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Why Do Interest Rates Move Together?
Class Notes

1. Discussion of exercise on the Components Model of interest rates.
   a. What does the components model say?
   b. What hypotheses did students formulate as part of the exercise?
   c. Did the data confirm or contradict the hypotheses?
   d. Overall, does the data confirm or contradict the components model of interest rates?

2. The Expectations Hypothesis of the Term Structure provides another reason why interest rates may move together.
   a. What is the term structure of interest rates?

   b. The intuition behind the terms structure is the idea that all bond hold strategies that move funds between the same two points in time are substitutes. If they are very close substitutes, then their expected yields should be the same because agents will always choose the strategy with the higher expected yield.

   c. Holding long maturity bonds is riskier than holding short maturity bonds.

   d. Many agents dislike risk and will therefore prefer less risky strategies for moving funds from the present to the future.
      i. The expectations hypothesis may not hold.

      ii. The liquidity premium hypothesis modifies the expectations hypothesis by taking risk into account.

   e. If the expectations hypothesis is approximately true, or if liquidity premia are constant, then the term structure of interest rates at a point in time can be used to forecast future interest rates.

The following figure is a screen shot of the U.S. Treasury web site that provides the Daily Treasury Yield Curve. It url is:
(http://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtml)

INTEREST RATE STATISTICS

Daily Treasury Yield Curve Rates

Historical Data

XML This data is also available in XML format by clicking on the XML icon

October 2006

Date	1 mo	3 mo	6 mo	1 yr	2 yr	3 yr	5 yr	7 yr	10 yr	20 yr	30 yr
10/02/06	4.67	4.88	5.02	4.90	4.66	4.59	4.56	4.57	4.62	4.83	4.76
10/03/06	4.71	4.90	5.02	4.90	4.67	4.59	4.56	4.57	4.62	4.83	4.76
10/04/06	4.78	4.93	5.00	4.87	4.60	4.53	4.50	4.52	4.57	4.79	4.72
10/05/06	4.81	4.94	5.03	4.90	4.65	4.58	4.55	4.56	4.61	4.84	4.76
10/06/06	4.76	4.95	5.05	4.94	4.74	4.67	4.64	4.65	4.70	4.92	4.84
10/10/06	4.81	5.00	5.10	5.00	4.82	4.75	4.71	4.71	4.75	4.96	4.88

* 30-year Treasury constant maturity series was discontinued on February 18, 2002 and reintroduced on February 9, 2006. From February 18, 2002 to February 8, 2006, Treasury published alternatives to a 30-year rate. See Long-Term Average Rate for more information.

Treasury Yield Curve Rates. These rates are commonly referred to as "Constant Maturity Treasury" rates, or CMTs. Yields are interpolated by the Treasury from the daily yield curve. This curve, which relates the yield on a security to its time to maturity is based on the closing market bid yields on actively traded Treasury securities in the over-the-counter market. These market yields are calculated from composites of quotations obtained by the Federal Reserve Bank of New York. The yield values are read from the yield curve at fixed maturities, currently 1, 3 and 6 months and 1, 2, 3, 5, 7, 10, 20, and 30 years. This method provides a yield for a 10 year maturity, for example, even if no outstanding security has exactly 10 years remaining to maturity.

Treasury Yield Curve Methodology. The Treasury yield curve is estimated daily using a cubic spline model. Inputs to the model are primarily bid-side yields for on-the-run Treasury securities. See our Treasury Yield Curve Methodology page for details.

For more information regarding these statistics contact the Office of Debt Management (202) 622-1118.
For other Public Debt information contact (202) 219-3350.

Daily Treasury Yield Curve Rates

Daily Treasury Long-Term Rates

Daily Treasury Real Yield Curve Rates

Daily Treasury Real Long-Term Rates

Weekly Aa Corporate Bond Index

1. The slope of the yield curve is said to be the difference between a long maturity rate (generally the yield on a 20 year treasury bond) and a short rate (the yield on a one or three month treasury bill). What happened to the slope of the yield curve during the first week in October?

2. Aside from a change in the slope, did the yield curve shift during the first week in October?

Holding Long Maturity Bonds is Riskier than Holding Short Maturity Bonds

Consider a bond that matures in M years and pays a coupon of constant value C at the end of each year. In reality, the typical bond pays coupons semi-annually but our simplification does not affect the results we will obtain.

If the annual coupon is constant, then there exists a simpler version of the present value formula for the price of the bond. Again, let the coupon be C dollars per year, let the par value of the bond (the amount paid to the bond holder at maturity) be F dollars and let the maturity date occur M years in the future. If R is the constant discount rate, the present value of the bond is:

$$PV = \frac{C}{1+R} + \frac{C}{(1+R)^2} + \ldots + \frac{C}{(1+R)^M} + \frac{F}{(1+R)^M}$$

$$= \frac{C}{R} + \left(F - C/R\right)\left(\frac{1}{1+R}\right)^M$$

To prove that the second line follows from the first, rewrite the present value as the difference between the present values of two perpetuities–the first starting in the present and the second starting in year t+M+1. Then use the rules for computing the present value of perpetuities to show that the result is true.

Let F = $1000 and C = $100. The following table gives present value for various values of R and M.

Present Value					
R\M	1	2	5	30	4
.08	$1018.52	$1035.67	$1079.85	$1225.16	$1250.00
.10	$1000.00	$1000.00	$1000.00	$1000.00	$1000.00
.12	$982.14	$966.20	$927.90	$838.39	$833.33

Question: Why do the data in the above table show that long maturity bonds are riskier than short maturity bonds?

The Expectations and Liquidity-Premium Hypotheses

Definitions

i_t = Today's (time t) interest rate on a one period (one year) bond.
$i_{n,t}$ = yield to maturity of an n-year bond at time t.
i^e_{t+m} = yield to maturity expected at t to occur at time t + m.

The Hypotheses

The *Expectations Hypothesis* of the term structure of interest rates is:

$$(1 + i_{n,t}) = [(1 + i_{1,t})(1 + i^e_{1,t+1})(1 + i^e_{1,t+2})\cdots(1 + i^e_{1,t+(n-1)})]^{1/n}$$

The hypothesis says that the gross yield at time t on an n-year bond equals the geometric average of the current one year rate and the one year rates expected for each year between time t and time n. There is an arithmetic approximation to the expectations hypothesis given by:

$$i_{n,t} = \frac{[i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + i^e_{1,t+(n-1)}]}{n}$$

The *Liquidity Premium* hypothesis modifies the expectations hypothesis by including a liquidity or risk premium in its explanation of long-maturity rates. The formula in its arithmetic form is:

$$i_{n,t} = \frac{[i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + i^e_{1,t+(n-1)}]}{n} + l_{n,t}$$

Holding constant expectations of future interest rates, the liquidity premium hypothesis says that the n-year rate is now greater than the average of expected one-period rates because there is a premium ($l_{n,t}$) added to compensate asset owners for increased risk and loss of liquidity.

Predicting Future Interest Rates

Using the above hypotheses one can forecast future one-year interest rates. For the EH the analysis proceeds as follows.

$i_{2,t} = [i_t + i^e_{t+1}] / 2$ $\qquad$ $i^e_{t+1} = 2\, i_{2,t} - i_t$

$i_{3,t} = [2i_{2,t} + i^e_{t+2}] / 3$ $\qquad$ $i^e_{t+2} = 3i_{3,t} - 2i_{2,t}$

$i_{4,t} = [3i_{3,t} + i^e_{t+3}] / 4$ $\qquad$ $i^e_{t+3} = 4i_{4,t} - 3i_{3,t}$ and so forth.

Exercise:

1. Use the current yield curve to forecast one-year interest rates for the next three years.
2. Explain how the liquidity premium hypothesis would alter those forecasts.