

STATISTICS 151 SECTION 1 MIDTERM 2 MARCH 19 2009

This is an open book exam. Course text, personal notes and calculator are permitted. You have 75 minutes to complete the test. Personal computers are not allowed. If you have any queries about the meaning of the questions, ask the teaching assistant for advice.

Answers are to be written in a blue book.

Before you begin, copy out the following pledge and sign it. If your blue book has a preprinted pledge, you may sign that instead.

Honor pledge: On my honor, I have neither given nor received unauthorized aid in this exam.

SHOW ALL WORKING — even correct answers will not get full credit if it's not clear how they were obtained. Incorrect answers will gain substantial credit if the method of working is substantially correct.

Answer all questions. The score for each part is indicated at the end of the question (100 points total).

1. In a case control study, patients with lung cancer were compared with a matched group of patients who did not have lung cancer. Among the lung cancer patients, 10.8% had prior exposure to asbestos. Among the patients who did not have lung cancer, only 1.7% had been exposed to asbestos. Over the whole population that roughly matches these patients for age, gender, medical history etc., approximately 1% of all people contract lung cancer.

Given that someone has been exposed to asbestos, what is the probability they contract lung cancer? [**20 points.**]

2. According to statistics from UNC Basketball, last season Tyler Hansborough had an 80.6% success rate in free throw shooting. Assume, on the basis of this, that the probability he makes a free throw is always 0.806, and that this is independent from throw to throw.

In a particular game, Tyler takes a total of nine free throws.

- (a) What is the probability that he makes all nine? [**4 points.**]
- (b) What is the probability that he makes exactly eight of the nine? [**6 points.**]
- (c) What is the probability that he makes at least seven of the nine? [**8 points.**]
- (d) Over a series of games, he takes 95 free throws. Let X be the number of those that he makes. What are the mean and standard deviation of X ? [**7 points.**]
- (e) Assuming that X has an approximately normal distribution, what is the probability that X is at least 80? [**7 points.**]
- (f) Coach Roy Williams thinks Tyler can do better, so as an incentive to practice more, he decides to offer Tyler a reward if he can make at least x (a number of be determined) of his next 95 free throws. Coach Williams doesn't want to make the challenge either too difficult or too easy, so he decides to choose the number x so that the probability Tyler gets the reward, assuming his long-run success rate of 80.6% doesn't change, is as near as possible to 0.1. What value of x should Coach Williams choose? [**8 points.**]

TURN OVER. QUESTION THREE ON THE OTHER SIDE.

3. A company offers the following insurance policy for computers.

If the computer requires a routine repair, the policy will pay out \$200. The probability of that in any given year is 0.1.

If the computer suffers a loss of hard drive and data, the policy will pay out \$500. The probability of that in any given year is 0.02.

If the computer is stolen or otherwise lost completely, the policy will pay out \$3000. The probability of that in any given year is 0.005.

These are considered to be disjoint events, i.e. we don't have more than one claim on any single policy in a given year.

- (a) Represent the sample space in the form of a table that lists all possible payouts x and their respective probabilities. [**10 points.**]
- (b) Calculate the mean of the payout that the company has to make in a single year on a single policy. [**6 points.**]
- (c) The company charges a premium of \$50. Based on your answer to (b), would you say this is a fair premium? Briefly explain why or why not. [**4 points.**]
- (d) Now suppose the company sells 10000 policies. Let \bar{x} be the mean payout that the company makes per policy. Find the mean and standard deviation of \bar{x} . [**Note:** The standard deviation of the payout on a single policy is \$228. You can assume that — you are not required to calculate it.] [**7 points.**]
- (e) Assuming the distribution of \bar{x} is approximately normal, what is the probability that the company makes a profit overall? [**6 points.**]
- (f) To what amount would the company have to raise the premium so that the chance of making a profit is 0.999? [**Note:** Throughout this question we are ignoring administrative costs and other fixed payments the company must make, so the premiums would in reality be higher.] [**7 points.**]

SOLUTIONS

1. Tree diagram (see figure, next page). If LC denotes the event of having lung cancer, NLC means no lung cancer, and A represents exposure to asbestos, then

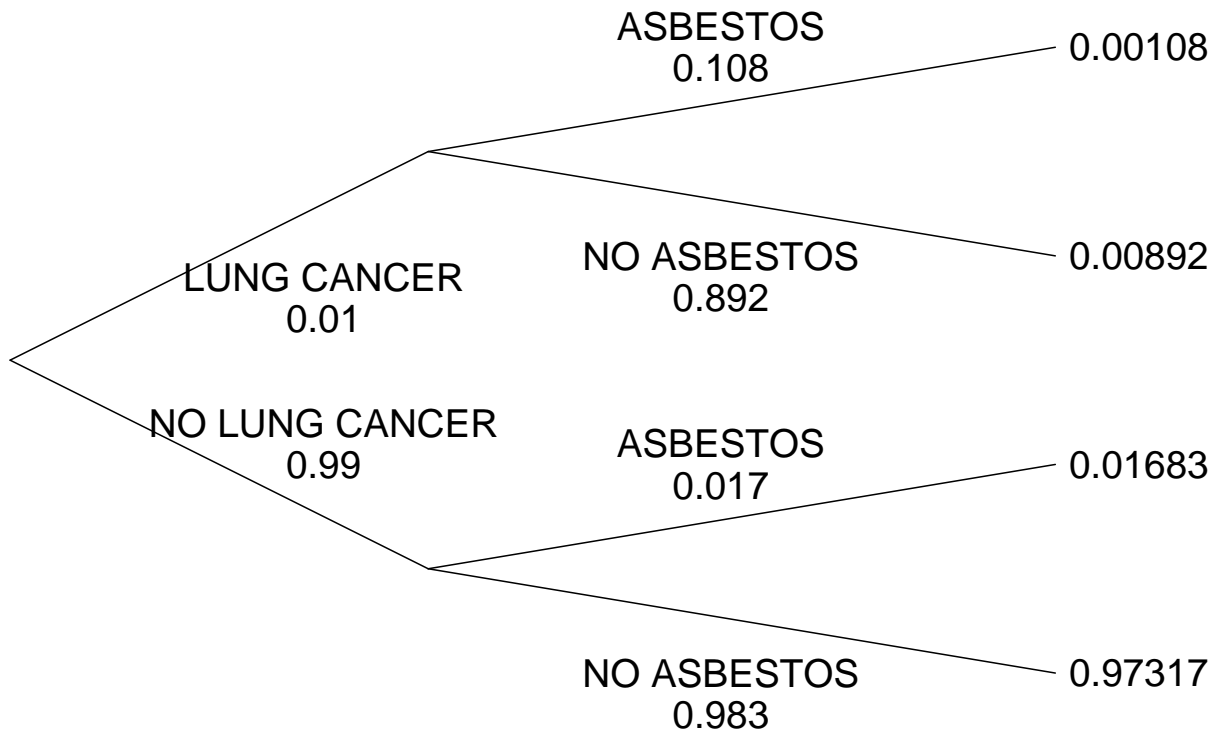
$$\begin{aligned}\Pr\{LC \text{ and } A\} &= 0.01 \times 0.108 = .00108, \\ \Pr\{NLC \text{ and } A\} &= 0.99 \times 0.017 = .01683, \\ \Pr\{LC | A\} &= \frac{.00108}{.00108 + .01683} = .0603.\end{aligned}$$

Among all the patients exposed to asbestos, approximately 6% will contract lung cancer.

2. (a) $(0.806)^9 = 0.144$.
 (b) $9 \times (0.806)^8 \times 0.194 = 0.311$.
 (c) The probability he makes exactly 7 free throws is $\frac{9 \times 8}{2} \times (0.806)^7 \times (0.194)^2 = 0.299$. Therefore, the probability he makes *at least* 7 is $0.299 + 0.311 + 0.144 = 0.754$.
 (d) Mean is $\mu = 95 \times 0.806 = 76.57$. Standard deviation is $\sigma = \sqrt{95 \times 0.806 \times 0.194} = 3.85$.
 (e) With $x = 80$, write $z = \frac{80 - 76.57}{3.85} = 0.89$ to two decimal places. According to the normal table, the probability that X is less than 80 is .8133 (say .81 to two decimal places). Therefore, the probability that X is at least 80 is $1 - .81 = .19$. (Note that in this and similar problems, we neglect the possibility that x is exactly 80.)
 (f) Referring to the normal probability table with a left-tail probability as close as possible to 0.9, z should be 1.28 to two decimal places. Therefore, $x = \mu + \sigma z = 81.498$. Since Tyler obviously cannot have a fractional number of successes, the coach should set the target to be $x = 82$.
3. (a) See the table:

Event	x	p	xp
No claim	0	.875	0
Repair	200	.1	20
Hard drive	500	.02	10
Total loss	3000	.005	15
Sum		1	45

- (b) According to the table, the answer is \$45.
 (c) The premium is greater than the expected payout, which is necessary for the company. But it's not much greater — it seems fair to the purchaser (maybe too fair...)
 (d) We have a mean \bar{x} when the individual payouts have mean $\mu = 45$ and standard deviation $\sigma = 228$. So the mean of \bar{x} is $\mu = 45$ and the standard deviation is $\frac{\sigma}{\sqrt{n}} = \frac{228}{\sqrt{10000}} = 2.28$.
 (e) We want to calculate the probability that \bar{x} is less than \$50. So the z statistic is $\frac{50 - 45}{2.28} = 2.19$. From the normal table, the left-hand tail probability associated with $z = 2.19$ is 0.9857. So the company has about a 98.6% chance of making a profit.
 (f) The z value associated with a left-hand probability of 0.999 is 3.08 (or 3.09 or 3.10 — since all three z values translate to 0.999 in the table, I'll accept any of them). Since \bar{x} still have mean 45 and standard deviation 2.28, we would have to raise the premium to $45 + 3.08 \times 2.28 = 52.02$ (dollars) to be certain of making a profit, with that probability.



Tree diagram for question 1.