Generalized Method of Moments and Inverse Control

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1. The Econometric Problem

2. GMM with Auxiliary Moment Restrictions

3. Examples
   a. Backward Looking Model
   b. Forward Looking Model

4. Unresolved Issues and Future Direction

1. The Econometric Problem

X_t	=	Vector of Endogenous Variables
ε_t	=	Vector of Exogenous Shocks
ρ	=	Vector of Structural Parameters
θ	=	Vector of Policy-Rule Coefficients

Structural Model

$$S(X_t, \varepsilon_t, \rho, \theta) = 0$$

Policy Rule

$$r_t = P(X_{t-1}, \theta)$$

1. The Econometric Problem

Estimate ρ and θ subject to the restriction that loss is minimized.

$$\text{Loss} = \Lambda = E_0 \sum_{j=1}^{\infty} \delta^j (X_j - X^*_j)' W (X_j - X^*_j)$$

X^* = Target for X_t,
W = Matrix of loss-function weights
δ = Central bank's time rate of discount

Brute Force Approach

Compute optimal θ at each evaluation of likelihood.

GMM Approach

Use auxiliary moment restrictions derived from the first order conditions of the optimal policy problem.

2. GMM with Auxiliary Moment Restrictions

Computing Loss

$$X_t = G\,X_{t-1} + \varphi_t$$

$$\Lambda = \text{trace}\ [\,W \cdot M\,]$$

$$M = (1-\delta)^{-1}\,[\,\Omega + \delta\,G\,\Omega\,G' + \delta^2\,G^2\,\Omega\,(G^2)' + \ldots\,]$$

Auxiliary Moment Restriction

$$\frac{\partial \Lambda}{\partial \theta} = 0$$

$$\frac{\partial \text{Vector}[M]}{\partial\,\theta_k} = \frac{\delta}{1\ \ \delta}\,[I - \delta\,G \otimes G]^{-1}\,[\frac{\partial\,G \otimes G}{\partial\,\theta_k}]\,[I - \delta\,G \otimes G]\ \text{Vector}[\Omega] = 0$$

3a. A Backward-looking Macroeconomics Model

$$y_t = a\,y_{t-1} - b\,(r_t - p_t) + u_t$$
$$p_t = \alpha\,p_{t-1} + \beta\,y_t + v_t$$
$$r_t = \theta_y\,y_{t-1} + \theta_p\,p_{t-1} + w_t$$

Definitions

y = output gap, p = inflation, r = short term interest rate

Advantages of this Model

There are analytic expressions for the reduced form coefficients and for the partial derivatives of loss with respect to policy parameters.

One can confirm optimal policy with the Ricatti equations.

Exact Identification.

3a. Findings Based on Monte Carlo Experiments

Optimality Hypothesis is True

1. GMM returns unbiased estimates of model parameters whether or not the optimality restriction is imposed.

2. When the optimality hypothesis is imposed, GMM returns unbiased estimates of the loss function weights. Convergence is less rapid.

Optimality Hypothesis is False

1. When the optimality hypothesis is not imposed, GMM returns unbiased estimates of parameters.

2. When optimality hypothesis is imposed,
   a. Estimates of structural parameters (rather than policy rule coefficients) are biased.
   b. W_y and W_r converge to zero.

3b. A Forward-looking Macroeconomics Model

$$y_t = \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t p_{t+1}) + u_t$$
$$p_t = \beta y_t + \alpha_1 E_t p_{t+1} + \alpha_2 p_{t-1} + v_t$$
$$r_t = \theta_y y_{t-1} + \theta_p p_{t-1} + \theta_r r_{t-1} + \theta_{y-1} y_{t-2} + w_t$$

Complexities Introduced

No Analytic Solution for Reduced form. The Blanchard and Kahn algorithm is used to compute reduced form.

No Analytic Expression for First Order Conditions. We derived numerical expressions for the partial derivatives.

Reduced form is over-identified.

3b. Findings Based on Monte Carlo Experiments

Optimality Hypothesis is True

1. GMM returns unbiased estimates of model parameters whether or not the optimality restriction is imposed.
2. Imposing the optimality restrictions leads to
   a. More accurate estimates of the structural parameters in the sense sample standard deviations are smaller.
   b. Unbiased estimates of the loss function weights.

Optimality Hypothesis is False

1. When the optimality hypothesis is not imposed, GMM returns unbiased estimates of parameters.
2. When optimality hypothesis is imposed,
   a. Estimates of structural parameters (rather than policy rule coefficients) are biased.
   b. W_y and W_r converge to zero.

3b. Findings Based on Monte Carlo Experiments

Tests of Over-Identifying Restrictions

If Q is the GMM criterion, $T \cdot Q$ is asymptotically χ^2 with degrees of freedom equal to the number of over-identifying restrictions.

When the Optimality Hypothesis is not Imposed

1. The number of rejections is too high in small samples.
2. The number of rejections converges to the expected number as sample size grows large.

When the Optimality Hypothesis is Imposed

1. When hypothesis is true, rejection is too frequent at all sample sizes.
2. When hypothesis is false, rejection rate is 100 percent at all test sizes and all sample sizes above 100.

4. Unresolved Issues and Future Directions

Apply the Method to a DGE Model

1. The mapping from structure to reduced form is more complex.
2. If the Central Bank maximizes social welfare the coefficients of W are known functions of the structural parameters.
3. Given a the structural-error covariance matrix, changes in the structural parameters change the reduced form covariance matrix.

Resolve a Scaling Issue

1. GMM involves a first pass with an arbitrary matrix of moment weights and a second pass with an optimal weight matrix.
2. For our experiments, second-pass weights associated with the partial derivative moments are much larger than second-pass weights associated with the "least squares normal equation" restrictions.

Take the Method to the Data

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