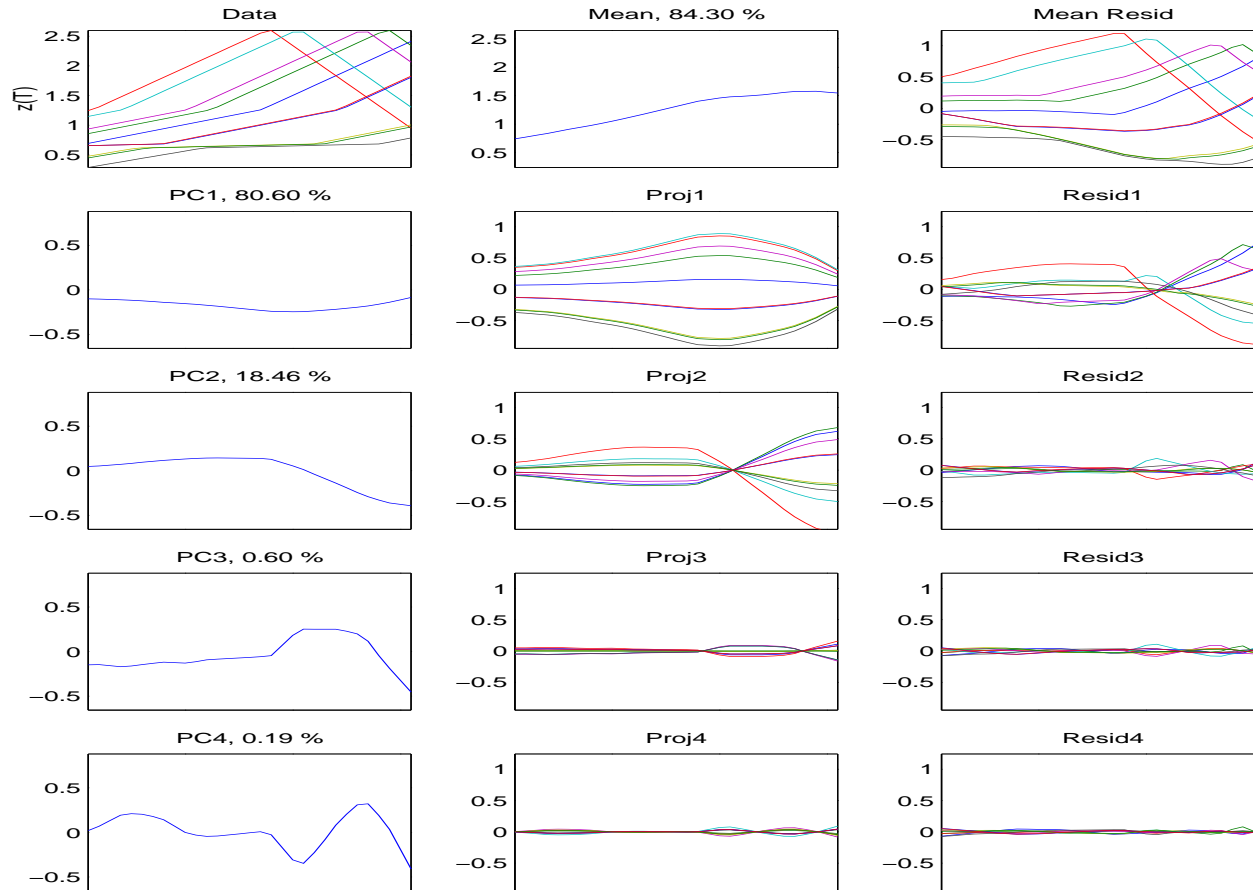
PCA

- 1. Subtract mean curve from data.
- 2. PC1 direction of greatest variability
- 3. PC2 direction of greatest variability \perp to PC1
- 4. PC3 direction of greatest variability \perp (PC1,PC2) ...
- Each variation is linear quantified by RSS.

Vertical Shift, Linear variation



Horizontal Shift, Nonlinear variation



PCA

No assumption on shape

Linear PC of G

Meaning of each PC?

RSS quantify a PC

Template Modes of Variations

Template shape of TPCs.

Nonlinear modes of variation of G .

Biological modes.

\widetilde{RSS} quantify a mode.

Model

$$Z_i(t_j) = R_i(t_j) + \epsilon_{i,j}, 1 \leq i \leq n \text{ and } 1 \leq j \leq d.$$

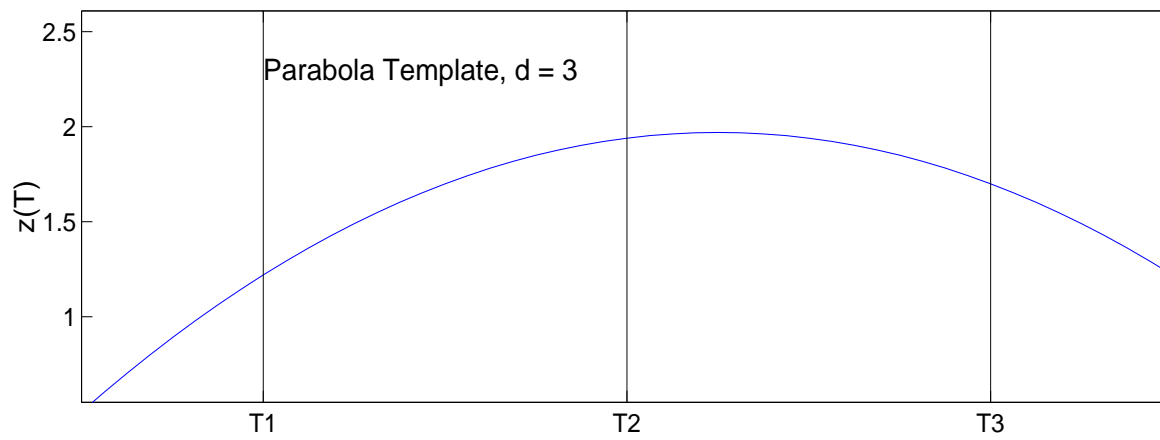
$$R_i(t) = w_i f(w_i(t - m_i)) + h_i$$

	Parameter	Variation
Vertical shift	h_i	linear
Horizontal shift	m_i	nonlinear
Gen-Spec	w_i	nonlinear

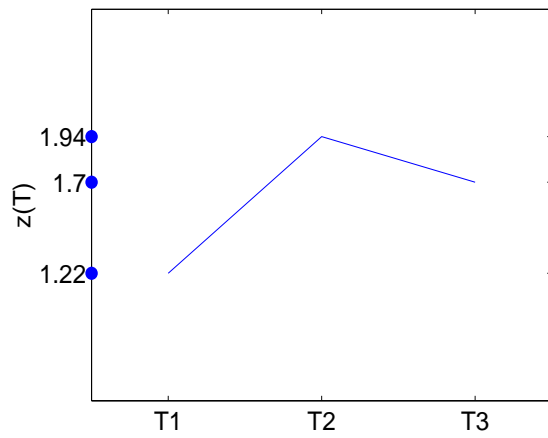
Outline

- Data
- Motivation and model
- Toy example
 - One variation (Vertical Shift, Horizontal Shift).
 - More than one variation (two, three, etc).
- Results

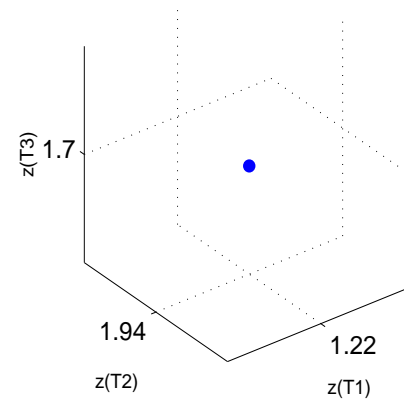
Curve space



$d = 3$



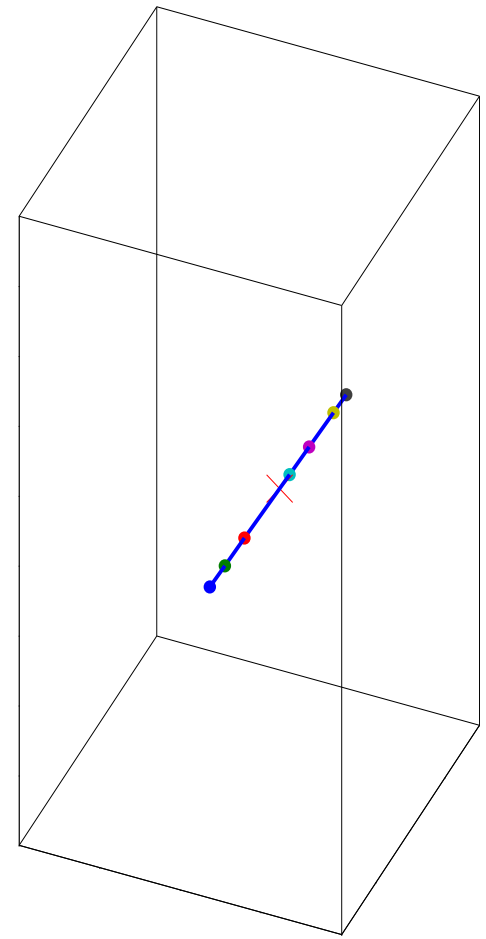
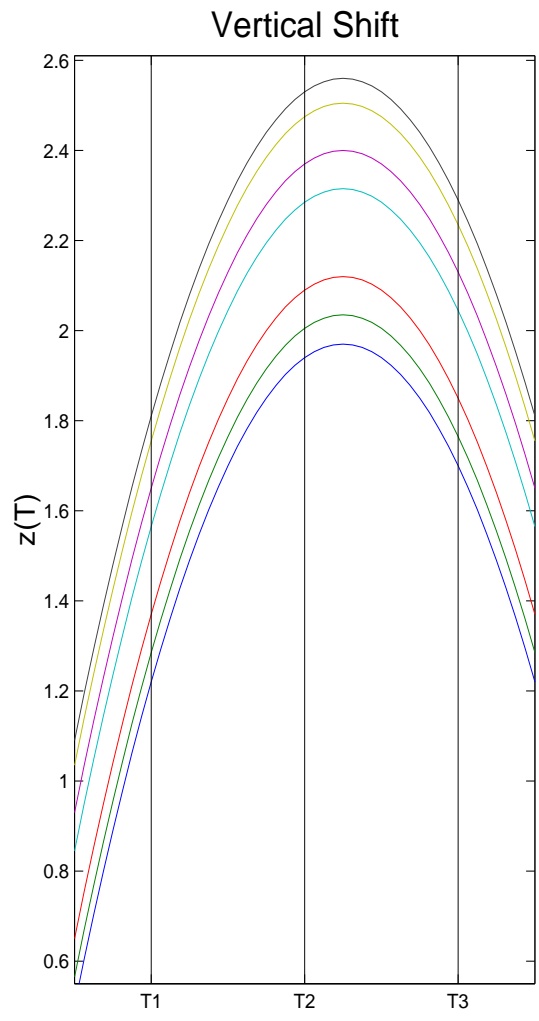
Point cloud space

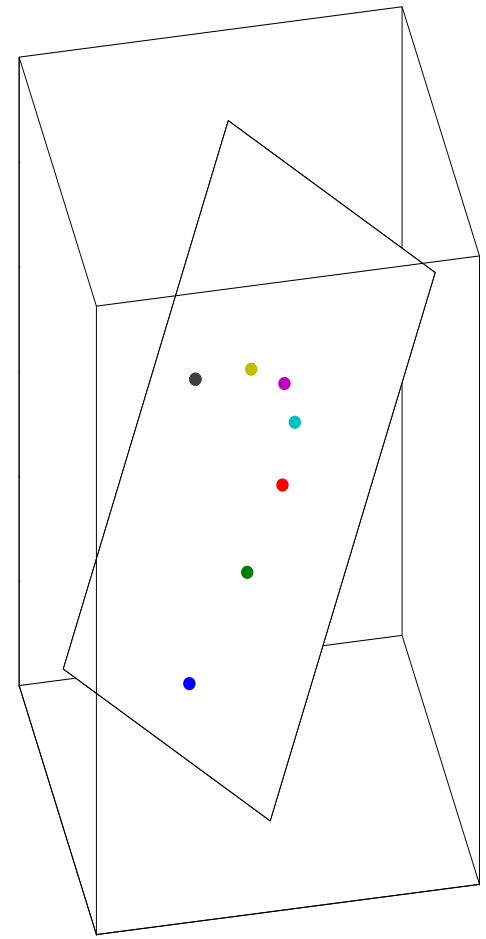
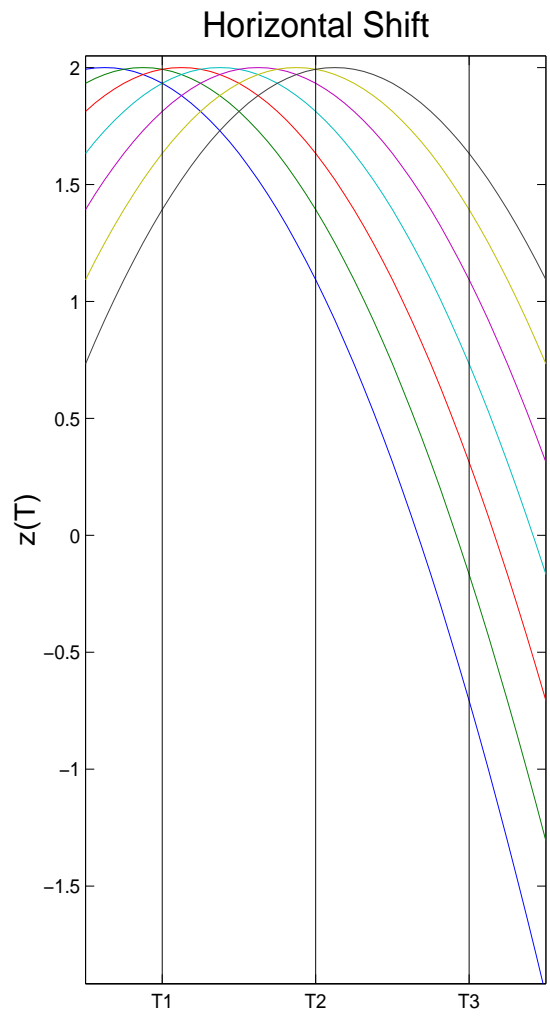


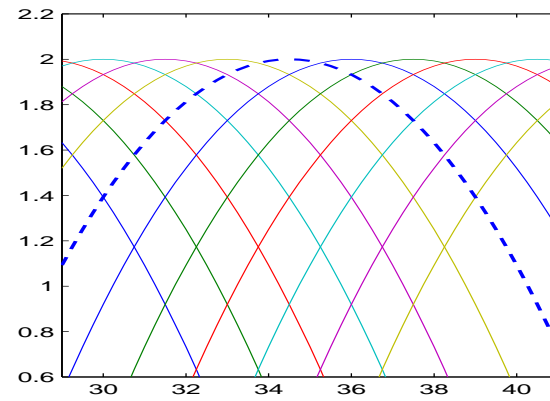
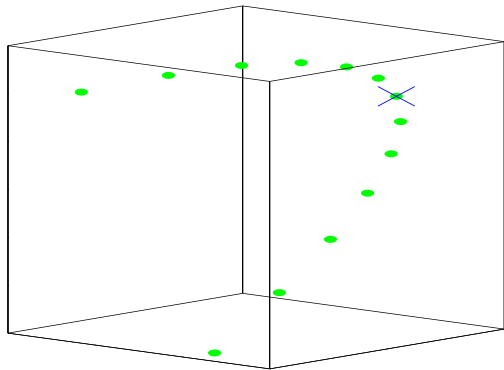
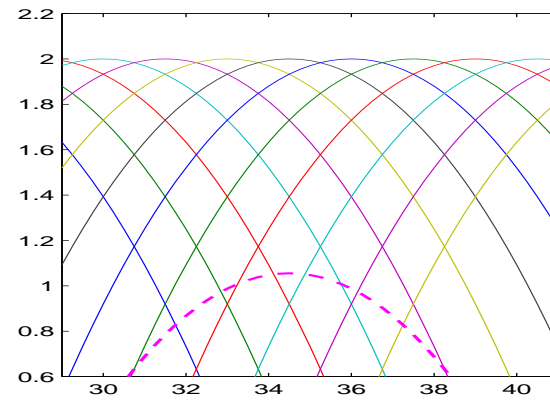
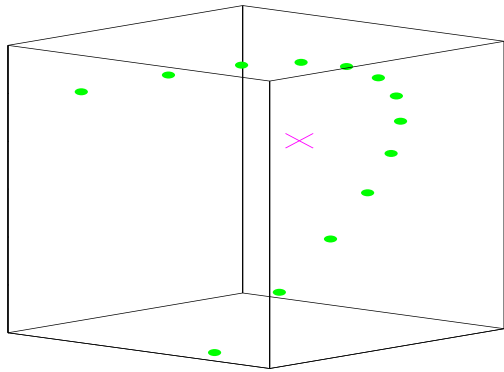
Toy example

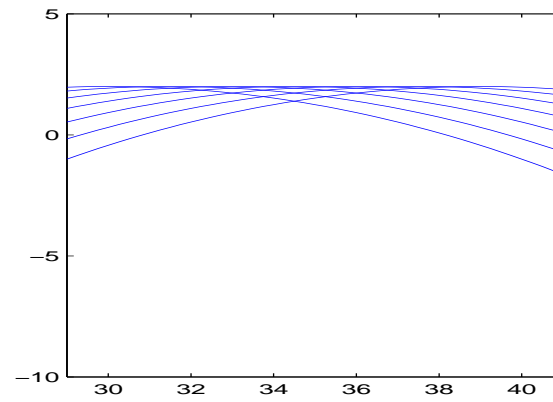
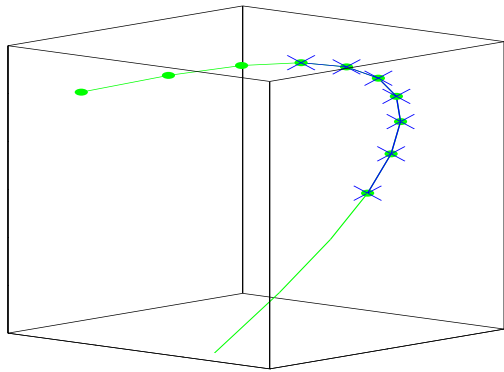
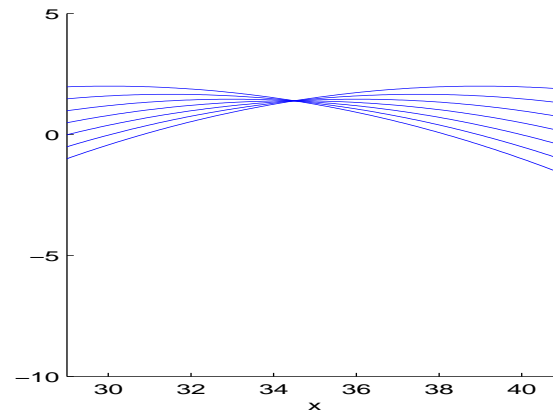
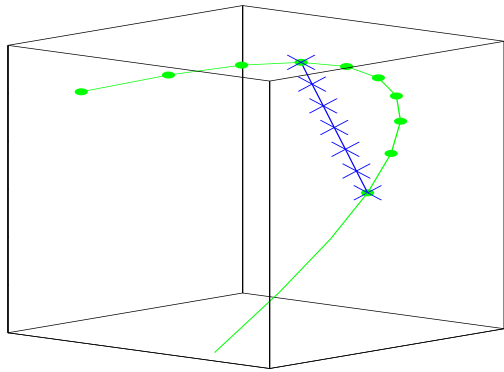
- $f(t) = -t^2 + c$.
- Model,

$$\begin{aligned}z_{i,j} &= w_i f(w_i(t_j - m_i)) + h_i + \epsilon_{i,j} \\ &= -w_i^3(t_j - m_i)^2 + w_i c + h_i + \epsilon_{i,j}.\end{aligned}$$









Linear case (conventional PCA), one mode

- Center of the projections: \bar{R} , and $\bar{R} = \bar{Z}$.
- Spread: $SSM = \sum_{i=1}^n \|R_i - \bar{Z}\|^2$.
- $RSS = \frac{SSM}{SSM+SSE}$ quantifies the variation.

Nonlinear case (Template Mode of Variation), one mode

- \tilde{R} is the sample geodesic mean.
- d_g is the geometric distance.
- \widetilde{RSS} quantifies nonlinear variations.

One mode of variation

Linear (PCA)	Nonlinear (TMV)
$RSS = \frac{SSM}{SSM+SSE}$	$\widetilde{RSS} = \frac{\widetilde{SSM}}{\widetilde{SSM}+SSE}$
$SSM = \sum_{i=1}^n \ R_i - \bar{Z}\ ^2$	$\widetilde{SSM} = \sum_{i=1}^n \left(d_g(R_i, \tilde{R}) \right)^2$
$\bar{Z} = \bar{R}$	$\bar{Z} \neq \tilde{R}$
$SSM \perp SSE$	$\widetilde{SSM}(\text{not } \perp)SSE$

More than one mode of variation

- Data fall in a manifold $d' \geq 2$: define new d_g
- Decompose model variation.
- Quantify each variation.

Two modes of variation: linear + nonlinear

- (h, m) **movie**. Define

$$d_g((h_1, m_1), (h_2, m_2)) = \sqrt{d_m(m_1, m_2)^2 + (h_1 - h_2)^2}.$$

- (h, w) **movie**. Define

$$d_g((h_1, w_1), (h_2, w_2)) = \sqrt{d_w(w_1, w_2)^2 + (h_1 - h_2)^2}.$$

Two modes of variations: nonlinear + nonlinear

(w, m) [movie](#), define

$$d_g((w_1, m_1), (w_2, m_2)) = \sqrt{\frac{1}{2} (M^2 + W^2)}, \text{ where}$$

$$M^2 = d_{m,(w=w_1)}(m_1, m_2)^2 + d_{m,(w=w_2)}(m_1, m_2)^2$$

$$W^2 = d_{w,(m=m_1)}(w_1, w_2)^2 + d_{w,(m=m_2)}(w_1, w_2)^2$$

Two modes of variations (w, m) (contd)

Other decomposition,

$$d_g((w_1, m_1), (w_2, m_2)) = \sqrt{\frac{1}{4} (D_m + D_w + Diff_m + Diff_w)}, \text{ where}$$

$$D_m = \left(d_{m,(w=w_1)}(m_1, m_2) + d_{m,(w=w_2)}(m_1, m_2) \right)^2$$

$$D_w = \left(d_{w,(m=m_1)}(w_1, w_2) + d_{w,(m=m_2)}(w_1, w_2) \right)^2$$

$$Diff_m = \left(d_{m,(w=w_1)}(m_1, m_2) - d_{m,(w=w_2)}(m_1, m_2) \right)^2$$

$$Diff_w = \left(d_{w,(m=m_1)}(w_1, w_2) - d_{w,(m=m_2)}(w_1, w_2) \right)^2$$

Three modes of variation (w,m,h)

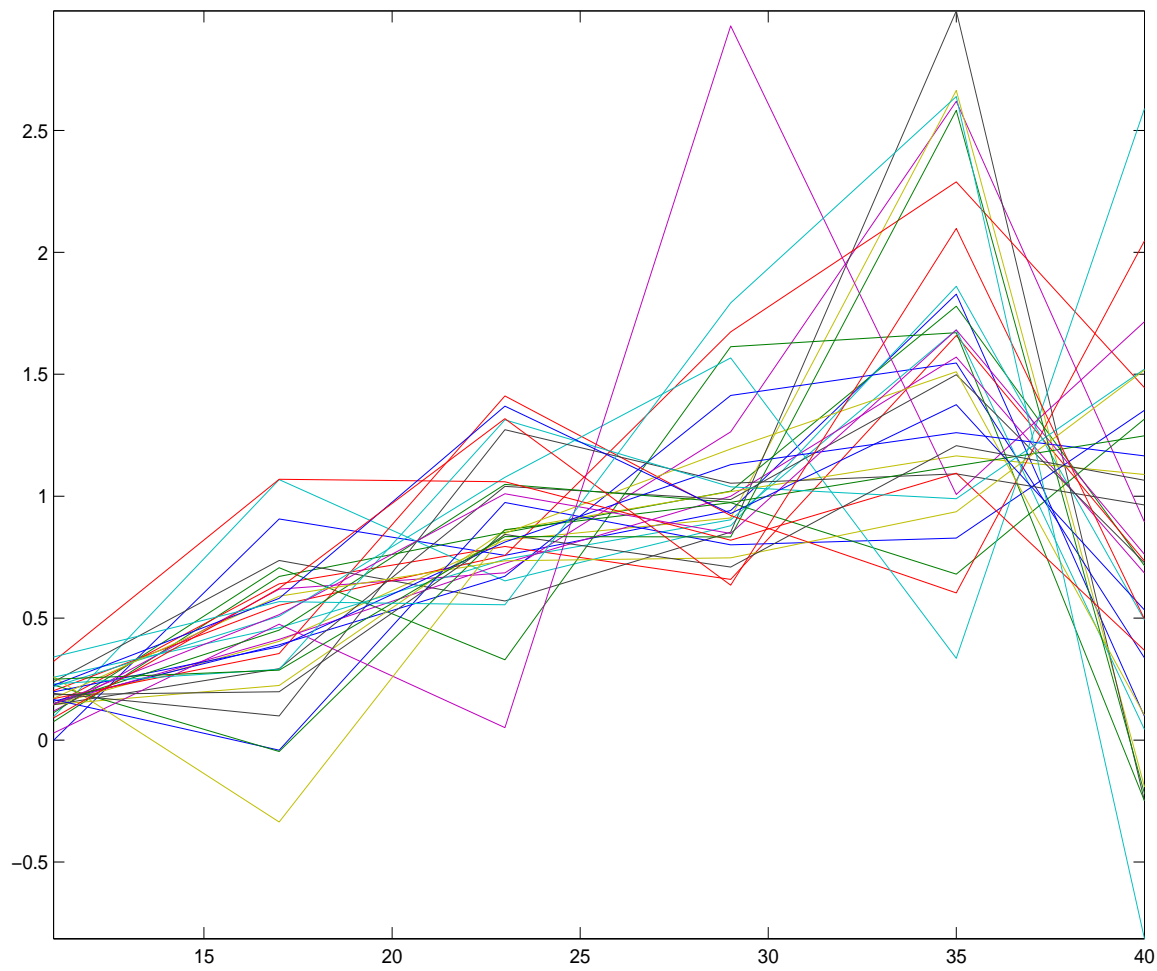
$$d_g((w_1, m_1, h_1), (w_2, m_2, h_2)) = \sqrt{d_g((w_1, m_1), (w_2, m_2))^2 + (h_1 - h_2)^2}$$

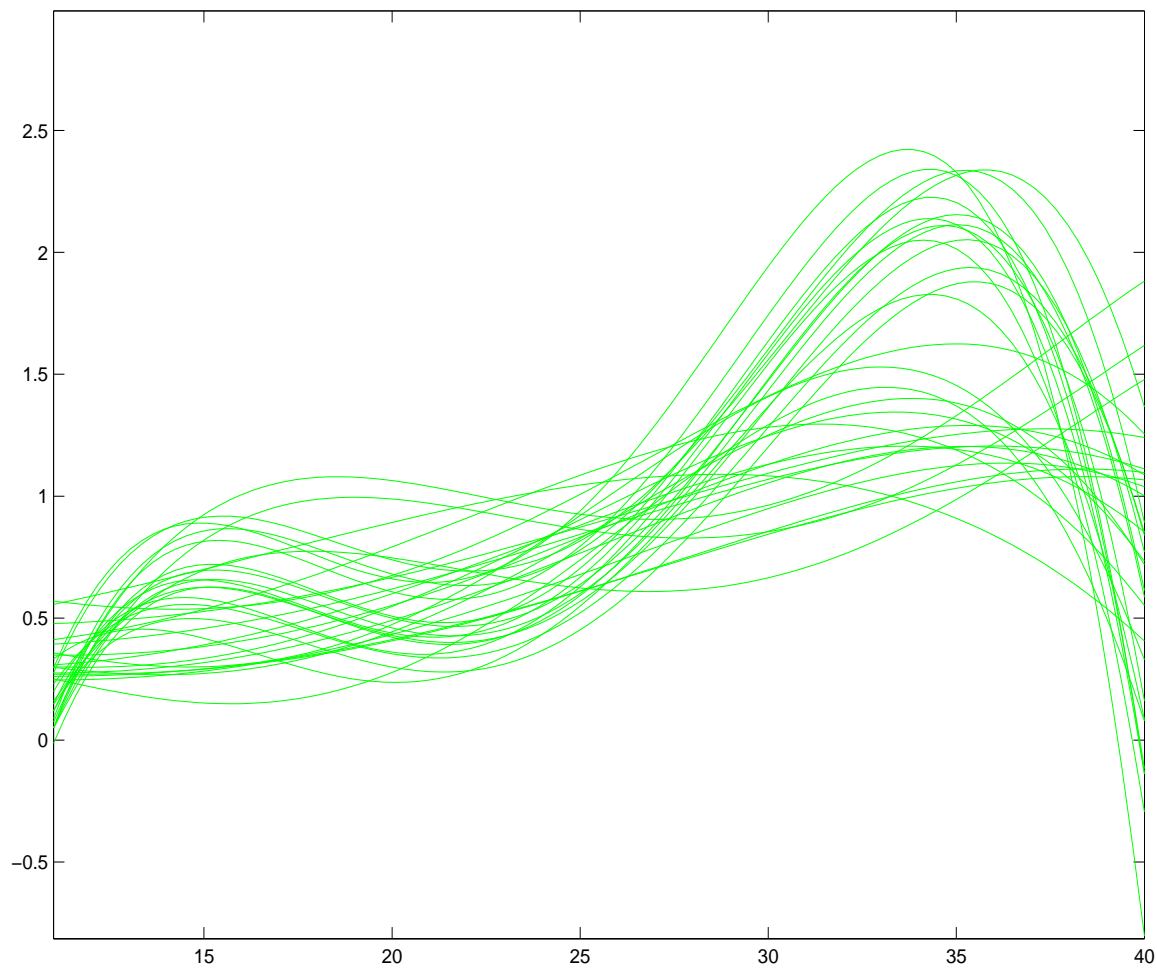
Geodesic mean, SS, and RSS

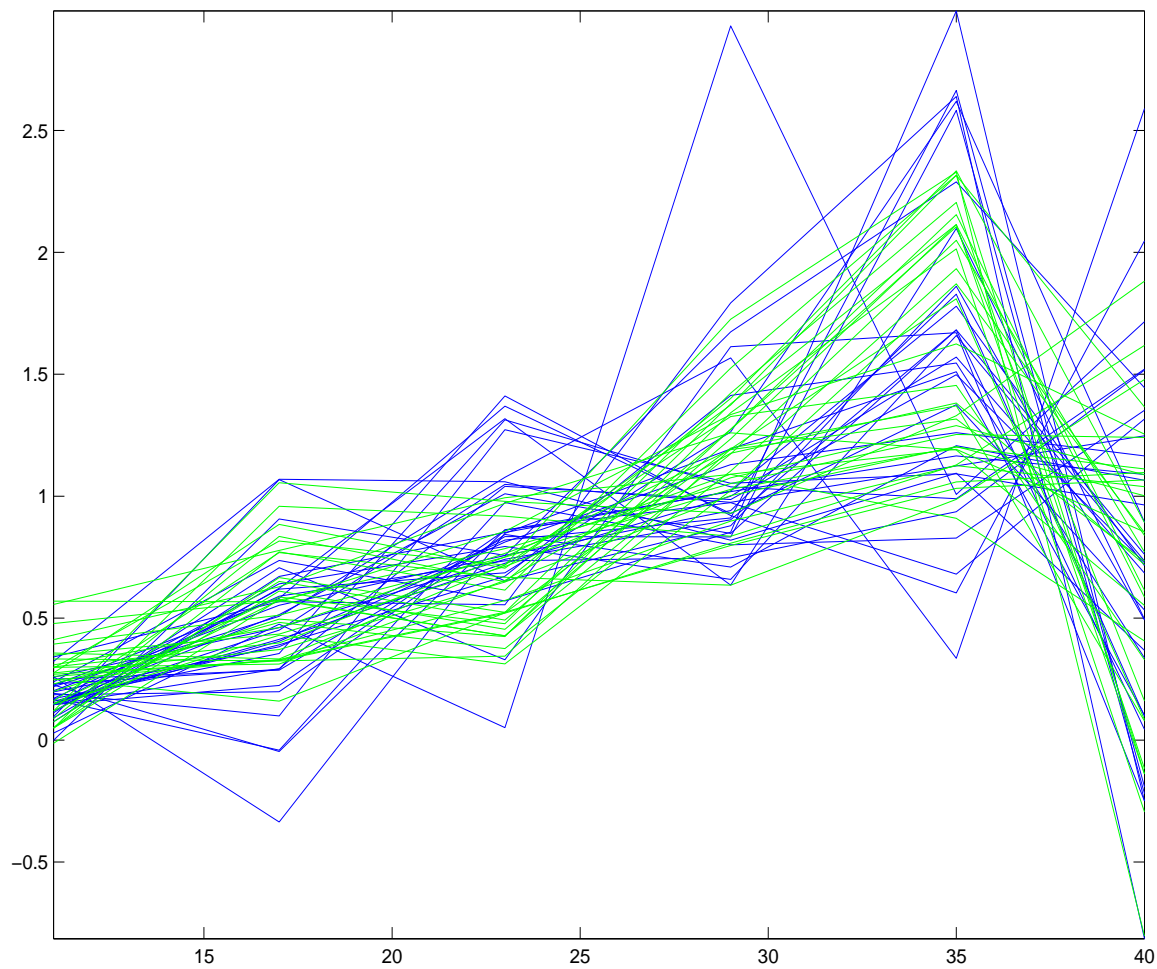
- Geodesic sample mean $\tilde{R} \doteq \text{Argmin}_a \sum_i d_g(a, R_i)^2$.
- Spread: $\widetilde{SSM} = \sum_i d_g(\tilde{R}, R_i)^2$.
- $\widetilde{RSS} = \frac{\widetilde{SSM}}{\widetilde{SSM} + \widetilde{SSE}}$.

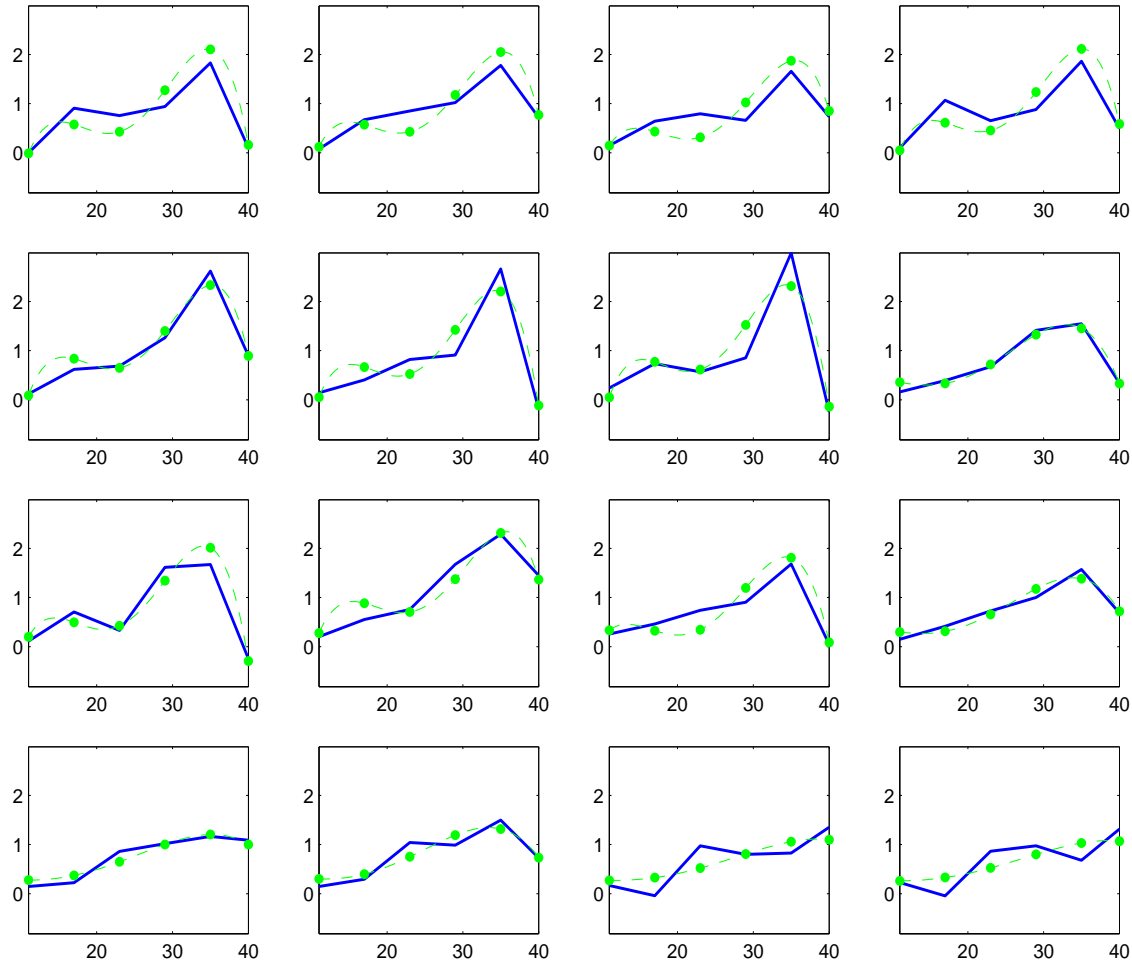
Outline

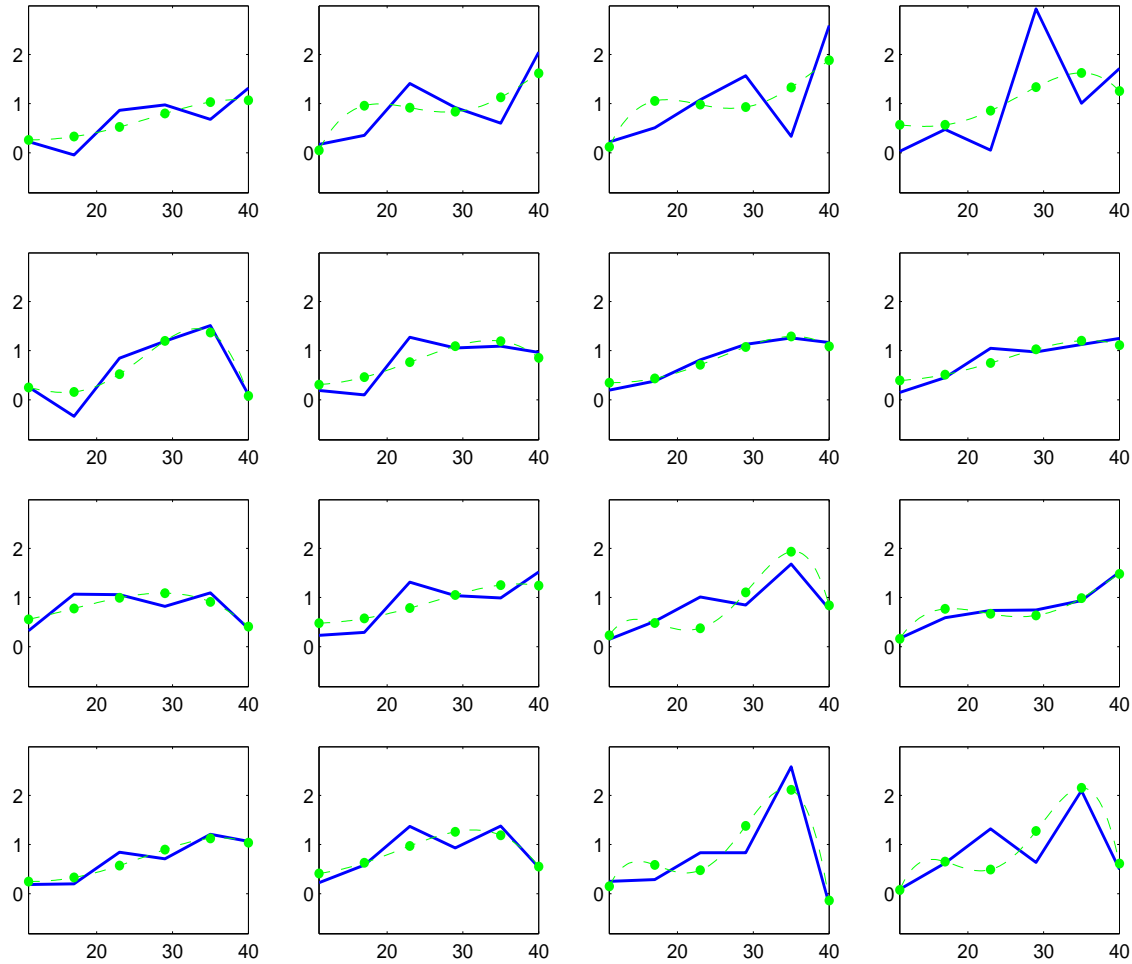
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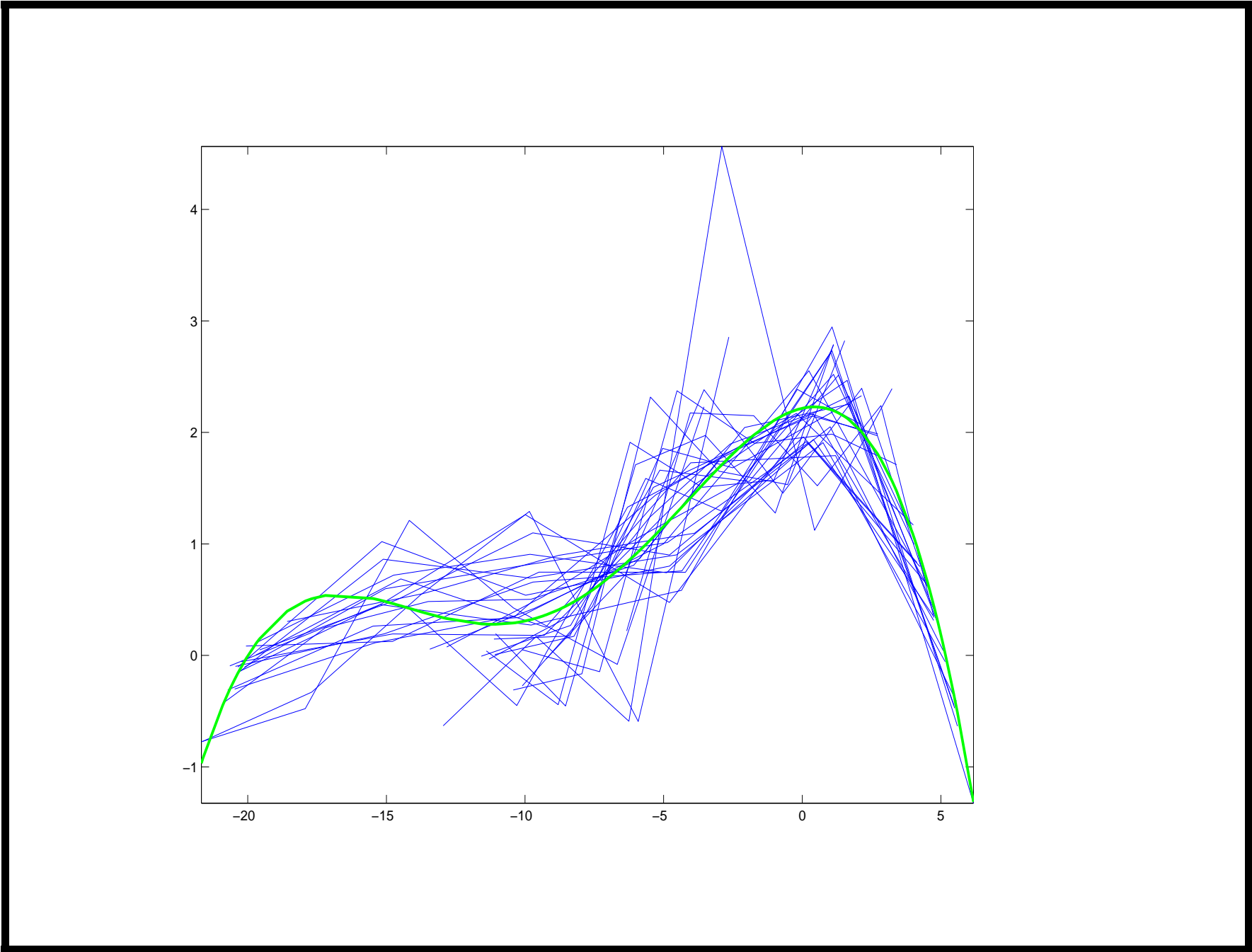












Mode of variation	\widetilde{RSS}
Generalist Specialist	43.40%
Horizontal Shift	20.85%
Vertical Shift	7.36%
Horiz-Gen.Spec	3.78%
Model total	75.28%
Error	24.72%

Conclusion for Evolutionary Biology: Selecting individuals with highest performance in a range of temperature will not result in the next generation in a population with highest performance over all ranges of temperatures.

Assuming a common shape of curves, the Template Modes of Variation allows to decompose curves' variance into linear and nonlinear modes and to quantify each mode. Linear modes are a particular case of this method of decomposition.

Current work

- Is low VS a common feature?
- Clustering?
- In stat, for $d' \geq 2$.
 - Conditions of existence and uniqueness of “center” along the manifold.
 - Convergence of the estimates from the data to true “center” and “spread” .
 - Sensitivity analysis of the decomposition to common shape.

THANK YOU