

Equity Returns and Business Cycles in Small Open Economies

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Abstract

This is the first paper in the literature to match key basic business cycle moments and long-run equity returns in a small open economy with production. These results are achieved by introducing three modifications to a standard real business cycle model: (1) borrowing and lending costs are imposed to increase the volatility of the intertemporal marginal rate of substitution; (2) investment adjustment costs are assumed to, similarly, increase the volatility of equity returns; and (3) GHH preferences are imposed to smooth consumption. We also decompose the contributions of productivity, the world interest rate, and government expenditure shocks to equity premium. Our results are based on data from Argentina, Brazil, and Chile.

JEL Classification: E32; E44; F41; G12; G15.

Keywords: Asset pricing; Country risk premium; DSGE; Real business cycle; Small open economy.

1 Introduction

The literature on equity premia in small open economies with production is rather thin in the twenty plus years following the seminal work of Mehra and Prescott (1985). The difficulty in generating the equity premium in such an economy is that the exogenous world interest rates, as one important driving force, are quite smooth. As a result, the intertemporal marginal rate of

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substitution (IMRS) is not volatile enough. Since a volatile IMRS is required to generate equity premia, simply extending both the Jermann (1998) and Boldrin et al. (2001) models from a closed economy to a small open economy fails to achieve what these models accomplished in the closed economy case. More specifically, in a small open economy characterized by those models, large changes in business cycle moments have only a trivial effect on equity premia. To handle this problem in light of the smooth world interest rate issue, we impose borrowing and lending costs to enhance the volatility of the marginal rate of substitution.

By introducing borrowing and lending costs, we break down the direct link between the volatility of world risk free rate and the volatility of the IMRS. With these new costs, we magnify the volatility of the IMRS by linking it to the borrowing and lending margin alongside the exogenous world interest rates. With this enhanced IMRS, adjustment costs generate sufficiently high volatility of equity returns, and Greenwood et al. (1988) (henceforth GHH) utility function depresses the volatility in generated consumption, which is in line with the data. As a result, with the three modifications, GHH preferences, capital adjustment costs, and borrowing and lending costs, we are able to generate equity premium as observed in the data and match the basic dynamics of the RBC.

Our work is related to three strands of the existing literature. First, it is related to the equity premium puzzle literature in a closed economy. The models in this literature ranges from consumption based to production based asset pricing models.¹ The consumption based asset pricing models apply different preferences. Mehra and Prescott (1985) is the first to show that the equity premium puzzle cannot be explained by constant relative risk aversion (CRRA) utility since consumption profile, based on historical data, is quite smooth. Habit formation utility and Epstein and Zin (1991) preferences solve the puzzle. For example, Campbell and Cochrane (1999) explain equity premia by linking the asset prices to deviations of consumption from the external habit. The mechanism is as follows. In comparison to CRRA utility, as consumption falls towards habit in business cycle trough, the curvature of the habit formation utility function rises more sharply, hence asset prices fall and expected returns rise more, accordingly. As a result, with habit formation, even though consumption is smooth, the IMRS could be quite volatile.

With Epstein and Zin (1991) preferences, the coefficient of risk aversion and the intertemporal elasticity of substitution (IES) are separated. As a result, the equity premium is not only a function of consumption profile, but also a function of volatile consumption-delivering portfolio returns, see Bansal and Yaron (2004) and Bansal (2008) among others.

However, altering utility function alone is not sufficient to explain equity premia in a production economy, see Constantinides and Duffie (1996). As shown in Jermann (1998) and Boldrin et al. (2001), the reason for this failure is the relatively low model generated volatility of the rate of return on equity. To explain the equity premium, it is necessary to introduce investment adjustment costs,

¹We selectively discuss some of studies relevant to our work. This review is by no means exhaustive.

which boost the volatility of the rate of return on equity. But this innovation will result in higher volatility of consumption growth in a CRRA setting. Introduction of habit formation depresses the volatility of the generated consumption, and hence reconciles the results with smooth historical consumption data. In a closed production economy, both Jermann (1998) and Boldrin et al. (2001) are able to match the moments of business cycles and equity premia with the investment adjustment costs and habit formation preferences. Without adjustment costs, Constantinides and Duffie (1996) and Tallarini Jr. (2000) are not successful.² Instead, our work focuses on a small open economy.

The second strand is the asset pricing in a small open economy. When the production economy is extended to a small open economy, again, the modification of the utility function and the investment adjustment costs fail to generate sufficiently high equity premia. The reason is that the observed volatility of the world risk free rate is relatively low.³ This feature of world interest rates depresses the volatility of IMRS in a general equilibrium model. Hence, the model can not generate the equity premia observed in the data.⁴ In this strand, Mendoza and Smith (2006) is probably the best representative. However, they focus on short run dynamics of equity premium due to the potential sudden stops. Besides, their model cannot generate equity premium in the long run since sudden stops are very rare events. In comparison, our research is focused on the long run dynamics of the equity premia.

The third strand is the literature about world interest rate and government expenditure shocks. As Mendoza (1991), Neumeyer and Perri (2005), and Uribe and Yue (2006) show, shocks to world interest rates are important in driving the business cycles in an emerging economy. As demonstrated in Bekaert and Harvey (2003) among other studies, government expenditure affects risk premia for emerging markets. Burnside et al. (2004) demonstrate that government expenditure shocks have a significant impact on business cycles. As a result, we include both world interest rates and government expenditure shocks.

The rest of the paper is organized as follows. Section 2 presents the benchmark economy and defines the equilibrium. Section 3 describes the data and the calibration. Section 4 discusses the results about both equity returns and business cycle moments. And Section 5 concludes.

²The major shortcoming of all these DSGE models of asset pricing is the “risk free rate volatility anomaly”. To explain equity premium, it is necessary to have highly volatile intertemporal marginal rate of substitution. However, the highly volatile intertemporal marginal rate of substitution will result in highly volatile risk free rates, an outcome which is at odds with the observed data.

³World interest rates are the most important external driving forces in a small open economy.

⁴Precisely due to this reason, introduction of cross-sectional variation of consumption alone fails.

2 Benchmark economy

We study a one sector economy in the benchmark model. This economy has three types of agents: the representative domestic household, firms, and the government. A joint exogenous stochastic process of productivity, world interest rate, and government expenditure drives the economy. We assume that in this economy, the government does not invest or produce any goods and services. It collects a lump-sum tax to finance its expenditure.

2.1 The representative household

The representative household chooses hours and consumption to maximize the lifetime expected utility given the budget constraint. The household receives profits, capital rents, and labor income from the firms. There are two means to smooth consumption: to purchase one-period international non-state contingent real bonds, and to invest. There are two real frictions in the model, investment adjustment costs as in Mendoza (1991) and borrowing and lending costs for the household as in Uribe and Yue (2006). Formally, the representative household maximizes the life-time expected utility as:

$$\max_{\{c_t, h_t, i_t, k_{t+1}, d_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t), \quad (1)$$

$$\theta_0 = 1, \theta_{t+1} = \beta(\tilde{c}_t, \tilde{h}_t)\theta_t, t \geq 0,$$

where \mathbb{E}_0 denotes the expectation operator conditional on information available at time $t = 0$. The variables θ_t , c_t , h_t , i_t , k_{t+1} , and d_t denote the subjective discount factor, consumption, hours, investment, capital, and net foreign debt position, respectively. The variables \tilde{c}_t and \tilde{h}_t denote the cross-sectional averages of consumption and labor supply, respectively, which the individual households take as given.

The functional forms of the GHH utility function and the subjective discount factor are given by:

$$U(c, h) = \frac{\left(c - \frac{h^\omega}{\omega}\right)^{1-\gamma} - 1}{1-\gamma},$$

$$\beta(c, h) = \left(1 + c - \frac{h^\omega}{\omega}\right)^{-\beta_1}.$$

GHH utility is suitable for dynamic programming. As long as $\beta_1 < \gamma$, these preferences guarantee that there exists a unique limiting distribution of state variables, and the consumption good in every period is a normal good as discussed in Mendoza (1991).

It can be shown that, as in Epstein and Zin and in contrast to CRRA preferences, GHH utility function allows for separation of the coefficient of risk aversion, γ , and the IES. Another feature of

GHH utility is that it rules out the wealth effect on the labor supply decision.⁵

Following Mendoza (1991), we modify the GHH utility such that the household endogenizes its subjective discount rate of time preference. Let consumption be a composite of final good consumption and the disutility of labor supply, then the endogenous subjective discount factor is decreasing in past consumption. As a result, the GHH utility implies that a mean reverting behavior exists in the consumption. Whenever the representative household changes the current consumption, not only the marginal utility of current consumption changes, but also the impatience level for the future consumption changes as well. Specifically, an increase in current consumption causes a decrease in the marginal utility. All else equal, this implies a comparable increase in the future consumption. However, the subjective discount factor decreases as well. All else equal, this leads to a decrease in future consumption. Together, this means that the future consumption will not increase as much as today's consumption does, indicating a reversion towards the mean.

The endogenous subjective discount factors is one way to modify the standard real business cycle models to assure stationary behavior, as noted in Schmitt-Grohé and Uribe (2003). It is important to note that this modified GHH utility is not suitable for incorporation of balanced growth as is done in Boldrin et al. (2001) or in Aguiar and Gopinath (2007). Hence, we do not consider the growth effect in our model⁶.

The representative household is subject to period budget constraint:

$$d_t + w_t h_t + r_t k_t \geq r_{t-1}^f d_{t-1} + \Psi(d_t - \bar{d}) + c_t + i_t + \Gamma_t + \Phi(k_{t+1} - k_t). \quad (2)$$

The variables r_t and w_t denote the capital return and wage rate. The variable r_t^f is the world risk free rate from period t to $t+1$. We discuss the dynamics of this variable in detail in our treatment of the driving force. The variable Γ_t denotes the government lump sum tax. The variable k_t denotes physical capital and its law of motion is given by:

$$k_{t+1} = (1 - \delta)k_t + i_t. \quad (3)$$

We assume that all equity are held by domestic households. In this economy, dividends are returns to capital ownership which may include physical, human capital, or even intangible assets such as patents. Equity is assumed to be equivalent to the capital stock.

The expression $\Psi(d_t - \bar{d})$ denotes borrowing costs, and the expression \bar{d} is the non-stochastic steady state value of the debt position. In absence of borrowing and lending costs, households can borrow or lend at the world interest rate freely. Practically, this is not a tenable situation for most small open economies. Only a handful of these countries have access to world debt markets with

⁵This model can not predict the negative response of hours to positive technology shocks, see Gali (1999).

⁶If we assume that the subjective discount factor is constant, then balanced growth is admissible.

trivial costs. Second, while the effective interest rate of a small open economy is influenced by the world interest rate strongly, it is also effected by the debt position of this economy. We follow Uribe and Yue (2006) in assuming a quadratic form for these costs which depends on deviations from the steady state debt and a positive cost coefficient, ψ . The functional form of $\Psi(d_t - \bar{d})$ is given by:

$$\Psi(d_t - \bar{d}) = \frac{\psi}{2} (d_t - \bar{d})^2. \quad (4)$$

The expression $\Phi(k_{t+1} - k_t)$ denotes investment adjustment costs. As it has been shown by Kydland and Prescott (1982) and Jermann (1998), investment adjustment costs are important factors in explaining the business cycles and equity premium. Following Mendoza (1991), we include that costs in our model and assume the functional form of investment adjustment costs, $\Phi(k_{t+1} - k_t)$, as:

$$\Phi(k_{t+1} - k_t) = \frac{\phi}{2} (k_{t+1} - k_t)^2. \quad (5)$$

The representative household is subject to the non-Ponzi-game condition:

$$\lim_{j \rightarrow \infty} \mathbb{E}_t \frac{d_{t+j+1}}{\prod_{s=0}^j r_{t+s}^f} \geq 0. \quad (6)$$

The condition rules out the possibility that the representative household borrows to finance its consumption without limit.

The household's utility maximization problem is characterized by the following conditions: five first order conditions and the law of motion of capital held in each period, the period budget constraint, and the non-Ponzi game condition. In particular, we are interested in two optimality conditions for debt (world risk free asset) and capital (risky asset).

2.2 The firms

There is a large number of identical, final good producing firms. Firms, which are fully owned by domestic households, produce the final goods by hiring labor, renting capital, and buying imported intermediate input from households. Each firm issues a single stock, which is traded domestically. This assumption about domestic ownership is made to simplify our model and also make it more tractable theoretically. Firms use constant return to scale production technology, given by:

$$y_t = z_t h_t^\alpha k_t^{1-\alpha},$$

where the variables y_t , z_t , k_t , and h_t denote output of final good, total productivity factor, capital, and labor, respectively. The productivity follows an exogenous AR(1) process which will be defined in Section 2.4. Since firms do not make the investment decision, firm's optimization problem

becomes static. They choose k_t and h_t to maximize the period profit, given z_t , r_t , and w_t :

$$\max_{\{k,h\}} \Pi_t = z_t h_t^\alpha k_t^{1-\alpha} - r_t k_t - w_t h_t.$$

The first order conditions for the firms are standard:

$$r_t = (1 - \alpha) z_t h_t^\alpha k_t^{-\alpha} \quad (7)$$

$$w_t = \alpha z_t h_t^{\alpha-1} k_t^{1-\alpha} \quad (8)$$

These optimality conditions have their usual interpretation. Profits are equal to zero since we have constant returns to scale technology.

2.3 The government

The government faces a stream of public expenditures, denoted by g_t ; that is exogenous, stochastic, and unproductive. These expenditures are financed by levying lump sum taxes, which we denote by Γ_t . The governments sequential budget constraint is then given by:

$$\Gamma_t = g_t, \quad (9)$$

for $t \geq 0$. To simplify the discussion we assume that the government expenditures follow an AR(1) process. We discuss this process in detail next.

2.4 The driving force

This small open economy is driven by the joint exogenous processes of productivity, world interest rate, and government expenditure. In particular the productivity, z_t , follows an independent process; while the world interest rates, r_t^f , and the government expenditures, g_t , are correlated. Independence of productivity from government expenditure and world interest rate is intuitively reasonable. Productivity is a technical process which, in the short run, is not related to international business cycles which impact world interest rates. Neither is it directly affected, in the short run, by government fiscal policies. Mathematically, the driving force is given by

$$\begin{pmatrix} \hat{z}_t \\ \hat{r}_t^f \\ \hat{g}_t \end{pmatrix} = \begin{pmatrix} \varrho_z & 0 & 0 \\ 0 & \varrho_r & 0 \\ 0 & \varrho_{gr} & \varrho_g \end{pmatrix} \begin{pmatrix} \hat{z}_{t-1} \\ \hat{r}_{t-1}^f \\ \hat{g}_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^z \\ \varepsilon_t^r \\ \varepsilon_t^g \end{pmatrix}, \quad (10)$$

where \hat{z}_t denotes the Hodrick-Prescott filtered logarithm of z_t , \hat{g}_t denotes the HP filtered logarithm of g_t , and \hat{r}_t^f denotes the HP filtered logarithm of r_t^f . All the shocks are assumed to be normally distributed. In particular, we assume that $\varepsilon_t^z \sim \text{NIID}(0, \sigma_z^2)$. The expression NIID denotes identical

and independent normal distribution. The variance and covariance matrix of world interest rates and government expenditure shocks is assumed to be:

$$\Sigma_{rg} = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_g^2 \end{pmatrix}. \quad (11)$$

Our specification of the exogenous driving force follows the literature. In particular, with respect to the government expenditure process, Burnside et al. (2004) and Ravn et al. (2008) have adopted similar processes. In our numerical exercise, the structural parameters, ϱ_z and σ_z are calibrated. Other parameters in Eq. (10) are estimated based on the available data by applying the ordinary least squares (OLS). Their values are given in Table 1.

For the convenience of discussion, we introduce one variable here: \mathbf{s}_t , which is given by:

$$\mathbf{s}_t = \begin{pmatrix} z_t & r_t^f & g_t \end{pmatrix}'.$$

The variable \mathbf{s}_t is a vector of the exogenous state variables at time t .

2.5 Competitive equilibrium

In the equilibrium all markets are cleared and:

$$\tilde{c}_t = c_t; \tilde{h}_t = h_t. \quad (12)$$

The competitive equilibrium is defined in the standard form, as a sequence of real allocations $\{c_t, h_t, i_t, k_{t+1}, b_t, \Gamma_t\}_{t=0}^{\infty}$ and prices $\{r_t, w_t\}_{t=0}^{\infty}$, given $\{b_{-1}, k_0, z_0, g_0\}$, the driving force, (10), in such a way that households maximize their utility, firms maximize their profits, the government balances its budget, and all markets are cleared.

3 Data and calibration

We use quarterly data from a variety of sources. To get equity returns, we use MSCI market return indices data from DataStream. We use log differences of market indices as country returns and then aggregate the returns to quarterly level. Bond rates are from EMBI data bank constructed by J.P. Morgan and are calculated per quarter. The differences between the US bond returns and the expected inflation rates are the proxy for the world risk free rates. The expected inflation rates are given by the average inflation rates in the previous 4 quarters, see Uribe and Yue (2006).

Real trade balances are from International Financial Statistics data bank. Nominal macroeconomic variables, Government expenditure, GDP, and investment, are from International Financial Statistics data bank. The length of samples is not the same, based on availability of the data for

different countries. The range is from the first quarter of 1996 to the third quarter of 2007 for Chile translating into 47 observation to the first quarter of 1991 to the third quarter of 2007 for Brazil with 67 observations.

We transform the local currency denominated nominal per capita macroeconomic variables to US Dollar denominated real variables. All the series are de-seasonalized. The cyclical components of the variables are obtained by using the HP filter.

To solve the model, we must select values for parameters which characterize the stochastic shocks, preferences, and technology. These values are chosen to keep the model roughly consistent with some empirical regularities observed in business cycle and equity returns in sample countries. There are two steps in the calibration process. In the first step, we choose the values for a set of parameters for each country, closely following the standard procedure in the business cycle literature. In the second step, for those parameters with weak *a priori* knowledge, we pick their values to maximize the model's ability to replicate a set of business cycle and asset pricing moments.

For some parameters calibrated in the first step, we follow Mendoza (1991) and set the risk aversion coefficient, $\gamma = 2$, the capital depreciation rate, δ , at 0.025, and the exponent of labor supply in utility, ω , at 1.455. The rest of parameters calibrated in this step have country-specific values. We use the sample means of the corresponding data to determine the non-stochastic steady state ratio of trade balance to GDP, s_{tb} , the ratio of government expenditure to GDP, s_g , and the interest rate, r^f . The labor income share to GDP, α , are taken from the literature. In particular, we follow Michalopoulos (1969) and set its value at 0.735 for Argentina, 0.71 for Brazil, and 0.676 for Chile.

The steady state marginal return to capital, μ_k , is calculated from the deterministic steady state optimal condition $\mu_k = r^f - 1 + \delta$. The share of investment in value added, s_i , is calculated by the following equation:

$$s_i = \frac{i}{y} = \frac{\delta \mu_k k}{\mu_k y} = \frac{\delta s_k}{\mu_k}.$$

From the setup of the problem, the determination of the steady state values of c and h are independent of β_1 . Thus, after we calculate the steady state values of c and h , the parameter β_1 can be calibrated by the deterministic steady state optimal condition:

$$1 = \left(1 + c - \frac{h^\omega}{\omega}\right)^{-\beta_1} R.$$

With the data, the calibrated β_1 for all countries are less than γ , a condition which must be satisfied to guarantee that the GHH preferences have a unique limiting distribution of state variables, and the consumption good in every period is a normal good, see Mendoza (1991).

Given the regression results, the four structural parameters in the second step are ψ , ϕ , ρ_z , and σ_z . We calibrate their values by matching the standard deviation of output, the standard deviation of investment, the first-order serial autocorrelation of output, and the equity return, as close as

possible. In particular, we follow the same method as in Jermann (1998) to obtain their values by searching over hundreds of thousands of grid points, each of them defined by the four parameters.

The values of all the parameters are listed in Table 2.

4 Results

We find that the benchmark model with borrowing and lending costs, investment adjustment costs, and GHH utility function is able to match the selected business cycle and the equity return.⁷ As shown in Table 3, the benchmark model can generate the standard deviation of output, the standard deviation of investment, the first-order serial autocorrelation of output, and the mean equity return. All the values in Table 3 are in percentages.

With respect to business cycle moments, they are in general more or less matched across countries. Compared to the data, our benchmark model slightly overestimates the standard deviation of output in Argentina and the standard deviation of investment in Chile, and underestimates the first order serial autocorrelation of output in Brazil. If we use the band-pass filtered data, the model can match σ_Y , σ_I , and equity return closely, but fails to match the $\rho(Y_t, Y_{t-1})$. The model tends to generate high $\rho(Y_t, Y_{t-1})$ while the BP filtered data give a low $\rho(Y_t, Y_{t-1})$.

Given the low level of world interest rate for the sample period (around 0.8% per quarter), all four countries demonstrate high returns per quarter. Our model matches these returns very well. The best match is Chile where numerical error up to the fourth decimal point is essentially equal to zero. Brazil is the worst in which the generated equity return is about 0.04 percentage point smaller than the observed sample return per quarter. Given the fact that no previous work is able to generate equity return in a small open economy, we consider these results as convincingly successful.

4.1 Intuition behind the equity return

We emphasize that this success is due to imposition of all three requirements: GHH utility (or habit formation preference or Epstein and Zin preference), borrowing and lending costs, and capital adjustment costs. Omission of any one of them causes failure in generating successful results. In particular, we explain why it is important to have the borrowing and lending costs because the necessity of the other two factors has been fully discussed in the literature.

Let λ_t and φ_t be the Lagrange multipliers associated with (2) and (3), respectively. The Euler

⁷The derivation of equity return and the second moments are shown in the Appendix.

equation (13) represents the first order condition with respect to the debt position:

$$\left(1 + \tilde{c}_t - \frac{\tilde{h}_t^\omega}{\omega}\right)^{-\beta_1} \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \frac{[1 - \Psi'(d_t - \bar{d})]}{r_t^f}. \quad (13)$$

This condition has two important features. First, in this small open economy, $\{r_t^f\}_{t=0}^\infty$ is an exogenous process. Second, the term, $1 - \Psi'(d_t - \bar{d})$, appears in the equation due to borrowing and lending costs introduced in (2). Both have important implications for the equity return.

One stylized fact about world interest rates is that they are quite smooth, see Campbell (1999). In absence of borrowing and lending marginal cost, $[1 - \Psi'(d_t - \bar{d})]^{-1}$, the exogenous and smooth world interest rates force the MRS to be smooth too. Intuitively, without the cost, the supply of foreign financial assets is quite elastic. Holding everything else constant, the effect of shocks will be absorbed by changes in international bond holding position to keep the consumption smooth. Thus, the MRS will be smooth, and the model would not generate equity premia. This is also a problem in the closed economy. Both Jermann (1998) and Boldrin et al. (2001) match the equity premia at the cost of generating excessive high volatility of risk free rates. This is contrary to the low volatility observed in the data. Campbell (1999) and Cochrane (2006) call this “risk free rate volatility anomaly”. Introduction of the borrowing and lending costs makes the MRS more volatile. It directly breaks down the link between the MRS and world risk free rate. By imposing this cost, the supply of international bonds becomes less elastic. Thus, holding everything else constant, it is hard to adjust the international bond position to responds to shocks. In this case, consumption is forced to be volatile, as is the MRS.

The second Euler equation, which is the optimality condition with respect to capital, is given by:

$$1 = \left(1 + \tilde{c}_t - \frac{\tilde{h}_t^\omega}{\omega}\right)^{-\beta_1} \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{[1 - \delta + \phi(k_{t+2} - k_{t+1}) + r_{t+1}]}{[1 + \phi(k_{t+1} - k_t)]}. \quad (14)$$

Eq. (14) shows the relationship between household’s MRS and returns on capital. Given that the MRS (SDF) is volatile, it also requires volatile equity returns to generate equity premium. As it is discussed in detail by Boldrin et al. (2001), this is achieved by introducing investment adjustment costs.

Eq. (13) and Eq. (14) are critically important equations that will generate sufficient equity premium. In particular, Eq. (14) is the same as that in a closed economy. However, Eq. (13) is the key to generating sufficient equity premium when moving from a closed economy to a small open economy.

Without the borrowing and lending costs, Eq. (13) reduces to a standard bond pricing equation which implies that MRS is forced to be smooth because of the smooth risk-free interest rates. If so, there is no way to generate equity returns comparable to what is observed in the data. To

demonstrate this point, we set $\psi = 0$ while keeping the values of other parameters. The numerical results are shown in Table 4.⁸

By comparing the results shown in Table 4 to those in Table 3, we argue that in a small open economy, it is important to have the borrowing and lending costs to be able to general equity returns. We do not have the so called “risk free rate volatility” puzzle, see Cochrane (2006), Boldrin et al. (2001), Jermann (1998), and others, because we take the exogenous smooth risk free rates as given.

Additionally, certain preferences are required to smooth consumption. Here we choose GHH utility. The literature has emphasized two: habit formation and Epstein and Zin preference. In this paper, we show that GHH utility is a plausible candidate in addition to the two widely discussed in the literature. This is another contribution of this paper.

We decompose the impact of productivity shocks, world interest rate shocks, and government expenditure shocks on equity premia. The decomposition results are shown in Table 5. All values are in percentages. Columns one to three show the impact of productivity shocks (σ_z), world interest rate shocks (σ_r^f), and government expenditure shocks (σ_g) on equity premia.

From Table 5, it is clear that, productivity uncertainty is the most important factor in determining the equity premium in the long run. Almost 92% of the equity premium in Argentina is explained by the compensation to the productivity uncertainty. In Brazil, this ratio is about 67%, and in Chile, it is about 87%. This finding is in line with the literature since in the long-run, productivity shocks are the most important driving force in the economy. The next important risk is the government expenditure uncertainty. In Brazil, 30% of the equity premium is due to the compensation to the uncertainty associated with government expenditure.

4.2 Discussion of the business cycle results

We report model generated impulse-responses of aggregate consumption and output to a productivity shock, a world interest rate shock, and a government expenditure shock. There are two reasons to discuss only consumption and output. First, in general, the business cycle implications of shocks and corresponding responses of variables of interest in such a model have been widely studied, see Mendoza (1991), Schmitt-Grohé and Uribe (2003), among many others. Second, we follow Boldrin et al. (2001) in a discussion of the sensitivity of consumption to the change of income and interest rate.

Figure 2 plots the responses of consumption and output to a 1% increase in the productivity. Current consumption increases by 2.26% (Argentina), 2.03% (Brazil), and 2.55% (Chile), from their corresponding non-stochastic steady state, respectively. Current output increases by 2.02% (Argentina), 1.95% (Brazil), and 1.87% (Chile), from their corresponding non-stochastic steady

⁸All values are in percentages.

state, respectively. Combining the non-stochastic steady state values of consumption and output, the above percentage responses transform into the following observation: 68% of the change of current output in Argentina becomes the change in its current consumption; 54% in Brazil, and 73% in Chile. These findings show that current consumption in our model significantly responds to a temporary contemporaneous change in the output. Thus, the excessive sensitivity of consumption to income identified in Campbell and Mankiw (1989) is not a puzzle according to our representative agent optimization model.

Figure 1 plots the response of consumption to a 1% increase in the world interest rate. The fact that response functions are quite flat implies that the trade off between current consumption and future consumption does not respond to a change in today's risk free rate. The intuition is because with GHH utility, the elasticity of intertemporal substitution (EIS) in consumption, compared to that associated with the constant relative risk aversion preference, is low. The derivation of EIS is shown in the Appendix. This is in line with the empirical finding in Campbell and Mankiw (1989) and the theoretical prediction in Boldrin et al. (2001). However, our results show that the factor-market inflexibilities are not necessary conditions to explain the small response of consumption to the changes of risk free interest rates.

Figure 3 plots the response of consumption and output to a 1% increase in government expenditure. The consumption drops because of the negative wealth effect. The output does not change initially but decreases later since the economy accumulates relatively less capital. We need to emphasize that definition of a government expenditure shock is hypothetical in our model, even though Burnside et al. (2004) and Ravn et al. (2008) have adopted similar processes for the government expenditure.⁹

4.3 Sensitivity Analysis - Working Capital Constraint

So far, output does not drop in response to a world interest rate shock. This is due to the fact that with GHH utility, there is no wealth effect on labor supply, see Chari et al. (2005). To force the model to produce output drop, it is necessary to introduce new mechanisms, such as working capital constraint. Here we extend the model by including working capital constraint following Uribe and Yue (2006), which takes the subsequent form:

$$WK_t \geq \eta w_t h_t, \tag{15}$$

⁹The most concrete definition and discussion of government expenditure shocks is given by Burnside et al. (2004). They discuss the impact of wars on government expenditure. In our study, we focus on Latin American countries which have not faced any serious armed conflict during the sample period.

where the variable WK_t denotes the amount of working capital. The representative firm's debt position, d_t^F , evolves as:

$$d_t^F = r_{t-1}^f d_{t-1}^F - y_t + w_t h_t + \mu_t k_t + \pi_t + WK_t - WK_{t-1},$$

where the variable r_{t-1}^f denotes the risk-free interest rate. Define the net liability of the representative firm as $a_t = r_t^f d_t^F - WK_t$, we can rewrite the budget constraint of the representative firm as:

$$\frac{a_t}{r_t^f} = a_{t-1} - y_t + w_t h_t + \mu_t k_t + \pi_t + \left(\frac{r_t^f - 1}{r_t^f} \right) WK_t. \quad (16)$$

Since the representative firm is owned by the representative household, the objective function of firms is defined by:

$$\max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \theta_t \frac{\lambda_t}{\lambda_0} \pi_t,$$

where λ_t denotes the marginal wealth utility of the representative household. The representative firm is also subject to the following non-Ponzi-game constraint:

$$\lim_{j \rightarrow \infty} \mathbb{E}_t \frac{a_{t+j}}{\prod_{s=0}^j r_{t+s}^f} \leq 0. \quad (17)$$

The introduction of working capital constraint will only change the optimal condition for labor demand. Labor demand is determined by the following equation:

$$w_t \left[1 + \eta \left(\frac{r_t^f - 1}{r_t^f} \right) \right] = \alpha_h z_t k_t^{\alpha_k} h_t^{\alpha_h - 1}. \quad (18)$$

Since any real valued process $\{a_t\}_{t=0}^{\infty}$ satisfies (16) and (17) will be optimal for the representative firm, we follow Uribe and Yue (2006) and set $a_t = 0$. Hence only one parameter (η) in Eq. (15) needs to be calibrated. We follow Uribe and Yue (2006) and set $\eta = 1.2$, which means that the representative firm needs to save money to be able to pay at least 1.2 quarter wage bills.

Table 3 reports the numerical results with the working capital constraint. In the table, "WK" denotes the model with working capital constraint. From the table, it is clear that the working capital constraint has a small impact on both business cycle and equity returns moments. Once we shut down the borrowing and lending costs channel, the introduction of working capital constraint cannot generate equity returns, as shown in Table 4. Comparing the results reported in this table, the working capital constraint only improves the model's ability in generating equity returns by a small margin.

5 Conclusions

This is the first paper in the literature to match key basic business cycle moments and long-run equity returns in a small open economy with production. These results are achieved by introducing three modifications to a standard real business cycle model: (1) borrowing and lending costs are imposed to increase the volatility of the intertemporal marginal rate of substitution (IMRS or SDF); (2) investment adjustment costs are assumed to, similarly, increase the volatility of equity returns; and (3) GHH preferences are imposed to smooth consumption. The logic behind the first modification is the presence of exogenous and smooth world interest rates.

Without the borrowing and lending cost, the supply of foreign financial assets is quite elastic. Holding everything else constant, the effect of shocks will be absorbed by changes in international bond holding position to keep the consumption smooth. Thus, the IMRS will be smooth, and the model would not generate equity premia. By imposing this cost, the supply of international bonds becomes less elastic. Thus, holding everything else constant, it is hard to adjust the international bond position in response to shocks. In this case, consumption is forced to be volatile, as is the IMRS. Regarding capital adjustment costs, a similar analysis holds. In absence of these costs, as in the standard RBC model, the supply of capital is perfectly elastic at a constant price. Holding everything else constant, the effect of shocks will be absorbed by changes in capital stock to keep the consumption smooth. Thus, the IMRS will be smooth, and the model again would not generate equity premia. By imposing capital adjustment costs, the supply of capital becomes less elastic, imposing volatility on the IMRS. The intuition for the use of GHH utility function is the same as that for habit formation in Boldrin et al. (2001).

Based on data from Argentina, Brazil, and Chile, we show that the model can replicate both business cycle moments and equity returns. In sum, we believe that this model makes significant progress in addressing the equity premium puzzle in a small open economy with production framework.

By decomposing the contributions of productivity, the world interest rate, and government expenditure shocks to equity premium, we find that productivity shocks are the most important contributor to the equity premia in a small open economy.

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Tables

Table 1: Estimated Parameters of the Driving Force

Country	ϱ_r	ϱ_{gr}	ϱ_g	σ_r	σ_g
Argentina	0.9986	1.2008	0.8206	0.001	0.1134
Brazil	0.9834	0.3565	0.7924	0.001	0.1225
Chile	0.9882	-0.0778	0.8296	0.001	0.0349

Reported values are OLS parameter estimates of the driving force in the economy as specified in $\hat{\mathbf{s}}_t = \varrho \hat{\mathbf{s}}_{t-1} + \varepsilon_t$ where $\hat{\mathbf{s}}_t$ represents HP-filtered deviations from the non-stochastic steady state of variables in $\mathbf{s}_t = \begin{pmatrix} z_t & r_t^f & g_t \end{pmatrix}'$. ϱ_r , ϱ_g , and ϱ_{gr} represent autoregressive terms in the driving force for world interest rate, government expenditure, and the interaction term for government expenditure and world interest rates, respectively. σ_r and σ_g represent standard errors of world interest rate and government expenditure processes.

Table 2: Structural Parameter Calibration

Country	ρ_z	σ_z	ϕ	ψ	γ	ω	δ	s_{tb}	s_g	α_h
Argentina	0.70	2.5e-3	10	4.00	2	1.455	0.1	1.00e-02	0.127	0.735
Brazil	0.50	4.0e-3	50	0.14	2	1.455	0.1	2.29e-02	0.195	0.710
Chile	0.55	3.5e-4	34	7.10	2	1.455	0.1	5.01e-02	0.115	0.676

Reported values are calibrated parameters in this study. ρ_z and σ_z are the first order autoregressive parameter and standard deviation of productivity process, respectively. ϕ and ψ are the cost parameters for capital adjustment costs and borrowing and lending costs, respectively. γ is the coefficient of risk aversion, and ω is the GHH exponent of labor supply. δ is the depreciation parameter. s_{tb} and s_g represent the non-stochastic steady state ratio of trade balance to GDP and the ratio of government expenditure to GDP, respectively. α_h is the share of labor from national income.

Table 3: Equity Returns and Selected Business Cycle Moments

Country		Business Cycle Moments			Equity Returns
		σ_Y	σ_I	$\rho(Y_t, Y_{t-1})$	$\mathbb{E}_t(r_{t+1})$
Argentina	Model	18.3	22.1	83	3.98
	WK	18.4	21.8	83	3.93
	Data	17.2	22.2	81	4.00
Brazil	Model	18.3	20.5	70	5.71
	WK	18.1	19.4	69	5.29
	Data	18.3	19.9	82	5.75
Chile	Model	6.71	8.72	83	2.01
	WK	7.23	9.10	85	1.97
	Data	6.73	10.8	85	2.01

All values are in percentages. σ_Y and σ_I are standard deviations of output and investment, respectively. $\rho(Y_t, Y_{t-1})$ denotes the first order autocorrelation of the output. $\mathbb{E}_t(r_{t+1})$ is the expected value of equity returns. Rows denoted by “Model” refer to our GHH utility-based model with capital adjustment and borrowing and lending costs. Rows denoted as “WK”, refer to imposition of working capital constraint to generate output drop, following the formulation in Uribe and Yue (2006). Rows denoted as “Data” report the unconditional sample moments.

Table 4: Equity Returns without Borrowing and Lending Constraint

Country		Equity Returns
		$\mathbb{E}_t(r_{t+1})$
Argentina	Model	0.98
	WK	1.00
	Data	4.00
Brazil	Model	1.14
	WK	1.13
	Data	5.75
Chile	Model	0.88
	WK	0.89
	Data	2.01

All values are in percentages. This table demonstrates the impact of shutting down the borrowing and lending constraint. By imposing $\psi = 0$ condition,

$$\left(1 + \tilde{c}_t - \frac{\tilde{h}_t^\omega}{\omega}\right)^{-\beta_1} \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \frac{[1 - \Psi'(d_t - \bar{d})]}{r_t^f}.$$

collapses to the usual RBC risk-free asset pricing equation.

Table 5: Decomposition of Equity Premia

Country	σ_z	$\sigma_{r,f}$	σ_g
Argentina	92	0	8
Brazil	67	3	30
Chile	87	8	5

All values are in percentages. This table reports the contribution of a shock to the three exogenous state variables (productivity, world interest rate, and government expenditure) to the equity premium level in each country. Each entry demonstrates the importance of a particular factor in determining the equity premium in the long run. Shocks are a measure for uncertainty about a particular factor.

Figures

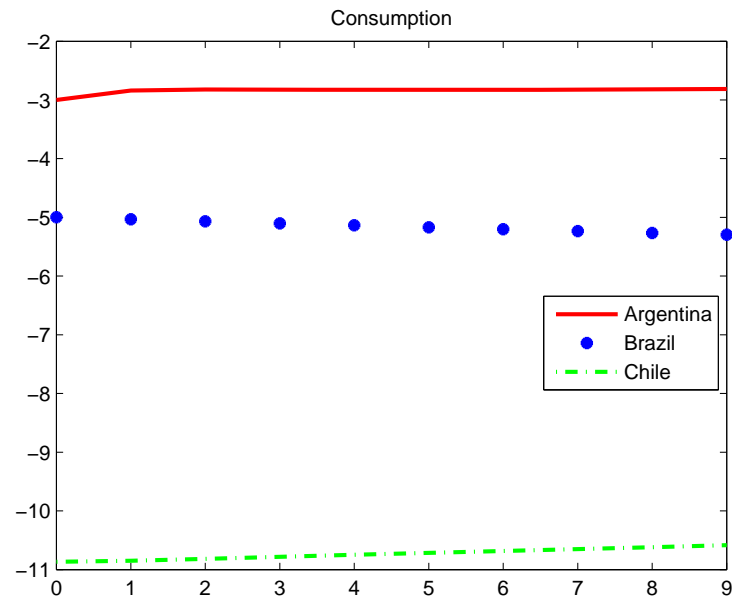


Figure 1: Response of consumption to a 1% world interest rate shock.

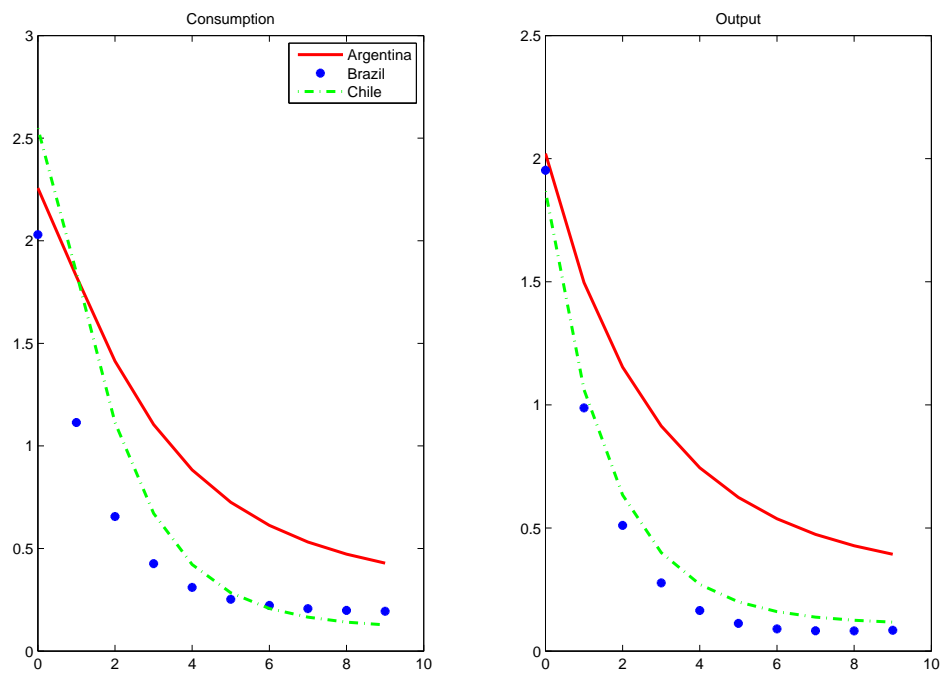


Figure 2: Responses of consumption and output to a 1% productivity shock.

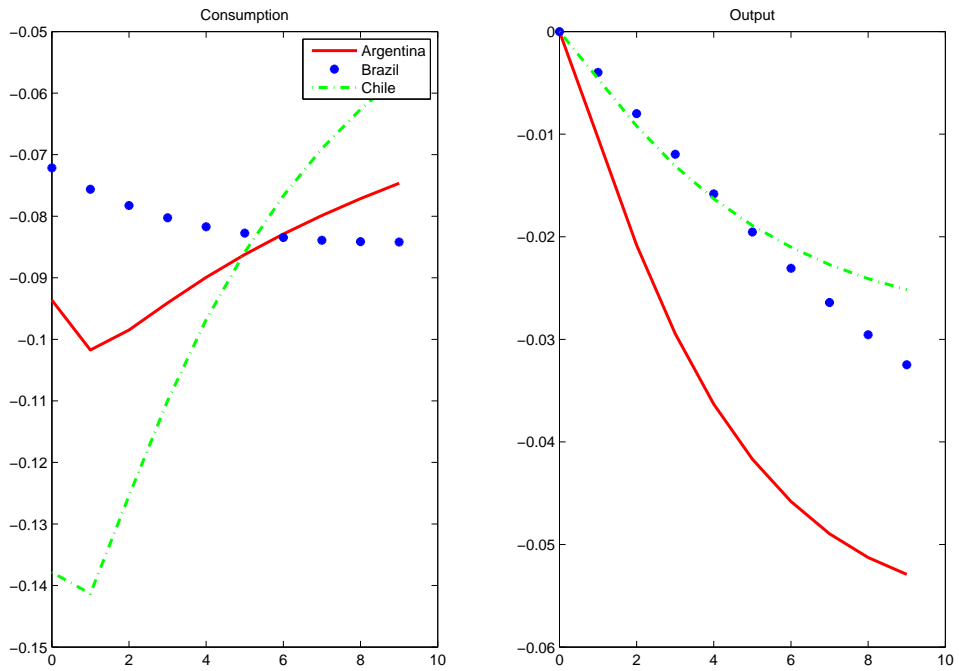


Figure 3: Responses of consumption and output to a 1% government expenditure shock.

Appendix

EIS (Elasticity of Intertemporal Substitution) for GHH Utility:

The Euler equation is given by:

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1}$$

Thus, we have

$$\frac{\partial \ln \left(\frac{U_{ct+1}}{U_{ct}} \right)}{\partial \ln R_t} = -1$$

Since we are interested in *EIS*, we need to calculate the value of $\frac{\partial \ln \left(\frac{C_{t+1}}{C_t} \right)}{\partial \ln R_t}$. Right now, we have $\frac{\partial \ln \left(\frac{U_{ct+1}}{U_{ct}} \right)}{\partial \ln R_t}$. To get what we want, we do a first order Taylor approximation of $\ln U_{ct}$ and $\ln U_{ct+1}$:

$$\begin{aligned} \ln U_{ct+1} &= \ln(U_c) + \left[\frac{\partial \ln U_{ct+1}}{\partial \ln C_{t+1}} \Big|_{C_{t+1}=\bar{C}} \right] [\ln C_{t+1} - \bar{C}] \\ \ln U_{ct} &= \ln(U_c) + \left[\frac{\partial \ln U_{ct}}{\partial \ln C_t} \Big|_{C_t=\bar{C}} \right] [\ln C_t - \bar{C}] \end{aligned}$$

It is easy to see that

$$\begin{aligned} \ln \left(\frac{U_{ct+1}}{U_{ct}} \right) &= \frac{\partial \ln U_c}{\partial \ln C} \ln \left(\frac{C_{t+1}}{C_t} \right) \\ EIS &= \frac{-1}{\frac{\partial \ln U_c}{\partial \ln C} \Big|_{C=\bar{C}}} \end{aligned}$$

Now consider the following expressions:

$$\begin{aligned} C - \frac{h^\omega}{\omega} &= R^{1/\beta_1} - 1 \\ \ln U_c &= -\gamma \ln \left(C - \frac{h^\omega}{\omega} \right) \end{aligned}$$

We know that at steady state

$$\frac{\partial \ln U_c}{\partial \ln C} = \frac{\partial \ln U_c}{\partial C} \cdot \frac{\partial C}{\partial \ln C} = \frac{-\gamma C}{C - \frac{h^\omega}{\omega}}$$

Gathering and plugging terms, we have:

$$EIS = \frac{-1}{\frac{\partial \ln U_c}{\partial \ln C}} = \frac{C - \frac{h^\omega}{\omega}}{\gamma C} < \frac{1}{\gamma}$$