Econ 423
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Interest Rates in the Long Run
Notes

The rate of interest is a price ground out by the forces of supply and demand. But the forces that act on interest rates are different in the long run than in the short run. In this set of notes, we focus on the long run and investigate what Irving Fisher meant when he said that the real rate of interest is determined by the "patience of the population and the productivity of capital."

Outline

1. What is the real rate of interest?
   a. How is the real rate defined?
   b. How can we estimate the real rate with U.S. data

2. In the long run, the real rate of interest is determined jointly by the patience of the population and the productivity of capital.
   a. The household's problem is to distribute income between the current and future periods.
   b. The firm's problem is to decide how much of its resources to commit to capital building (investment in the economic sense of the term).
   c. In general equilibrium, the plans of households and firms must be compatible.
   d. The long run real rate of interest is the real rate of interest that makes the supply of saving equal to the demand for investment funds.

What is the real rate of interest?

Define R, the nominal rate of interest, to be the annual yield to maturity of a debt instrument. For example, R might be the yield to maturity of a U.S. Treasury bond.

The real rate of interest, r, is the nominal rate of interest minus the expected annual rate of inflation over the life of the debt instrument.

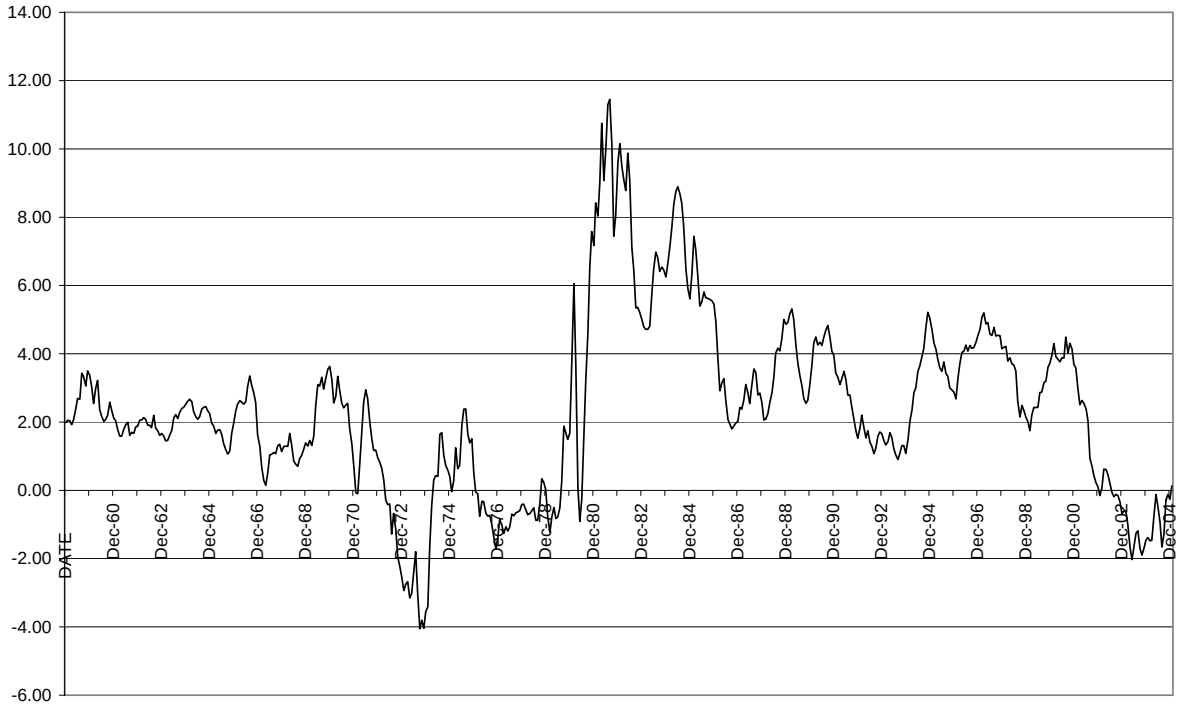
If the annual expected rate of inflation is $\mathbf{B}$, $r = R - \mathbf{B}$. For now, we assume that interest income is not taxed.

If interest income is taxed at rate $\mathbf{J}$ and if interest expenses are deductible, then the real rate of interest is defined as $r = (1-\mathbf{J})R - \mathbf{B}$.

The following graphs show the one year T bond rate, the inflation rate, and an estimate of the real rate of interest from 1959 to 2005.

One Year Bond Rate and Inflation Rate

Real Rate of Interest



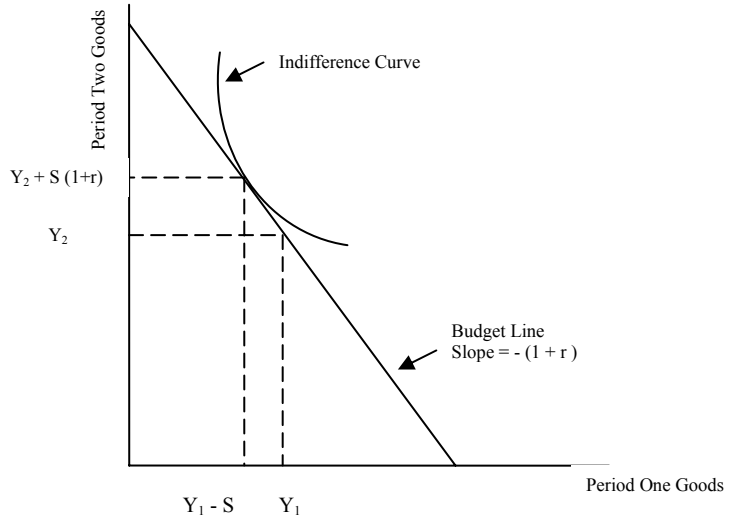
Fisher's Theory

In his famous book, *Theory of Interest*, Irving Fisher argued that the equilibrium rate of interest is determined jointly by the patience of individuals and the productivity of capital. These notes set out a simple version of Fisher's model.

Suppose an economy with two agents–a representative household and a representative firm. There is a single all-purpose composite commodity called "goods." Agents consume and produce goods in each of two periods. The real rate of interest is the only price in the model. Households and firms take the real rate of interest as given and respond optimally. For households, the response is a saving decision, for firms an investment decision. In equilibrium, decisions of the households and firms jointly determine the real interest rate.

The Household

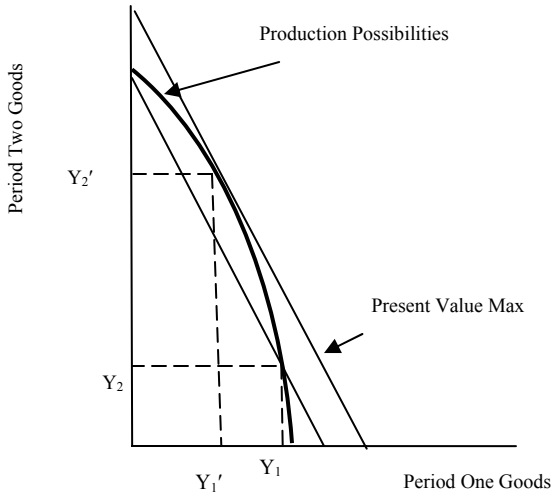
The household derives utility from consuming goods. Each period, it receives an endowment of goods and can borrow and lend at real interest rate r. The household chooses the point on its budget line that maximizes utility. The solution to the household's problem is characterized as a tangency between budget line and indifference curve. The following diagram depicts the household's opportunity set and its optimal saving decision.



The Firm

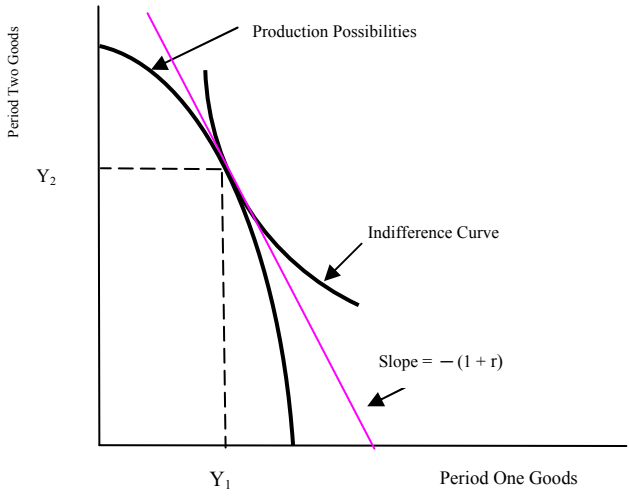
The firm owns a production process that yields y_1 in period one and y_2 in period two. The firm can change the mix of period-one and period-two goods by using some of its period-one output to build capital which allows the firm to produce more period-two goods. By investing, the firm lowers its output of period one goods and raises its output of period two goods. The production possibilities frontier of the firm is shown in the diagram. If capital is more productive, then the firms PPF is steeper.

The present value of the firm's output is the horizontal intercept of a line with slope equal to -(1+r) that passes through the firms production point. The firm chooses the point on its PPF that maximizes present value. The optimal production decision is characterized by tangency between the PPF and an iso-profit line with slope equal to -(1+r). The optimal production decision of the firm is shown in the diagram.



General Equilibrium

The equilibrium real rate of interest is determined by the patience of households (as represented by the shape of the indifference curve) and the productivity of capital (as represented by the shape of the firm's PPF). The interest rate sends signals to households and firms. A high interest rate signals households to save more. A high interest rate signals firms to use its transformation technology less. Equilibrium requires that period-one and period-two goods choices of households and firms are the same. The following shows how compatibility of household and firm plans determines the equilibrium rate of interest. The equilibrium real rate of interest is the only "trading ratio" between period-one goods and period-two goods that provides a budget line that induces households to save a sufficient amount to finance the investment that the firms wish to make. It is likewise the only interest rate that provides a present value line that makes optimal an investment level of the firm that is compatible with the households saving plan.



Study and Discussion Questions

As you study these notes please consider the following questions, some of which we will cover in class and some that will be left for you to answer on your own.

1. In what sense is the real rate of interest a "forward looking" variable? The real rate depicted on page 2 is a perfect foresight estimate of the real rate. How does it differ from an estimate of the real rate that an economic agent would make in real time?
2. As the real interest rate increases, how does the budget line in Figure A change?
3. What is a correct interpretation of the horizontal intercept of the budget line?
4. Is the household in Figure A saving or borrowing in period one? How do you know?
5. In Figure B, how can one depict the present value of a firm that does not use its technology?
6. Should the Figure B firm use its technology if it faces an interest rate shown in the iso-profit line? Why?
7. Why does it make sense that the firm invests less when the real interest rate is high?
8. Why does tangency between the household indifference curve and the firm PPF determine a unique, equilibrium real rate of interest?
9. In what sense, are the decisions of the household and the firm mutually compatible at the equilibrium rate of interest?
10. What would happen to the equilibrium rate of interest if an invention improved the firm's technology?
11. Why might the real rate of interest depart from the long-run equilibrium in the short run?

Fisher Model Mathematics

Household

The household receives income of y_1 in period one and y_2 in period two. It derives utility from consuming goods according to the utility function $U = U(c_1, c_2)$. The problem of the household is to choose consumption levels that maximize utility subject to the budget constraint

$$c_1 + \frac{c_2}{1 + r} \leq y_1 + \frac{y_2}{1 + r}$$

To solve this problem we first use the budget constraint to express c_2 as a function of y_1, y_2, and c_1 and then take the derivative of utility with respect to c_1. A necessary condition for an optimal choice of period one consumption is that the derivative equal zero.

$$U = U(c_1, (1 + r)y_1 + y_2 - (1 + r)c_1)$$

$$\frac{dU}{dc_1} = U_1 - U_2(1 + r) = 0$$

$$\Rightarrow \quad -\frac{U_1}{U_2} = -(1 + r)$$

where U_1 and U_2 are the marginal utility associated with c_1 and c_2. To maximize utility the household chooses a consumption plan that equates the marginal rate of substitution to $-(1 + r)$.

Firm

The firm owns an "orchard" that yields y_1 in period one and y_2 in period two. It also owns a technology that allows it to convert goods in period one into goods in period two. The technology is represented by a production function $z = f(x)$ that gives the amount of period two output (z) given period one input (x). The problem of the firm is to choose the input level that will maximize the present value of its output. That is the firm chooses y_1' to

$$\textbf{maximize} \quad PV = y_1' + \frac{y_2'}{1 + r} \quad \text{s.t.} \quad y_2' = y_2 + f(y_1 - y_1')$$

Setting the derivative of PV with respect to y_1' equal to zero yields $-f' = -(1 + r)$ where f' is the marginal product of the firm's technology.

General Equilibrium

In general equilibrium, household and firm plans must be compatible–the firm must produce what the household consumes. Also, both firm and household must face the same interest rate. The general equilibrium is defined by the condition

$$-f' = -\frac{U_1}{U_2}$$