MATH 681 Introductory Topology : midterm one

Allow yourself **two hours** to sit this exam. It should be attempted under usual exam conditions, i.e., do not consult the textbook or your notes, and do not discuss the problems with anybody. Please hand in your solutions by **Wednesday 17th February**.

1. Let (X, d) be a metric space with induced topology $\mathcal{T}$. Let $\mathcal{T}'$ be the finite complement topology on X. Prove that $\mathcal{T}$ is finer than $\mathcal{T}'$.

2. Let U be an open subset of a topological space X. Is U equal to the interior of its closure $\text{int}(\bar{U})$? Prove or find a counterexample.

3. Let $f : X \to Y$ be a map between topological spaces that is continuous, closed, and onto. Prove that f is a quotient map.

4. Let $\mathbb{R}^2$ and $\mathbb{R}$ have their standard topologies. Define a map

$$\begin{aligned} p : \mathbb{R}^2 &\to \mathbb{R} \\ (x, y) &\mapsto \sqrt{x^2 + y^2}. \end{aligned}$$

   Prove that p is a closed map.

5. Let $\mathbb{R}^3$ have the standard topology, and let

$$X = \{(x, y, z) \in \mathbb{R}^3 | \text{ at least two of } x, y, \text{ and } z \text{ are rational } \}$$

   with the subspace topology. Is X connected? Explain your answer.

6. Let X be a topological space and let $Y = X \cup \{p\}$, where p denotes some element *not* in X. Define a collection $\mathcal{T}$ of subsets of Y by $U \in \mathcal{T}$ if

   - either $p \notin U$ and U is open in X,
   - or $p \in U$ and $Y - U$ is a closed compact subspace of X.

   a) Prove that $\mathcal{T}$ is a topology on Y.

   b) Prove that $(Y, \mathcal{T})$ is compact.