MATH 681 Introductory Topology : homework assignment eleven

1. Suppose that $U \subset \mathbb{R}^2$ can be written as the union of open sets $U_1, \ldots, U_k$ such that $(U_1 \cup \ldots \cup U_i) \cap U_{i+1}$ is connected for $1 \leq i < k$. Show that a one-form ω which is exact on each U_i must be exact on all of U.

2. For each point $P = (x_0, y_0)$ define a one-form on $\mathbb{R}^2 - P$ by

$$\omega_P = \frac{-(y - y_0)dx + (x - x_0)dy}{(x - x_0)^2 + (y - y_0)^2}.$$

   a) Let P and Q be two points in $\mathbb{R}^2$, and let L be the line segment joining P to Q. Show that $\omega := \omega_P - \omega_Q$ is exact on $\mathbb{R}^2 - L$.

   b) Find a function f on $\mathbb{R}^2 - L$ such that $df = \omega$.

3. Let $U = \mathbb{R}^2 - \{(x, y) | x^2 + y^2 = 1 \text{ or } 0\}$, i.e., U is $\mathbb{R}^2$ with the unit circle and the origin removed. Find an explicit basis for $\mathrm{H}^1(U)$.