

Kubak Crystal Caves Problem
 (#3, page 205, Frank and Bernanke)

The point of this problem is to design an allocation system that maximizes total welfare of the agents described in the problem. The following table provides arrival times and reservation prices.

Name	Arrival Time	Reservation Price
Herman	1:48	20
Jon	1:50	14
Kate	1:53	30
Jack	1:56	15
Penny	1:57	40
Fran	1:59	12
Faith	2:06	17

The key to answering this question correctly is to think about social costs and social benefits. The social costs include wear and tear to the cave and the warden's time, but these are the same for each scheme. One cost that is not the same for each scheme is the cost of payments made by the warden to volunteers. The social benefits are of only two kinds: the benefit of visiting the cave to those who get to do so and the payment received by those who volunteer to give up their place. While it makes sense to consider surplus as the difference between reservation price and opportunity cost when figuring out what decision each visitor will make, from a social point of view the benefit of visiting the cave is always the visitor's reservation price to do so.

- a. The social benefit created by "free admission" and first-come-first-served allocation is thus $20 + 14 + 30 + 15 = \$79$
- b. The warden offers a reward of (just over) \$15 to induce Jon, Jack, and Fran to give up their place in the queue. The volunteers are free to come back another day but receive no guarantees about admission. The warden pays each volunteer the same reward. Of course, the three named people are those whose reservation prices are all at or below \$15. .

The social benefit has three parts: Those who visit the cave get their reservation prices, those who give up their place each get \$15, the warden loses \$15. The total benefit is:

$$(20 + 30 + 40 + 17) + 45 - 45 = \$107$$

- c. One problem with this allocation is that it is not self-financing. The warden or whoever supplies the funds, are worse off then they were under the first-come-first-served rule.

Suppose the warden knows from past experience that he must raise about \$45 to cover the cost of payments to the volunteers. He abandons the first-come-first-served policy and charges everyone \$7.00 for a cave visit. Since all the visitors in the above table have reservation prices above \$7, they all wish to visit and pay the \$7 as they arrive. Once the warden sees that he has seven potential visitors, he offers a reward of \$15 for volunteers. Jon, Jack, and Faith volunteer.

$$\text{Total benefit is now } (13 + 23 + 33 + 10) + (45-21) + 4 = \$107$$

This example serves to demonstrate that the increase in social welfare is obtained by choosing a scheme that permits those with high reservation prices to visit the caves. There are other schemes besides this one that would produce the same allocation and achieve the same social welfare.