

Modeling the Price of A Share of Common Stock

1. Market Fundamentals Model

The mainstream model of share prices is the “fundamentals” model. The model says that shares are valued because and only because they are a claim to expected future dividends.

Assume:

- I. Agents are risk neutral so that they care only about expected returns.
- II. Agents have common beliefs about what determines the value of a share.
- III. Agents forecasts are rational. The agents know as much as the economist.
- IV. Agents are price takers.

Let $P_{z,t}$ be the price of a share of company Z at time t and let $D_{z,t}$ be the dividend paid by Z at time t. Then the fundamentals model says that the price of a share is the present value of expected future dividends.

$$P_{z,t} = \frac{E_t(D_{z,t+1})}{(1+R)} + \frac{E_t(D_{z,t+2})}{(1+R)^2} + \dots = \sum_{s=1}^{\infty} \frac{E_t(D_{z,t+s})}{(1+R)^s}$$

One version of the fundamentals model adds hypotheses about how firms earn profits and pay dividends. Given those additional hypotheses, the fundamentals model predicts how the price of a share will related to the parameters of the dividend model. For simplicity of notation we drop the Z-subscript so that $P_t = P_{z,t}$. Suppose the firm pays dividends equal to a fixed proportion of profits and profits are equal to a “standard” return to capital plus a random component. The firm uses retained profits to increase its capital stock.

Let Q_t be profit (per share) at time t, K_t = capital (per share) at time t, and let U_t be an unforecastable random shock to profits.

$$\begin{aligned} D_t &= \delta Q_t \\ Q_t &= \rho K_t + U_t \\ K_{t+1} &= K_t + (1-\delta) Q_t \end{aligned}$$

Plugging the equations into the present value expression for stock prices permits derivation of the following:

$$P_t = \frac{\delta \rho (1 + (1-\delta) \rho)}{R - (1-\delta) \rho} K_t = f \left(K_t, \rho, R, \delta \right)$$

If $R = .10$, $\delta = .25$, and $\rho = .12$

$$\frac{P_t}{K_t} = \frac{(.25)(.12)[1 + (.75)(.12)]}{.10 - (.75)(.12)} = \frac{.0327}{.01} = 3.27$$

2. The "Bubbles" Model

Equation (1) implies:

$$P_t = \frac{E_t(D_{t+1})}{(1 + R)} + \frac{E_t(P_{t+1})}{(1 + R)}$$

Suppose we modify assumptions II and III to permit diversity of opinion. The fundamentals model says that the expected value of next period's stock price $E_t(P_{t+1})$ must be "justified" by forecasts of dividends. The bubbles model weakens that requirement and says that if anything happens that makes people think the stock will be more valuable in the future, then the current price will rise as a result. The expectations of the traders are self-fulfilling.

3. The "Random Walk" Model

Reinstate II and III. Suppose we measure time in days rather than months or years. Then R , the per period rate of interest, is a small number. Because it is unlikely that a dividend will be paid in the next period $E_t(D_{t+1})$ will be zero for most stocks and time periods. Then, the above equation reduces to:

$$P_t = E_t(P_{t+1})$$

This property is called the Martingale property. One example of a martingale is a random walk such as the following equation for stock prices where V_t is a "white noise" random variable.

$$P_t = P_{t-1} + V_t$$

4. The Capital Asset Pricing Model (CAPM)

Drop assumption I and assume that people require compensation for bearing risk. Rearrange the bubbles-model equation to derive $R_{z,t}$, the expected one-period return from holding a share of Z .

$$R_{z,t} = \frac{E_t(P_{z,t+1}) - P_{z,t}}{P_{z,t}} + \frac{E_t(D_{z,t+1})}{P_{z,t}}$$

How large should $R_{z,t}$ be? In a risk neutral world, the required return on stocks would be approximately the required return on long-maturity treasury bonds. CAPM suggests that accounting for risk aversion provides an alternative answer.

$$R_{z,t} = R_t + \beta_Z (R_{M,t} - R_t)$$

where R_t is the "risk free" rate (Treasury Bond yield), $R_{M,t}$ is the average return on an optimally diversified portfolio and $\beta_Z (R_{M,t} - R_t)$ is the risk premium built by the market into $R_{z,t}$. The parameter β_Z measures the correlation between returns on the Z share and the overall market return.

Stock Pricing Exercise

In this exercise students will work with the fundamental value hypothesis of stock market prices to determine the appropriate price for the stock of a fictional company. They will also determine what sort of changes in the economic environment would change the price of the fictional stock and by how much that price would change.

The XYZ company is a bio-technical products company that is about to make an initial public offering of shares. The current earnings of the company are zero. The consensus view of investment analysts is that the prospects for future earnings of XYZ are well described by the following table of scenarios.

Scenario	A	B	C	D	E	F
Probability of Scenario	1/6	1/6	1/6	1/6	1/6	1/6
Permanent Level of XYZ Earnings per share	\$0.00	\$2.00	\$2.00	\$5.00	\$5.00	\$15.00

In scenario A the firm never has earnings while in scenario C earnings are expected to be permanently \$2.00 per share. Suppose the yield to maturity of 20 year treasury bonds is 5.00%.

Questions

- What price will the fundamental theory of stock prices predict for the shares of XYZ? How did you arrive at your answer.
- What would be the effect on the IPO price of XYZ shares if, on the day before the IPO, XYZ surprisingly announced that it had sold a patent for a new product and thereby earned \$5.00 per share? Your answer should depend on whether this announcement affects the table of scenarios.
 - Suppose the table of scenarios remains unchanged. How should the share price change?
 - Suppose analysts interpret the announcement to mean that the probability of scenario D has risen to 1/3 while the probability of scenario A has fallen to zero. How should the share price change?
- What would be the effect on the current price of XYZ shares if the treasury bond rate falls to 4.50%? Explain your reasoning.
- Why might the actual price of shares of XYZ be greater than the share price you concluded in part A?