

# Complex Dependencies in the Alliance Network

Skyler J. Cranmer      Bruce A. Desmarais      Elizabeth J. Menninga

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## Abstract

The multifaceted and strategic interactions inherent in the formation of international military pacts render the alliance decisions of states highly interdependent. Our aim here is to model the network of alliances in such a way as to capture the effects of covariates *and* account for the complex dependencies inherent in the network. Regression analysis, due to its foundational assumption of conditional independence, cannot be used to analyze alliance decisions specifically and interdependent decisions generally. We demonstrate how alliance decisions are interdependent and define the problems associated with the regression analysis of nonindependent dyads. We then show that alliances can naturally be conceived of as constituting a network, where alliance formation is an inherently interdependent process. We proceed by introducing the exponential random graph model for analyzing interdependence in the alliance network *and* estimating the effect of covariates on alliances.

The inherent complexity of alliance relationships presents analysts of international politics with an equally complex task in thoroughly explaining the realizations of these relationships. We argue that the study of interdependent processes like alliance formation are complicated both theoretically and empirically by the fact that states account for the behaviors of other states when determining their own behaviors. In other words, if states consider the alliance relationships of other states when deciding whether and how to adjust their alliance portfolios, then both our theories and our empirical tests must account for that consideration. While theoretical work has examined the interdependent nature of alliances for some time, the empirical study of alliances has lagged behind because of a lack of empirical tools for testing and accounting for the interdependent nature of alliance links. The existing empirical tool that, for lack of a better alternative, has been widely used is regression with a relational (dyadic) variable as the outcome. Regression analysis – counter to many of the substantive theories in the field – treats individual relationships between states as conditionally independent. This causes two closely related problems: (1) because the theoretical story represented by the empirical model is inaccurate, misspecification bias is induced and (2) confidence intervals are too small because of the smaller number of *independent* observations under study than the number assumed in the regression modeling process.

Our aim is to address a substantial empirical challenge facing the study of alliances: explaining one or more alliances with reference to one or more of the *other* alliances extant in the international system, all while accounting for important state and dyad level covariates. We show that if the international system is treated as a network of nations, both theoretically and empirically, then the major problems with regression on the dyad can be resolved without abandoning the ability to estimate the effect of covariates on the individual alliances embedded in the network. We demonstrate our approach in the context of alliances, but the problems and solutions we discuss are by no means restricted to the study of alliances, but rather can easily be applied to international conflict (see e.g. Cranmer and Desmarais

(2011)) or any other relational outcome.

We argue that the study of alliance relations requires an empirical framework that can solve the two problems discussed above while bringing our statistical methodology closer to our substantive theories. We propose that network analysis, as a general approach, can solve the problems in a single, unified, empirical framework. Network analysis allows for the modeling of complex interdependencies between states while controlling for variables at the state and dyadic levels. Network analysis is a broad topic, and it is not our intention to introduce every aspect of it. Instead, we will introduce a single network analysis method which, due to its versatility, is particularly useful for the study of alliances: the class of statistical models known as exponential random graph models (ERGM). We discuss and demonstrate how the ERGM approach solves the two empirical problems under consideration. The ERGM approach to modeling alliances can do more than correct potential biases in estimated covariate effects; it also allows us to model theoretically interesting and precisely formulated endogenous interdependence structures critical to the formation of the alliance network. In other words, the ERGM technique can be used to (a) correct biases in coefficients that result from unmodeled dependencies in a way that (b) produces appropriate confidence intervals given the volume of data and (c) advances our theoretical and empirical knowledge by explicitly modeling those dependencies.

We analyze alliances as a network phenomena and find some substantial differences between the established effects of a number of covariates and their effects once previously unmodeled dependencies have been taken into account. For example, contrasting standard logistic regression results to ERGM results, we find significant sign changes in the coefficients for joint democracy, major power status, and a history of conflict. We also discuss theoretical explanations for network dependencies and find, empirically, that state-level “popularity” and a tendency for states to cluster are supported by the data.

We begin by considering established theoretical explanations for alliance formation based on covariates measured at the state and dyadic levels. We point out that it is substantively and empirically problematic to think of these processes as independent because a state's choice of allies often depends on the existence of other alliances. We also illustrate how regression on dyadic relationships can artificially shrink standard errors and produce nonsensical codings for multinational phenomena. We then discuss how network analysis generally, and exponential random graph models specifically, create an opportunity for us to resolve these problems theoretically and allow us to directly solve them empirically. Lastly, we perform a ERGM analysis of the alliance network and show that, through our network approach, we produce a superior model to classical models both in terms of model fit, predictive accuracy, and substantive theory.

## **Theory and Empirics in the Study of Alliance Formation**

Most hypotheses in the alliance literature do not directly address the complex dependencies in the network of alliances. Rather, they involve a single independent variable and its hypothesized causal effect on alliance outcomes. Examples of such hypotheses abound and concern the important theoretical effects of dyadic attributes such as joint democracy (Leeds 1999; Lai and Reiter 2000; Gibler and Wolford 2006), conflict histories (Lai and Reiter 2000), geographic contiguity/proximity (Gibler and Vasquez 1998; Gibler and Wolford 2006), and trade (Rosecrance 1986; Keohane and Nye 1989; Oneal and Russett 1999), or state level attributes such as major power status and military insecurity (Waltz 1979; Walt 1987) on alliance outcomes. Some work has also modeled alliances as an outcome generated by the relationship of states in a dyad's relationships with other states: Walt (1987) postulates that alliances are more likely between states that share a common enemy and both Lai and Reiter (2000) and Gibler and Wolford (2006) find empirical support for this hypothesis.

If most hypotheses do not directly concern interdependencies in the alliance network and the analyst's interest is limited to evaluating her hypothesis given the data, why then must our empirical analyses – at minimum – control for dependencies in the alliance network? The first reason is theoretical. While, assuming temporarily that dependencies do exist in the network of alliances (this assumption is justified at length below), hypotheses concerning covariate effects will be appropriately specified if (and only if) the hypothesized effect is orthogonal to the dependencies, the general theory from which the specific hypothesis is derived will be misspecified if it omits important dependencies from its explanation of alliance formation.

The second reason to be concerned about dependencies, even if the hypothesis of interest is not, is that omitting dependencies from the empirical analysis will cause bias in the coefficients and standard errors of interest that can range from the relatively trivial to the catastrophic. Much of the recent work on alliances has taken an approach to empirical analysis that uses regression models of dyadic alliance outcomes to estimate the effects of covariates (see e.g. Leeds (1999), Lai and Reiter (2000), Gibler and Wolford (2006) among many others). Such a design presents two closely related empirical challenges: (1) the inability of classical regression models (least squares, logit, etc. . . ) to model complex interdependencies because of the unavoidable assumption of independence of observations conditional on the covariates and (2) the problematic “multiplication” of data in dyadic analyses. It is important to clarify what we are referring to when we mention “dyadic analysis” or “dyadic design”. By this we mean pooling directed or undirected dyadic outcomes between pairs of states into a sample and treating them as if they are independent of each other given the exogenous covariates in the analysis (a required assumption of regression models). We do not object to, and indeed advocate, the use of dyadic data, but in our approach dyads are integrated into a network instead of analyzed as independent observations.

## Problem 1: Nonindependence of Alliance Relationships

The use of regression methods on alliance relations violates the critical assumption of observational independence that these models require; if a dyad of states conditions its alliance behavior on the behaviors of other dyads of states, then the basic assumption that allows for the creation of a joint likelihood is violated. While regression models are often robust to violations of their assumptions, if only by chance, such models are particularly non-robust to the violation of independence brought on by relational data. In fact, when independence is violated, regression models are biased in a way that can lead the researcher to believe the covariates have much stronger effects on the outcome of interest than they actually do.

The core of the problem is that regression models assume the data are independent and identically distributed (i.i.d) conditional on the covariates. The “independent” part of the i.i.d assumption implies that the observed relationships in the data constitute an exchangeable random sample (Gelman et al. 2004). In other words, the labels on the observations, unless used to define independent variables, contain no relevant information. As a result, we should be able to shuffle the rows of a data matrix without affecting the inferences drawn from the data. Observations in the data set can exchange places without affecting inference because each observation is assumed to be independent of every other observation. More specifically, the joint likelihood of  $\theta$  conditional on the outcome  $\mathbf{y}$  and data  $\mathbf{X}$  can only be established by assuming exchangeability and thus independence:

$$L(\theta|\mathbf{y}, \mathbf{X}) = \prod_{i=1}^n f(y_i|\mathbf{x}_i, \theta), \quad (1)$$

if the observations  $i$  are not independent, the product over every  $f(y_i|\mathbf{x}_i, \theta)$  cannot – by the axioms of probability – be taken to produce a valid likelihood. The direct implication of independence is that, for example, the fact that state  $i$  is involved in alliance dyads  $ij$  and  $im$  (assuming dyads are the unit of observation in equation 1) is restricted to say nothing

about the likely values of  $ij$  and  $im$  outside of that which is determined by  $\mathbf{x}_{ij}$  and  $\mathbf{x}_{im}$ . If the exchangeability assumption is violated, then the likelihood function is *necessarily* misspecified. In non-relational settings, such as survey research, where respondents are unlikely to know/affect one another, independence is usually a reasonable assumption. In the context of alliance data, however, assuming that alliance ties are not affected by other alliances (i.e. NATO is *completely unaffected* by the Warsaw Pact) is not generally reasonable. More generally, exchangeability and i.i.d are violated when standard regression models are run on relational data.

To justify the i.i.d assumption, one must assume that states (or dyads) can have similar alliance generating processes based on similar values of their covariates, but *states do not influence one another* (e.g. the dyadic alliance between the US and UK during the Cold War was entirely unrelated to other contemporaneous alliances). To expand this example, the treatment of each dyad within NATO (8,582 of them since NATO's founding) as conditionally independent alliances requires that each dyadic outcome be independent of all others; implying that the alliance behavior of NATO members *must* be independent of the behavior of Warsaw pact countries *and* other NATO countries. Obviously, such an assumption would never be made substantively or voluntarily by an analyst as it flies in the face of centuries worth of scholarship on international politics.

Depending on the degree of inappropriateness of the exchangeability assumption, idiosyncrasies in the independent variables that are associated with the omitted factors, such as the basic structure of the network of alliance connectivity between states, will be exploited to fit the interdependencies that have not been accounted for in the model. For instance, if a state is likely to be allied to those with whom its allies are allied (“the friend of my friend is my friend”)<sup>1</sup>(Siverson and Emmons 1991) *and* alliance groups happen to share similar

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<sup>1</sup>A phenomenon called ‘triangle closure’ that has been the subject of some debate in the alliance literature. Maoz et al. (2007) have found intransitivity in alliance structures when looking at the enemies and allies of one’s enemies, but Cranmer, Desmarais and Kirkland (2011) and Maoz (2010, ch. 3) have found evidence of

covariate values (i.e. all democracies), the influence of triadic group formation on the shape of the alliance network will be completely assigned to covariate effects, even though these covariates may exercise no independent influence on the alliance network. Generally, in the face of misspecification, the properties of MLE (asymptotic efficiency and unbiasedness) no longer hold (Greene 2003).

## Problem 2: Data “Multiplication” in Dyadic Analyses

The second problem with regression analysis on dyadic alliance data is the problem of what we call data “multiplication” in dyadic designs.<sup>2</sup> The problem of data multiplication is that the number of “observations” in a dyadic dataset is often large (much larger than the number of states in the system – the instrumental actors that generate the data), thereby shrinking standard errors and making it progressively (and non-linearly) harder to conclude that the effect of a given covariate is *not* statistically significant. This is a problem because, if the number of observations in the dataset is artificially large, then it becomes quite likely that we will erroneously accept that a significant effect exists for a given variable when, in fact, it does not.

When we say “data multiplication” we mean that an *undirected* dyadic dataset has  $N = \binom{n}{2}$  observations where  $n$  is the number of states in the system. This means that if there are two states in the system and a third is added, two new observations (dyads) are created. If there are 240 states in the system, and one is added, 240 new observations are created. So, the number of observations included in the statistical analysis is nonlinearly increasing in the number of states in the system; see Figure 1 for a graphical representation. More specifically, if there were 250 states in the system-year, there would be 31,125 dyads in the

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transitivity in alliance networks.

<sup>2</sup>As discussed above, “dyadic design” here refers to regression on dyads, not the basic dyadic form of the data.

system-year; were those states observed over a 100 year period, the dataset would contain more than three million observations. This is exacerbated in analyses using directed dyads which, depending on how they are recorded, can have up to  $2\binom{n}{2}$  observations.

[Figure 1 about here.]

In dyadic analysis, the longitudinal observation of a relatively small number of countries translates into a massive  $N$ , which poses a serious problem for traditional statistical techniques. The  $p$ -values and significance levels determined through classical statistical methods are calculated from the standard errors for the parameters. Standard errors are strongly influenced by the number of observations; as  $N$  increases the standard error necessarily decreases. As standard errors decrease, parameters for covariates achieve significance at traditional levels. With the large  $N$  often associated with dyadic studies, minor effects will be able to achieve unrealistically low  $p$ -values (Erikson, Pinto and Rader 2009). This problem is magnified in directed dyadic datasets and can call into question results derived from dyadic level analysis. Is the parameter significant because it truly drives the outcome of interest? Or is it significant because almost everything is significant at 0.05 with 1 million observations?

A further concern with dyadic analysis is the handling of multinational alliances. Standard dyadic setups can often inflate the number of actual alliances in the data. Suppose we are considering a dyadic dataset where the outcome variable is defense pacts (an indicator variable coded 1 if the two countries in a given dyad had a defense pact in a given year). In other words, a link in the network occurs when two countries are allied. But if three countries were involved in a single alliance (i.e. Germany, Austria-Hungary and Italy in the Triple Alliance) three links (three “conditionally independent” events) in the network are created. In the context of a logistic regression model, that means that three binomial successes are generated by a single alliance. This problem is compounded the more states become involved in the multinational alliance. For example, using a standard dyad-year dataset (all dyads in

a year), NATO had 12 member states at its founding in 1949 – 66 conditionally independent alliances – and currently has 28 members, thus producing 378 dyadic alliances per year. Summing the number of dyadic alliances produced annually by NATO (accounting for its six enlargements), the NATO alliance as treated by regression models has 8,582 individual alliances that are entirely independent of each other beyond the values of their covariates.<sup>3</sup>

Our concern here is that the dyadic treatment of such events can not only lead one to erroneous conclusions about the frequency of alliances but also bias statistical analysis by inflating the number of events we are trying to model. We suggest that this is problematic and a method of analyzing international politics should be able to treat multinational alliances more appropriately.

We are not the first to have considered the data multiplication problem in the dyadic analysis of international relations. King and Zeng (2001) advocate a choice-based sampling approach to logistic regression analysis of rare-events in international relations. Essentially, the researcher can save resources and efficiently exploit the variance in the population by only analyzing a small, but more balanced sample of 0/1 outcomes, effectively avoiding regression on the massive low-variance populations associated with many dyadic outcomes of interest in international relations. Poast (2009) extends this idea to  $k$ -adic events – those constituting multilateral international events involving more than two states. It is advocated that conventional regression analysis be performed on bootstrap samples from the larger population of  $k$ -ads, where the dependent variable is a 0/1 indicator of whether a multilateral event occurred at the  $k$ -ad level. Note that, because weighted or conventional regression analysis is performed post-sampling, underlying both of these methods is the assumption that the units in the sample, whether they be dyads or  $k$ -ads, are independent of each other. For this reason, we view these methods, though useful in certain situations,

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<sup>3</sup>Data multiplication is a problem in other areas of international relations as well. World Wars I and II together are represented as 567 conditionally independent dyadic wars and comprise 53.44% of the wars in the 20th century. The WTO has involved 153 member states and generated 160,816 dyadic trade agreements.

as inappropriate for the study of alliances.

In a related study, Erikson, Pinto and Rader (2009) advocate the use of randomization or permutation tests in dyadic analyses. A permutation test is performed by randomly permuting the values of the independent variable of interest about the observations in the sample and repeating the analysis to construct the distribution of test statistics under the null-hypothesis of zero relationship. Permutation tests relax the assumption that the parametric model is correct, but assume that the observations are exchangeable, which implies independence (Good 2002). Though helpful for robusting against the inappropriate choice of covariates or link functions, the randomization tests advocated by Erikson, Pinto and Rader (2009) are not appropriate for the study of dependent data.

## Networks of Nations

Here we argue that alliance relations can be naturally thought of as a network process and introduce some basic network terminology.<sup>4</sup>

To illustrate the complex structures alliance networks can take on, and to demonstrate the effects those structures can have on international conflict and cooperation, consider World War I. Historians and political scientists alike have often claimed that the alliance *structure* in the decades preceding the war was a primary cause of it (Williamson 1988). Prior to the outbreak of war, the alliance system among the great powers of Europe formed two (non-connected) subnetworks. The formation of the network structure so many say contributed to the war began in 1879 with the Dual Alliance between Germany and Austria-Hungary. This dyadic alliance came to include Italy with the Triple Alliance of 1882. The alliance relations (“links” in network jargon) between these three states (nodes) now formed a triangle: an

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<sup>4</sup>While this discussion focuses on alliances as the outcome of interest, it is easily generalized to other relational outcomes (e.g. conflict, trade, etc. . .).

alliance network structure likely to provide member states increased utility from a better defense (Cranmer, Desmarais and Kirkland 2011).

In response to the emergence of the Triple Alliance – as diplomats of the day believed that balanced power would promote stability (Williamson 1988; Levy 1981) – rival states to the Triple Alliance began forming their own alliance network. France, which had been isolated in terms of great power alliances, formed a defensive alliance with Russia in the Franco-Russian Alliance of 1893. Nearly a decade later, Britain began rapidly forming alliances: the Anglo-Japanese Alliance of 1902, the Entente Cordiale with France in 1904, and the Anglo-Russian Entente of 1907. The triadic alliance of Britain, France and Russia came to be called the Triple Entente. These alliances (and Britain’s alliance with Belgium; the Treaty of London (1839)) were the primary means by which the conflict between Austria-Hungary and Serbia diffused into what was, at the time, the worst bloodshed in human history.

This example demonstrates the nonindependence of dyadic and triadic alliances in two important ways. First, alliance dyads clearly cannot be conditionally independent of one another because, for example, the Triple Entente was formed to balance the Triple Alliance. In other words, realizations of the outcome variable were generated because of other realizations of the outcome variable; by definition, covariates could not capture the dependences among the dyadic links because that alliance generating process is endogenous to the alliance network. It follows that any model assuming independence of observations is *necessarily* misspecified and would thus produce biased results.

Second, the dependencies in this example have structure, structure that can be modeled. The dependencies in the example run deeper than the fact that Entente powers conditioned their behaviors on the behavior of the Alliance powers. For both the Alliance and Entente, triadic structure seems to have played a role in the generation of alliances. This is not an unprecedented idea: Cranmer, Desmarais and Kirkland (2011) argue that triadic alliance

structures are appealing for member states because they may increase the credibility of alliance commitments. This is to say, it seems that alliance structure within (as well as between) alliance groups was nonindependent.

It is also necessary, for the purposes of our discussion, to distinguish between alliances that were made separately from other alliances, and alliances that were made independently of other alliances. The decisions of the UK and France to enter into an alliance were clearly made separately from the decision of Germany and Austria-Hungary to enter into an alliance; the decisions were made at different times by different governments. Yet the fact that these decisions were separate, does not imply that they were independent. The two decisions would be independent if either alliance would have existed and persisted regardless of whether the other alliance existed or persisted. Obviously, this is a counterfactual we are unable to check conclusively, but a wealth of historical information implies that these separate decisions were far from independent; the Germans and Austria-Hungarians were concerned about French and Russian power while Britain and France's alliance seems to have been motivated by concerns about the Triple Alliance. It is ultimately an empirical question whether a given alliance was formed independently of others or not, but in an environment where every state knows the alliance portfolio of every other state<sup>5</sup> and states behave at least semi-strategically, it seems likely that states consider existing alliances when deciding their own alliance behavior. While both dependence and independence are assumptions about the data generating process, dependence is (usually) a much weaker assumption than independence. For example, the method we advocate for inference on network data below assumes nonindependence, but reduces to logistic regression if no dependencies can be identified. As such, the presence or absence of interdependence is an empirical question that one may answer empirically by assuming nonindependence and discovering independence if it exists in the data.

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<sup>5</sup>Secret alliances have existed but are thought to be rare.

Network analysis is a powerful and flexible way to study alliances. In the context of international relations, the  $n$  states in the system are called nodes and are typically represented graphically with a point. The relationship between any two nodes is called a link and is typically represented as a line between the two points. Links capture one particular aspect of the relationship between nodes. For example, as we are considering alliances, a link would exist between any two states that are allied by whatever definition of an alliance we choose to apply (e.g. any alliance, defense pacts, etc...). In a trade network, a link would exist between any two nodes that trade together and would typically be weighted by how much they trade together. Links can be directed (i.e. one node initiates a conflict with another node) or undirected (i.e. two nodes are in a conflict with one another). Put simply, recording the relationships (links) between states (nodes) is the first step toward a network framework for studying international politics and is already institutionalized in the field, but is typically called “dyadic” data rather than “relational data,” “network data,” or “link lists”; indeed, the dyad is the basic relational unit involved in most network analysis.

Relational data, as we have with dyadic alliance data, can naturally lead to a network perspective on the relational outcome. Consider the simple case of a system with only three states in it –  $A$ ,  $B$ , and  $C$  – and the alliance ties between them. An intuitive way to begin thinking about this system is at the dyadic level: does there exist an alliance between  $A$  and  $B$ ? When we consider, however, theoretical arguments about the balance of power or capability aggregation, it makes progressively less sense to explain the relationship (link) between  $A$  and  $B$  without reference to  $A$  and/or  $B$ ’s relationship with  $C$  – a simplification necessary for dyadic analysis with conventional regression models. If, for example, all three states are allied to one another, the triangular structure produced (where three nodes all have links with one another) is called a transitive triple in network jargon and captures the adage that the “friend of my friend is my friend.” It should be noted that this transitive structure cannot be captured by simple dyadic analysis.

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

As we consider an increasing number of states (nodes), the utility of the network approach becomes more and more apparent. Links are not independently distributed according to a conditional-upon-covariates Bernoulli distribution;<sup>6</sup> rather, states organize themselves into clusters and by structures which can affect a variety of international phenomena. As can be seen in Figures 2-4, the alliance network looks nothing like a network with randomly distributed links; dense clusters of alliances emerge bridged by certain nodes. The complexity of the alliance network is apparent and increasing over time.

## Statistical Inference on Networks

### A Multitude of Approaches to Inference on Networks

There are many ways to approach the problem of statistical inference on network data. Perhaps the most intuitive approach is to include descriptive network statistics in traditional regression models. A wide variety of descriptive network statistics are available and measure useful concepts at the network (system), link (dyad), and node (state) levels. One can compute state level statistics (which vary by state) such as the state's *degree* (i.e. connectedness) in a given network or its *betweenness* role (i.e. the degree to which it serves as a bridge between disconnected clusters) to the rest of the network, and include the vectors of those

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<sup>6</sup>If that were the case, a logistic regression model would be well suited to the data and could be applied without violating model assumptions. Also, as mentioned above, if links in the network were indeed i.i.d, the network analysis method we recommend would reduce to logistic regression and would continue to be well specified.

statistics as predictors of a state level outcome in a standard linear or generalized linear model.<sup>7</sup> There also exists a suite of descriptive statistics defined on the dyad; or one may be interested in a dyadic measure of state-level statistics (i.e. the centrality ratio of two states). For example, Maoz et al. (2006) compute a statistic called structural equivalence (which measures the similarity of tie structures) for each dyad in the international system and use the structural equivalence scores to predict international conflict with a logistic regression. One can even compute network-level statistics or aggregate state/dyadic statistics for use in system-level regressions (Maoz 2009). The challenge here is that some network structures, such as triadic effects, are difficult/impossible include in a regression model.

Latent space models are a powerful way to condition out network dependences and produce “clean” estimates of covariate effects in network data (Hoff, Raftery and Handcock 2002; Hoff and Ward 2004). The most widely used latent space model was developed by Hoff, Raftery and Handcock (2002) and accounts for network dependencies by projecting each node in the network into a  $k$ -dimensional latent space. The nodes are projected such that their coordinates in the space will be closer together the more likely they are to share a tie; thus capturing the interdependencies in the network without modeling them explicitly. More and more complicated interdependencies can be captured by increasing the dimensionality of the latent space. Since its development, the latent space model has come to be well used in international relations (Ward, Siverson and Cao 2007; Cao, Prakash and Ward 2007; Ward and Hoff 2007; Ahlquist and Ward 2009).

Lastly, a literature on dynamic networks developed principally by Tom Snijders (Snijders 2001, 2005, 2006; Snijders, van de Bunt and Steglich 2010) presents a technique best known by the name of its software implementation: SIENA. The SIENA approach, which can include a wide variety of parameters to capture network structure, examines the changes in

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<sup>7</sup>A detailed treatment of descriptive network statistics is beyond the scope of this discussion. For extensive treatments of the topic, see Wasserman and Faust (1994) and Maoz (2010).

networks and does so from an actor-oriented perspective.<sup>8</sup> Specifically, a link in a change network occurs only when a tie is formed or lost between periods, not when the tie remains present or absent over time. SIENA is actor-oriented in so far as it defines a utility function at the node level for changes that nodes can make in their link profile. SIENA has been fruitfully applied in applications ranging from policy networks to alliance networks (see for example Berardo and Scholz (2005); Scholz, Berardo and Kile (2008); Pickup, Manger and Snijders (2008); Cranmer, Desmarais and Kirkland (2011)).

While all these approaches can be fruitful for different applications, we employ a fourth alternative called the exponential random graph model. Our choice should not be taken to deny the utility of alternative approaches, but rather reflects a desire to move away from independence assumptions completely and to directly model network effects which will be of substantive interest to international relations scholars in most empirical applications.

## **The Exponential Random Graph Model**

Exponential random graph models (ERGMs) are a popular, powerful, and highly flexible approach to inference on networks. First proposed by Wasserman and Pattison (1996) and then expanded by Park and Newman (2004) and Snijders et al. (2006) among others, the ERGM makes full use of the general network structure described above. The key difference in the way one must think of inference in an ERGM context versus a regression context is that the ERGM treats the entire network as a single observation. In other words, instead of considering each dyadic tie as a conditionally independent observation, the ERGM considers the whole network of ties to be one observation from a complex multivariate distribution – thus making it completely free of independence assumptions – and then asks what the

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<sup>8</sup>In its first versions, the SIENA software included an exponential random graph model (ERGM) like those discussed below, but as it developed, a separate ERGM package was created and SIENA focused exclusively on the actor-oriented dynamic model.

probability of observing that network is compared to a random network with the same number of nodes.

The objective in modeling with the ERGM is to select features of the observed network that differentiate it from a random *uniform* draw from the set of every possible ( $2^{\binom{N}{2}}$ ) network that could have been observed with the same number of nodes (e.g. the presence of ties aligns with the higher values of a covariate, there are a few definable dense clusters, there is a high degree of reciprocity in a directed network, etc.). The features that differentiate the observed network from a random network are included as a set of statistics computed on the network. We can denote these statistics  $\Gamma(y)$ . If we consider  $y$  to be the observed network of interest and  $y^*$  to be a random network on  $n$  nodes, we can write the probability of observing the network we did compared to all other possible networks as

$$P_{\theta}(y^* = y) = \frac{\exp\{\theta^T \Gamma(y)\}}{\sum_{\text{all graphs } y^*} \exp\{\theta^T \Gamma(y^*)\}}, \quad (2)$$

where  $\theta$  is the vector of parameters to be estimated, and  $\Gamma(y)$  is a vector of network statistics.<sup>9</sup> Two general types of statistics can be included in the model: statistics that take covariates at the state or dyad level as their arguments (thus accounting for factors exogenous to the network) or statistics that take dyadic values of the outcome network as their inputs (thus accounting for factors endogenous to the network).

The endogenous statistics are designed by the analyst, in much the same way as are exogenous covariates, to operationalize theoretically interesting relational processes (e.g. reciprocity, clustering, density etc.). Similar to a regression coefficient, a parameter in the ERGM is estimated to correspond to each statistic. A positive (negative) parameter value means we are likely to observe networks with larger (smaller) values of that statistic (e.g.

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<sup>9</sup>For a technical review and extension of ERGMs geared towards political network applications, see Cranmer and Desmarais (2011). For the only published applications of ERGMs to political science data that we are aware of, see Thurner and Binder (2009), Thurner and Pappi (2009), and Cranmer and Desmarais (2011).

number of reciprocated links, number of triangles, number of links etc.) than would be expected if the network were drawn at random from a uniform distribution of networks. Cranmer and Desmarais (2011) present a particularly intuitive example of such a substructure hypothesis in the context of international relations. Specifically, in the international conflict network, they hypothesize that the network will contain few triangles in which A is at war with B, B with C, and A with C. Simply speaking, it would be redundant and inefficient for two countries at war with the same third country to fight each other. In line with their hypothesis, Cranmer and Desmarais (2011) find a strong negative effect of the number of triangles in the conflict network.

Exogenous covariates can be included by specifying  $\Gamma(y)$  to capture the same relationships in the data that are tested using covariates in a regression model.<sup>10</sup> In the regression framework, a hypothesis that  $X$  will have a positive effect on  $Y$  is stated as an expectation that the probability that  $y_{ij} = 1$  will increase as  $x_{ij}$  increases. In the ERGM context, this is equivalent to the expectation that the links in  $Y$  will align with higher values in  $X$ . The value of  $\Gamma_X(Y)$  will be higher if the ones in  $Y$  correspond to the higher values in  $X$ . If  $X$  is expected to have a positive effect on  $Y$ , this, again, can be stated in ERGM form as an expectation that the likelihood of observing a particular instance of  $Y$  increases with the degree to which the ones in  $Y$  align with the higher values in  $X$ . By including statistics to capture covariates, the analyst can test the usual covariate-effect hypotheses alongside theories of complex interdependence in international networks. Indeed, if only exogenous covariates matter, and none of the more complex sub-structure statistics are important, the parameters assigned to the covariate effects will be exactly equal to coefficients in a logistic regression.

Though helpful to recognize the special-case relationship between ERGM and logistic

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<sup>10</sup>Let  $X$  be an  $\binom{n}{2}$  element matrix that represents a dyadic covariate where the  $ij^{th}$  element of  $X$  corresponds to the  $ij^{th}$  dyad in  $Y$ . We can then account for the additive effect of  $X$  on the individual components of  $Y$  with the following statistic:  $\Gamma_X(y) = \left[ \sum_{i=2}^n \sum_{j=1}^{i-1} y_{ij} x_{ij} \right]$ .

regression, the similarities end at the effect of purely exogenous covariates. Other systematic determinants of network structure, such as the tendency toward triadic closure, cannot be accommodated in the regression framework because they are *endogenous* to the very outcome being modeled – the network. When it comes to endogenous effects, which imply relationships among the ties in the networks, the ERGM can be used to evaluate whether the emergent network structures (e.g. a large number of closed triangles, the absence of reciprocated ties etc.) are unique features of the network under study or simply appear due to random chance or the distribution of some exogenous covariate. A parameter, which can and often is reported in a table in the same way as an exogenous covariate effect, is estimated for each statistic included in the model. If that parameter is significantly different from zero, it can be interpreted that the corresponding statistic significantly effects the probability of observing a particular instance of that network, controlling for the other statistics in the analysis.

This simultaneous estimation of the effects of network structures on the probability of observing the entire network dually differentiates ERGM from the regression approach to dyadic analysis and from conventional network analysis approaches. In the dyadic design, dyads are treated as conditionally independent. In an ERGM, dyadic outcomes are used to construct a network, which is then structurally compared to the population of possible networks that could have been observed – shrinking the number of independent observations from any number of dyads to exactly one network. Within the network analysis literature, many boutique models have been designed to capture singular emergent structures within networks. This is done by comparing a theoretical conception of a random network, which does not contain this structure, to a network that does contain it, to develop a statistical characterization of the difference. Excellent examples of this can be found in the literature on clustering coefficients (Watts and Strogatz 1998; Watts 1999) and that on preferential attachment (Barabasi and Albert 1999; Newman 2005). ERGM advances beyond this approach

in that (1) the random network with respect to a particular statistic automatically corresponds to the distribution of networks when the parameter corresponding to that statistic is zero, and, unlike any of the structure-specific models, (2) ERGM can be used to estimate the importance of structural effects *controlling for the importance of other structures and exogenous covariates*. The relationship between simple and multiple regression constitutes an excellent analogy for that between conventional network analysis methods and ERGM.

The ERGM is a flexible method in so far as it easily accommodates effects from all three levels of analysis, and its ability to model complex network structures is limited only by the analyst's ability to specify them. The ERGM is also a powerful method in that it gives the analyst a great deal of leverage over complex networks while requiring only two relatively mild assumptions. First, the ERGM requires that only  $\Gamma(y)$  differentiates the likelihood of observing different networks. This assumption is akin to the standard assumption of correct model specification. While we concede that correct specification is a difficult criteria to meet, the ability of the ERGM to integrate state-level, dyadic, indirect, and structural effects brings us into the realm of feasibility whereas traditional analysis at only one level of analysis is more likely to result in omitted variable bias. The second assumption required by ERGM is that the observed network  $y$  is average on all  $\Gamma(y)$ . While this is something of a strong assumption, it should be noted that, since we only observe one international system, our best guess at (or only data on) the average value of any given statistic on the network is the statistic we observed.

The disadvantages of ERGMs are that (1) they can only accommodate binary links, (2) they are unusually sensitive (compared to standard regression models) to missing data, and (3) the analyst must explicitly specify the network effects (unlike in the latent space model where accounting for the interdependence is semi-automated). This last point is non-trivial. One of the advantages of the latent space approach discussed above is that it does not require the analyst to have theoretical expectations about certain network structures. While

a disadvantage of the ERGM is that the analyst must develop expectations about network effects, it is also something of an opportunity in so far as the ERGM directly engages the network. In other words, if the analyst does in fact have a network hypothesis which cannot be captured with standard models or any of the alternatives listed above (hypotheses such as that the “friend of my friend should be my friend” and the “enemy of my enemy should be my friend”), the ERGM allows the analyst to do so.

We use an extension of the ERGM developed by Hanneke and Xing (2007), Hanneke, Fu and Xing (2010), and Desmarais and Cranmer (2010*b*) called the Temporal ERGM (TERGM), which allows the ERGM to analyze longitudinally observed networks. The TERGM conditions the realization of the network in each year on previous realizations of that network (up to an order specified by the researcher), thus accounting for temporal dependencies. The joint probability of observing all the networks we observed, one network for every year, can then be obtained by taking the product over the probabilities of each of those networks. We estimated the TERGM using a bootstrap pseudolikelihood approach developed by Desmarais and Cranmer (2010*a*), which has been shown to produce consistent point estimates and confidence intervals.<sup>11</sup>

## **ERGM Analysis of the Alliance Network**

We analyze alliances starting with a theoretical model loosely based on Lai and Reiter (2000) and Gibler and Wolford (2006). We extend this specification to include network effects. Our aim here is not to replicate Lai and Reiter (2000) and Gibler and Wolford (2006) exactly, but rather to use their model specifications, which are based on the most prominent theories of alliance formation, as a jumping off point to show the problems associated with the

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<sup>11</sup>While a full technical review of the TERGM extension to the ERGM is beyond the scope of this discussion, interested readers are referred to Desmarais and Cranmer (2010*b*) for such a discussion in the context of political networks.

regression analysis of alliances and correct these problems by extending this baseline model to incorporate network dependencies. Our primary goal here is to show how network analysis can work in harmony with specifications developed out of the more traditional regression framework. Our secondary goal is to show that, once independence assumptions have been abandoned and complex network structures included, the results for dyadic effects can change substantially.

As an outcome variable for our model, we use the alliance network. Each relationship included in the network is coded 1 if two states in a dyad are allied, as recorded by the Alliance Treaty Obligations and Provisions (ATOP) dataset (Leeds et al. 2002). We perform analyses that use the network defined on all alliance types (offensive, defensive, neutrality or consultation) as the outcome variable as well as some in which we restrict the analysis to the network of defense pacts only based on the finding by Lai and Reiter (2000) that defense pacts have different predictive structures than alliances writ large.

## **Exogenous Covariate Effects**

The role of jointly democratic dyads and political system similarity in the alliance formation process is one of the better studied aspects of alliance scholarship. Siverson and Emmons (1991) sparked the question with their finding that, in much of the 20th century, jointly democratic dyads were more likely to be allied than non-jointly democratic dyads. This seminal finding has been the basis of great debate. Simon and Gartzke (1996) agree that, in certain windows of time, democracies are more likely to ally, but argue that this is largely a product of the Cold War and is not robust over a longer span of time. Using a formal model, Leeds (1999) predicted that jointly democratic dyads should indeed be more likely to ally than any other dyad type, but that jointly autocratic dyads should be more likely to ally than mixed democratic-autocratic dyads. Lai and Reiter (2000), and later Gibler and

Wolford (2006), empirically test the proposition that not only are jointly democratic dyads more likely to ally, but that similarity of political systems should also be a positive predictor of alliance formation: both find fairly robust support for the role of political similarity but differ in that Gibler and Wolford (2006) find joint democracy to be a negative predictor of alliances before 1945 and find no effect for political similarity during that period. Gibler and Wolford (2006) explain this result by claiming that democracies are not actually more likely to ally, but that several large multinational alliances during the Cold War made jointly democratic dyads more common.

Following Leeds (1999), Lai and Reiter (2000) and, to a certain extent, Gibler and Wolford (2006), we posit the following two hypotheses specified at the dyadic level:

*Dyadic Hypothesis 1:* Joint democracy should positively predict alliances.

*Dyadic Hypothesis 2:* Political similarity should positively predict alliances.

We operationalize dyadic hypothesis 1 by including an indicator for joint democracy coded 1 if both states in the dyad have a Polity IV democracy score greater than 6 (Marshall et al. 2002). The literature suggests we should expect a positive effect from this indicator. To operationalize dyadic hypothesis 2, we include the absolute difference in Polity IV scores of the dyad. Similar systems will have smaller absolute differences and different systems will produce larger absolute differences. As such, we expect a negative effect of this variable on alliance relations (as differences increase, alliances become less likely).

A sizable body of literature addresses alliances that serve strategic purposes. Our next set of dyadic hypotheses examine how alliances relate to the security of the state.

The relationships between geographic proximity and international conflict and cooperation are interesting and nuanced. While, for states that are not powerful enough to project their power over substantial distances, geographic proximity is often a necessary condition for war, proximity also provides states an incentive to cooperate. Walt (1987) argues that states

that are geographically proximate are more likely not only to recognize regional threats, but to be sufficiently invested in defense against such threats that they will have a tendency to band together for common security. Gibler and Wolford (2006) pursue a different line of reasoning. They argue that border disputes are a common cause of conflict and that an alliance between states sharing a border indicates satisfaction with the territorial status quo and removes territorial threats from the dyad.<sup>12</sup> Gibler and Vasquez (1998) also find support for the pacifying role of alliances between states with territorial disputes. Moreover, close proximity may be necessary for states incapable of power projection to aid one another.

In light of the theoretical grounds for expecting a relationship between geographical distance, contiguity and alliance behavior, we consider the following two hypotheses:

*Dyadic Hypothesis 3:* Physical contiguity should positively predict alliances.

*Dyadic Hypothesis 4:* Geographic proximity should positively predict alliances.

We operationalize these hypotheses by including an indicator for contiguity (coded 1 if the states in the dyad are physically contiguous) and a variable measuring the distance between capitals respectively. These variables are taken from the Correlates of War Direct Contiguity Dataset (Stinnett et al. 2002). We expect a positive effect for the contiguity indicator (alliances are more likely when states are contiguous) and a negative effect for distance (states are less likely to ally as the distance between them increases).

From a strategic perspective, it makes little sense for rival states to ally with one another. Alliances between rivals should be unreliable or ill fated (such as the Reassurance Treaty between Germany and Russia prior to World War I) and neither state would want to increase the security of the other through alliance (Lai and Reiter 2000). Conversely, two states that share an enemy ought to be more likely to ally. Walt (1987) argues that when states are

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<sup>12</sup>Somewhat tangentially, this is the basis for their expectation of joint democracy being a negative predictor of alliances (as discussed above): territorial threats are null in jointly democratic dyads because such dyads do not engage in conflict, therefore alliances between democratic states are usually not necessary/useful.

threatened by others, they can use alliances to balance the threat. As such, states sharing a common enemy are likely to ally in order to balance that enemy, thus minimizing the probability that either of them will come under attack. Lai and Reiter (2000) also suggest that alliances between states sharing an enemy may be more reliable. As such, the following two hypotheses bear consideration:

*Dyadic Hypothesis 5:* States with a history of conflict are unlikely to ally.

*Dyadic Hypothesis 6:* States with a common enemy are likely to ally.

We operationalize these in a straightforward manner: for dyadic hypothesis 5, we include an indicator coded 1 if the states in the dyad have engaged in a militarized interstate dispute in the preceding 10 years, and for dyadic hypothesis 6, we include an indicator coded 1 if both states in the dyad have engaged in a militarized dispute with a (same) third state in the preceding 10 years. These variables are taken from the Correlates of War (COW) Dyadic Militarized Interstate Incident Data (Ghosn and Bennett 2003). We expect a history of conflict to have a negative affect on the probability of alliance formation, and a common enemy to have a positive effect.

The final dyadic effect we consider is economic interdependence. A widespread liberal hypothesis is that economic interdependence greatly reduces proclivities towards conflict and supports cooperation (Rosecrance 1986; Keohane and Nye 1989; Oneal and Russett 1999; Lai and Reiter 2000; Gibler and Wolford 2006). We incorporate this idea by considering the following hypothesis:

*Dyadic Hypothesis 7:* Higher levels of trade increase the probability of alliance.

We operationalize this hypothesis by including the total dyadic trade as measured by the COW trade data set (Barbieri, Keshk and Pollins 2008). Considering the trade hypothesis also adds a wrinkle to our research design: reliable trade data are only available after 1955,

while the rest of our variables go back to 1900, if not 1816. To accommodate the trade hypothesis with the limited data available to us, we run a second set of regressions using the same basic model specification but including trade, effectively limiting these models to the period 1955-2000.

As pointed out briefly above, and Maoz et al. (2005) examine in greater detail, network analysis generally and ERGM specifically allow us to naturally and seamlessly incorporate effects from multiple levels of analysis. We consider two effects at the state level. First, Waltz (1979) argues that major powers will be likely to seek alliances. This is the case, he argues, because major powers will seek to maintain and expand their spheres of influence and balance other major powers seeking to do the same. Major powers will also be able to have useful alliances over greater distances (and thus more states they can ally to) because of their ability to project power far from their borders. As such, we hypothesize that

*Monadic Hypothesis 1:* Major powers are likely to have more alliance partners.

We accommodate this hypothesized effect by including an indicator for whether a state is a major power as defined by the COW.

Second, we expect states to seek more alliances when they are under greater threat. The simple logic is that the more security a state believes it needs, the more likely it will be to try to secure itself through alliances (Walt 1987). From this proposition follows our second state-level hypothesis:

*Monadic Hypothesis 2:* States under greater threat will be more likely to form alliances.

We operationalize this by including a measure of the number of militarized disputes in which a given state has engaged in the previous 10 years (Ghosn and Bennett 2003). We expect this measure to have a positive effect on alliances.

## Endogenous Structure in the Alliance Network

We now expand upon the theoretical foundations and empirical specifications developed above and consider, both theoretically and empirically, endogenous structures we believe to be instrumental to the network generating process. We should note that such considerations represent a full departure from what is possible with standard regression models.

We begin with the most basic of network effects: structures commonly called “ $k$ -stars” in the network literature. The simplest form of  $k$ -star is the 2-star. A 2-star occurs when a state  $i$  shares a link with (is allied to) two other states,  $j$  and  $h$ , but  $j$  and  $h$  are not connected to each other (Frank and Strauss 1986). In the context of the alliance network, a 2-star would be formed when, for example, the UK was allied to Belgium and Japan prior to the outbreak of World War I, but Belgium and Japan were not allied to one another. Intuitively, a 3-star is formed when node  $i$  is connected to nodes  $j$ ,  $h$ , and  $\ell$ , but  $j$ ,  $h$ , and  $\ell$  do not have links with one another. Any number of links can be added to  $i$  to form a star of arbitrarily many links called a  $k$ -star. The first two cells of Figure 5 illustrate 2-stars and 3-stars respectively.

[Figure 5 about here.]

Stars are included in an ERGM as structural statistics (one of the statistics composing the statistic vector  $\mathbf{\Gamma}(\cdot)$  in equation 2). The statistic can be expressed flexibly for  $k$ -stars as

$$S_k(y) = \sum_{1 \leq i \leq k} \binom{y_{i+}}{k}, \quad (3)$$

where the  $+$  sign indicates summation over all  $i$  (Snijders et al. 2006). In essence, the star statistic counts the number of  $k$ -stars present in the observed network and then, in the estimation process, uses this number to put greater weights on hypothetical networks that have the same, or a similar, number of  $k$ -stars.

In substantive terms, stars capture “popularity effects,” sometimes referred to as “preferential attachment,” in the alliance network. Stars capture the tendency of multiple states to ally with the same state, expressly because that state is allied with many states. In other words, a “popular” state will have several other states allying with it. A nascent literature on alliances as a network phenomenon has already suggested that such popularity effects exist and has found empirical support for them (Cranmer, Desmarais and Kirkland 2011). Essentially, alliances can be seen as assets, and the presence of a large number of alliances with a candidate ally increases the potential value of allying with that candidate state. As such, we evaluate the following as our first network hypothesis:

*Network Hypothesis 1:* Popularity effects exist in the alliance network.

We operationalize this hypothesis by including 2-stars in our model. It is only necessary to include 2-stars (rather than, 3, 4, and  $k$ -stars) because there are two 2-stars in a 3-star and so on. As such, the 2-star statistic should adequately capture popularity effects should they exist. Indeed, many authors have noted the ability of the 2-star statistic to capture popularity effects within the ERGM framework (Wang et al. 2009; Lusher and Ackland 2010).

It is also often the case that two nodes are likely to be connected if they are both connected to a third node. Specifically, if one adds a connection between the two nodes with only one connection (nodes of degree 1) in a 2-star, a triangle is produced. This is illustrated in cell three of Figure 5. This phenomenon, commonly referred to as “triangle-closure,” is a formalization of the adage “a friend of a friend is a friend.” A triangle statistic is easily included in an ERGM specification as follows (Frank and Strauss 1986; Snijders et al. 2006):

$$T(y) = \sum_{i \leq j < h \leq n} y_{ij} y_{ih} y_{jh}. \quad (4)$$

In the context of the alliance network, this phenomenon can be seen twice in the Triple Alliance and the Triple Entente in which Germany, Austria-Hungary, and Italy, and France, Russia, and the United Kingdom were all allied to each other respectively. Cranmer, Desmarais and Kirkland (2011) suggest that, in the context of alliances, closed triangles are a particularly desirable structure because commitments are more credible and communal security is better maximized than with other structures. In other words, they suggest that the utility for some state  $i$  is higher when it is a member of a triangular alliance than it is when  $i$  is the twice-allied node in a 2-star, even though  $i$  has the same number of dyadic connections in both configurations. In several tests using the SIENA technique for dynamic networks mentioned above (Snijders 2001, 2005, 2006; Snijders, van de Bunt and Steglich 2010), Cranmer, Desmarais and Kirkland (2011) found robust support for the hypothesis that triadic closure is a major driver of alliance formation.

We build on the intuition proposed by Cranmer, Desmarais and Kirkland (2011), but our specification is somewhat different from theirs. They were concerned with the *formation* of new alliances, where it makes sense to model the addition of a single link at a time (the tendency towards triadic closure). We are concerned, however, with the network of alliances itself. If we apply Cranmer, Desmarais and Kirkland's (2011) theory to the network of existing alliances, it predicts that dense clusters of alliances should emerge. Indeed, Figures 2-4 show that states do seem to group together in terms of the density of their connections. As such, we propose the following hypothesis:

*Network Hypothesis 2:* A preference of states to form dense clusters will manifest in the alliance network.

To operationalize this hypothesis, however, we require a more nuanced measure than triangles. We will use the geometrically weighted edgewise shared partners (GWESP) statistic.

The geometrically weighted edgewise shared partners (GWESP) statistic is born of a

generalization of the triangle statistic. The concept of the  $k$ -triangle was first proposed by Snijders et al. (2006) and later refined into the GWESP statistic by Hunter and Handcock (2006) and Hunter (2007). To begin breaking down the meaning of the statistic, consider the “edgewise” component of GWESP; it means that a link (often called an “edge” in the network literature; in our case an alliance) must be present between two nodes for the statistic to be defined. So, suppose that a link exists between nodes  $i$  and  $j$ . Based on this link ( $y_{ij}$ ), the  $k$ -triangle statistic measures how many other states, both  $i$  and  $j$  are linked to. For example, if  $i$  and  $j$  are allied to each other and are also both allied to five other states, this would produce a 5-triangle and would look like Figure 5 (Snijders et al. 2006, 118). In other words, the  $k$ -triangle counts the number of allies that any given allied dyad (link) has in common (hence the “edgewise shared partners” part of the statistic’s name).

To proceed from the  $k$ -triangle concept to the GWESP statistic itself, one need only to consider giving probability weights to the different possible values of the  $k$ -triangle. For example, it should be more common that a pair of allied states has only one ally in common than it should be for them to have 50 allies in common. As Snijders et al. (2006, 116) describe the impetus for the  $k$ -triangle statistic, “What is needed to capture these ‘clique-ish’ structures is a transitivity-like concept that expresses triangulation also within subsets of nodes larger than three, and with a statistic that is not linear in the triangle count but gives smaller probabilities to large cliquelike structures.” This weighting system can be accomplished by applying a geometric weighting scheme to  $k$ : the lower the number of shared partners, the more likely that configuration should be and the higher the number of shared partners, the less likely that configuration should be.

If two allied states  $i$  and  $j$  have  $k$  allies in common, we can express their edgewise shared partners as  $EP_k(y)$  (using notation from Hunter (2007)). Hunter (2007, 12-14) points out

that we can relate the edgewise shared partners statistic to the  $k$ -triangle as follows:

$$T_k(y) = \sum_{i=k}^{n-2} \binom{i}{k} EP_i(y) \quad (5)$$

for  $2 \leq k \leq n - 2$ . All that remains then is to add a geometric weighting scheme to this statistic by introducing the parameter  $\theta = \log \lambda$  and specifying the GWESP statistic as,

$$GWESP(y; \theta) = e^\theta \sum_{i=1}^{n-2} \left\{ 1 - (1 - e^{-\theta})^i \right\} EP_i(y). \quad (6)$$

We include the GWESP statistic in our model to capture alliance clusters by the adage that “the friends of my friends are my friends.” We expect GWESP to be a positive predictor of the alliance network. This may seem odd to those familiar with network analysis outside of the ERGM framework. There are, after all, many thoroughly-developed algorithms designed to decompose the clustering in a network (see Fortunato (2010) for an excellent review of clustering and community-detection algorithms). Our use of GWESP highlights the difference between conventional network analysis and ERGM. Clustering algorithms are designed to identify dense clusters, not to determine whether there is a tendency to form clusters *controlling* for other determinants of cluster existence in the network. For instance, it could be the case that the presence of ties in the network are associated with the shared value of a categorical independent variable. A clustering algorithm would identify clusters around the unique values of this covariate, and it would appear that there was a significant degree of clustering in the network, when no such tendency actually exists – a classic story of omitted variables. Within the ERGM, this covariate can be included in the model, and if there is no tendency to cluster beyond homophily around this covariate, GWESP will have no effect (Snijders 2006).

[Table 1 about here.]

## Results

The results from our models are presented in Table 2. Two groups of models are presented in this table. The first four models (columns) are those for which the dependent variable is any alliance link, including offense pacts, defense pacts, neutrality agreements, and consultation agreements. The second four models (columns) are those for which the dependent variable is restricted to defense pacts only. As discussed above, two general specifications are evaluated: one for which the years under consideration are 1816-2000 and trade is not included, and one for which trade is included, but the period of analysis is restricted to 1955-2000 due to the availability of the trade data. To demonstrate the possibly important differences in substantive results between the network analysis and traditional regressions, we estimate an ERGM with the temporal extension proposed by Desmarais and Cranmer (2010*b*) for each specification as well as a corresponding logistic regression with the most similar specification possible (the network statistics we have included in the ERGM cannot be included in a logistic regression).

Our intention is to compare and contrast the results from dyadic logit and TERGM. Before we consider the results from the two approaches it is important to assess which model fits the data better – weighing in on which specification more closely approximates the underlying data generating process. Unfortunately, common information-theoretic measures of model fit such as the AIC or BIC cannot be used for comparison because the TERGM must be estimated with a bootstrap pseudolikelihood approach (see Desmarais and Cranmer (2010*a*) for technical details). Instead, we can simulate a series of alliance networks based on the parameters we have estimated in each of the models and then compare the simulated networks to the observed network to assess model fit.

We use the receiver operating characteristic curve (ROC curve) to non-parametrically judge which model best classifies the binary alliance outcomes. ROC curves plot the propor-

tion of correct positive (i.e. alliance present) predictions against the proportion of incorrect positive predictions (Pete, Pattipati and Kleinman 1993; Pepe 2000; Sing et al. 2009). The closer the ROC curve is to the top-left corner of the plot, the better the model predicts. A useful summary measure for the ROC is the area under its curve; a perfectly predicting model's ROC curve will integrate to one. Thus, the area under the curve is bounded between zero and one with higher numbers indicating better model fit. ROC curves for the specifications we considered are presented in Figure 6.

[Figure 6 about here.]

As is clear from the ROC plots and the areas under the various curves, the TERGM models fit the data remarkably better than the logistic regression models. For each of our four TERGM specifications, the model does a near-perfect job of predicting the alliance network. While the fit of the logistic regression model is not deplorable, it predicts non-trivially worse than the network models. This is further evidence to suggest that leaving network characteristics unmodeled can seriously jeopardize the validity of one's results. Since the TERGM appears to capture a process much closer to the actual data generating process, when TERGM and logistic results diverge – areas that we highlight below – it is safe to assume that the TERGM provides the more valid result.

Table 2 shows the results of the TERGM and dyadic logit specifications. When considering the network effects, we obviously cannot compare what the TERGM found to logistic regression because it is impossible to include such parameters in a regression. We do, however, seem to have learned something interesting about the alliance generating process.

The 2-star network parameter has a significant positive effect for all specifications, though the magnitude of the coefficient is somewhat small. Substantively this means that popularity effects in the alliance network are small, but real. In other words, some states are simply more popular or “sociable” in terms of their alliance behavior than others. While this seems

to be the case, the small coefficient indicates that either popularity effects are minor but widespread or that only a few states in the system are especially popular. Judging by Figures 2-4, it appears that the latter explanation is more likely: certain states, like the US, Canada, Turkey and France are especially popular.

The strongest result of any parameter considered is the GWESP statistic. GWESP produces a large, positive and significant effect in every specification in which it is included. This supports the hypothesis that states have a strong incentive to create dense alliance clusters. We can take this result as further support for Cranmer, Desmarais and Kirkland's (2011) hypothesis that the utility of closed triadic alliances is greater for each member than the sum of their dyadic alliances. Generalizing this triadic formation hypothesis with the GWESP statistic, it seems that these "synergy effects" have a pronounced effect on the way alliance ties are structured.

Table 2 shows some startling differences between the logistic regression and TERGM specifications. Earlier, we discussed how unmodeled network effects will have their explanatory power falsely attributed to covariates. Since the TERGM actually reduces to a logistic regression when no network effects are included (Snijders et al. 2006), we can understand the differences between the logit coefficients and the TERGM coefficients as the degree to which the logistic regression is biased by the nonindependence of dyads and the omission of important network effects. Furthermore, recalling our discussion of how dyadic analysis artificially increases the number of observations and shrinks standard errors, we can interpret the differences between the standard errors from the logistic regression and TERGM as the degree to which logit produces false confidence in the statistical significance of effects.

[Table 2 about here.]

We see the most alarming differences between logistic regression and the TERGM for the joint democracy indicator variable. While the variable is significant in all eight models,

the signs of the effect are reversed as we move from logit to the TERGM. Logistic regression consistently finds the *positive* and significant effect discussed by Siverson and Emmons (1991), Leeds (1999), Lai and Reiter (2000), and Gibler and Wolford (2006) while the ERGM consistently finds a significant *negative* effect. What is more, the magnitudes of the effects differ substantially; sometimes by as much as 478%. Logistic regression, consistently across all specifications, overestimates the magnitude of the erroneously signed coefficient. In substantive terms, the TERGM result can be seen as support for Gibler and Wolford's (2006) hypotheses that democracies are unlikely to ally because they do not pose a threat to each other (and thus alliances do not remove threats).<sup>13</sup> We are forced to conclude that, because of dyadic nonindependence and the omission of network effects, logistic regression would, in this case, lead us to *exactly* the wrong substantive conclusion.

We also find divergent results for the effect of the major power indicator. For both specifications with the time range 1955-2000, logistic regression and the TERGM both find significant effects, but produce differently signed coefficients. The TERGM finds a negative effect across both specifications, indicating that major powers are less likely to ally, whereas logistic regression finds a positive effect. The difference is less pronounced for the models with time ranges of 1816-2000; logit continues to find positive effects, but instead of significantly switching signs, the effects simply fall out of significance with the TERGM and cannot be reliably distinguished from zero.

The results for threat also vary based on the time frame under examination. For the models examining 1816-2000, logistic regression finds a positive effect (significant for the specification restricted to defense pacts), whereas the TERGM finds a negative effect (significant for all alliances). However, for both models considering the shorter time frame (1955-2000), we see a significant sign change between the TERGM and logit; in both cases,

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<sup>13</sup>Gibler and Wolford (2006) find this expectation of a negative effect supported for alliance formation, but not for alliance connections as we do; they found the positive effect produced by logistic regression.

logistic regression finds a significant negative effect where the TERGM finds a significant positive effect. Substantively, this indicates that states with a greater history of threat are indeed more likely to seek alliances, supporting monadic hypothesis 2. Logistic regression, however, would have falsely led us to the exact opposite substantive inference.

There are similarly vexing differences in significance between logistic regression and the TERGM for the distance variable: for all but one specification, the effect of distance is negative and significant with logistic regression, but indistinguishable from zero with the TERGM. In substantive terms, this means that with logit, we run the risk of falsely inferring support for the idea that non-contiguous states near one another are more likely to be allied; perhaps to counter regional threats (Walt 1987). In terms of effect strength, however, the effect is so small that in some cases the coefficient is rounded to zero even when reporting five decimal digits. The substantive conclusion we are left with is that distance is not an important predictor of alliances.

The results for our conflict history variable are similarly worrisome. For models considering all alliances, the TERGM produces positive coefficients and is significant over the long time period but not for the second half of the 20th century. Logistic regression, however, produces a negative and significant effect for the long period and finds a positive but insignificant effect for the short time period. When considering only defense pacts, the TERGM finds consistent negative effects, significant for the post-WWII period but not for the long period. Conversely, logit finds a significant negative effect for the long period and an insignificant positive effect for the short period.

In spite of the notable differences between TERGM and logistic results, the results for three of the variables agree nicely between the logistic and TERGM specifications. First, the contiguity indicator has a positive and significant effect, as expected, and similar magnitudes across all eight models. This, taken with the partial lack of results from the distance measure,

suggest support for Gibler and Wolfort's (2006) hypothesis that alliances between contiguous allies removes territorial threats and less support for Walt's (1987) hypothesis about regional threats.

Second, our political similarity measure also produces consistent results between logit and TERGM specifications. Each model we considered finds a negative and significant effect with similar magnitudes for the absolute difference in polity scores in the dyad. This indicates a preference among states to ally with states that are approximately similar to them politically.

Third, the results for trade are consistent between logit and the TERGM. While a uniformly positive and significant effect is found for each of the four models which include trade, the magnitude of the coefficient is extremely small; the strongest effects only occur in the fourth place behind the decimal. So, while we can distinguish these results from zero statistically, substantively we find that the effect of economic interdependence, as measured by trade, has a nearly nonexistent effect on the tendency of states to ally.

The effect of a common enemy nearly agrees between logistic and TERGM specifications. It has a positive and significant effect in all eight models, indicating that states are indeed drawn together by a common foe, but the magnitudes of the effects differ substantially between specifications. The coefficients from the logistic regressions are uniformly larger than those produced by the TERGM. In some cases they are several hundred percent larger. This is disconcerting; while both models agree that the effect is likely real, logit overestimates the size of the effect dramatically because of biases induced by dyadic nonindependence and an inability to control for network effects.

We must conclude then that not only does the use of logistic regression on the alliance network (1) risk faulty inference on the covariates as a result of dyadic nonindependence, but that (2) we can gain important substantive insight (such as the roles of popularity and clustering) by modeling network effects in a TERGM and (3) using regression over a network

model can forfeit a non-trivial amount of predictive power.

## Concluding Thoughts

We have presented a discussion of the empirical challenges, nonindependence of observations and data multiplication, that accompany the analysis of international alliances and proposed a method that overcomes these challenges. The broader point we hope to have made in this discussion is that the traditional regression framework is problematically restrictive and lies at the core of these problems. We have argued that network analysis provides a general, and much more flexible, framework for analyzing alliances and have also argued that alliances (as well as many other phenomena of interest to conflict processes scholars) can naturally be thought of as network phenomena. We then presented one of several ways network analyses can be conducted: the exponential random graph model (ERGM) and its temporal extension (TERGM). Lastly, we provided a detailed exposition of the inferential differences between traditional regression models and the ERGM. We found, as we expected we might given our initial discussion of the problems with dyadic approaches to the analysis of alliances, that several of the effects found by logistic regression were incorrect, including a number of significant sign changes and losses of significance. We also found that the network properties we introduced, 2-stars and GWESP, were strong and relevant predictors of the alliance network, including them produced a model that fit the data remarkably well. In short, our analysis showed that network approaches to the study of alliances allows us to test hypotheses that are untestable in the regression framework and that controlling for the structure of the network can non-trivially alter the inferences we make about the effects of covariates.

We hope that this study will be a jumping-off point for the further analysis of international politics as a complex network phenomenon. We hope that through the network treatment of international politics, theoretical gaps can be closed, potentially faulty inferences can

be avoided, and – more than anything – new frontiers for the theoretical and empirical understanding of conflict and cooperation will become available.

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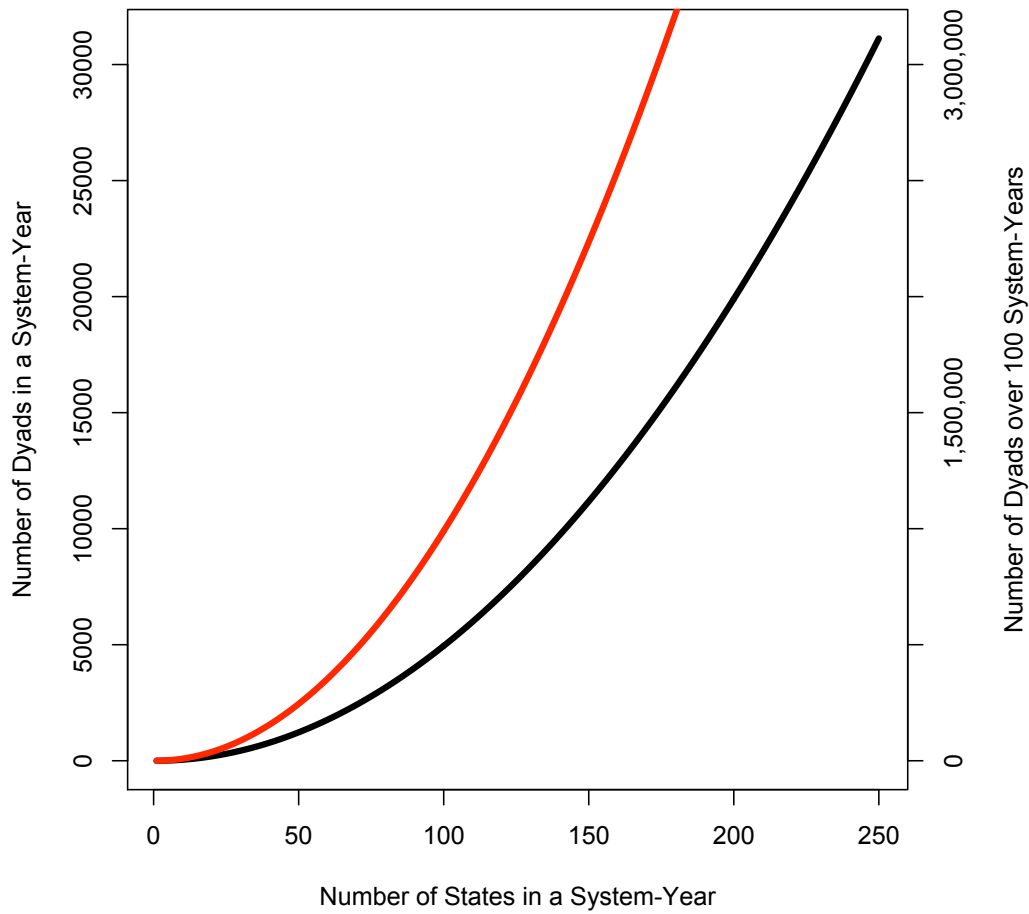


Figure 1: Data multiplication in dyadic analyses. The number of “observations” in a dyadic data set are nonlinearly increasing by a function of  $N = \binom{n}{2}$  where  $n$  is the number of states in the system-year. The black and grey lines represent the number of undirected and directed dyads respectively as a function of  $n$ .

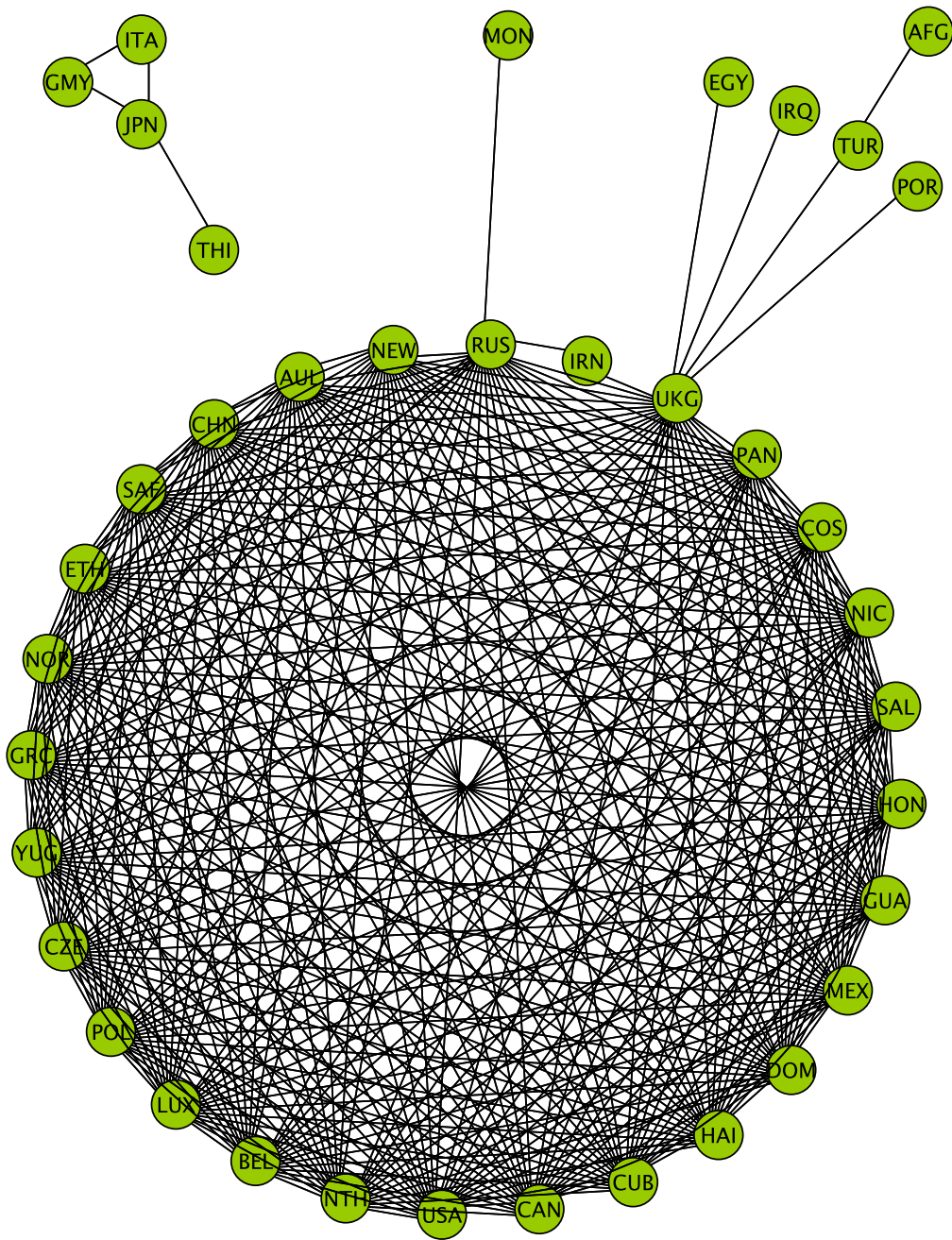


Figure 2: The network of defense pacts in 1942.

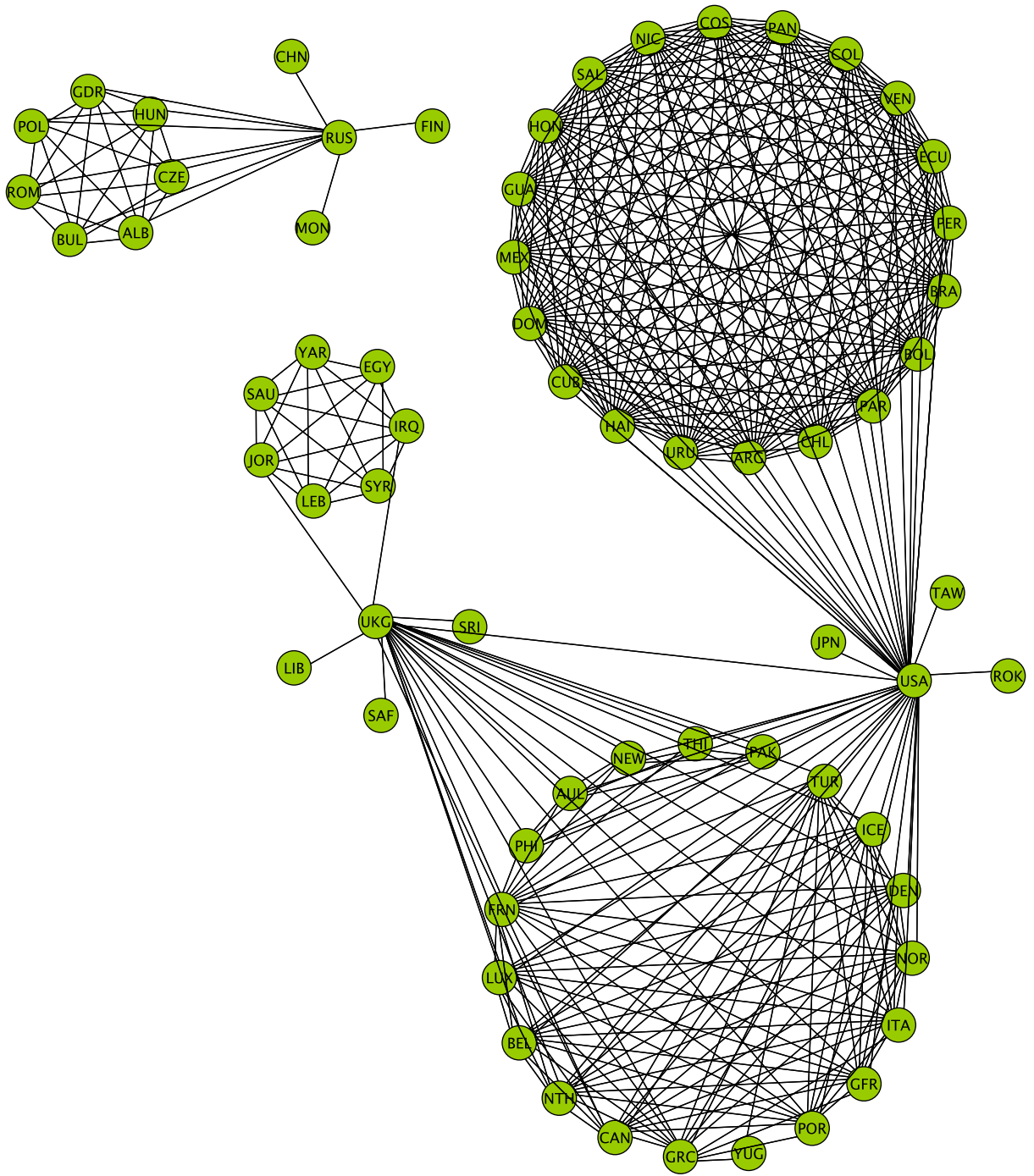


Figure 3: The network of defense pacts in 1956.

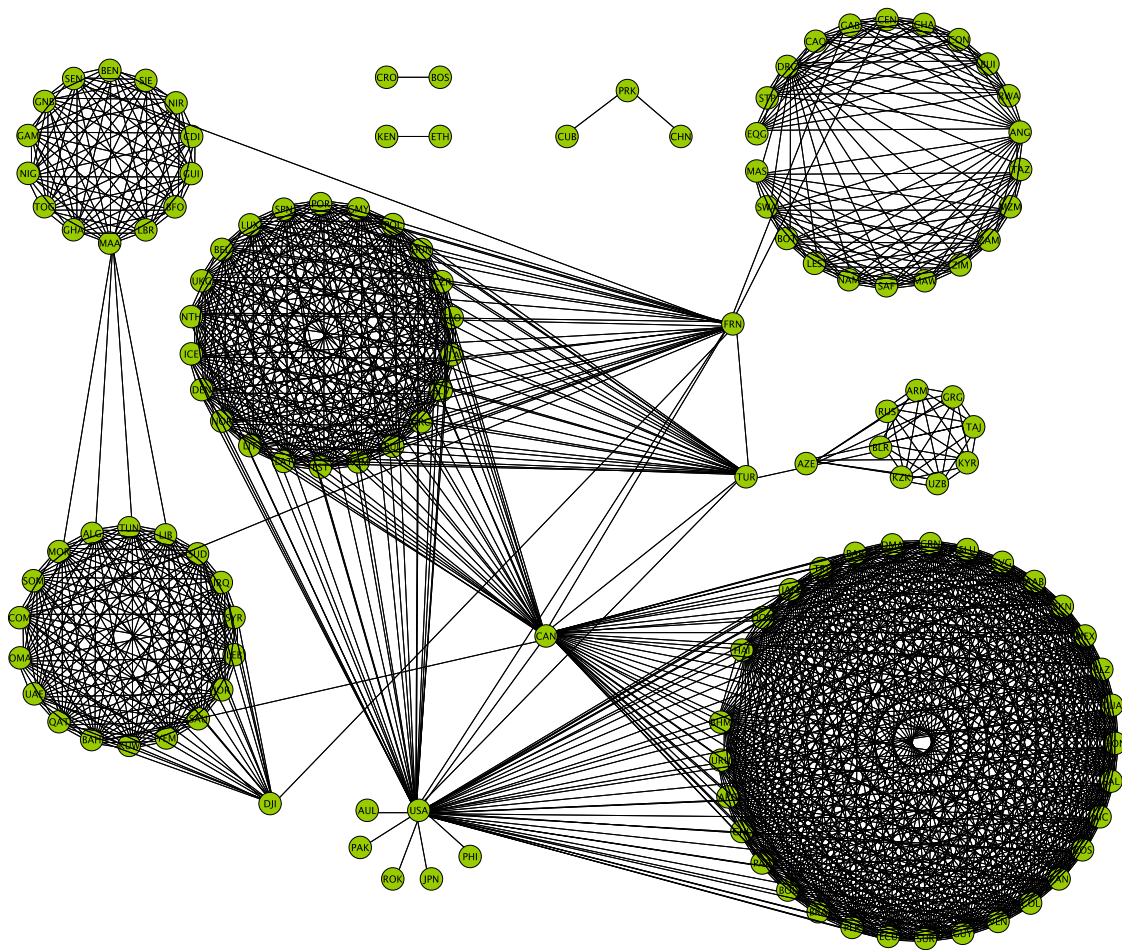
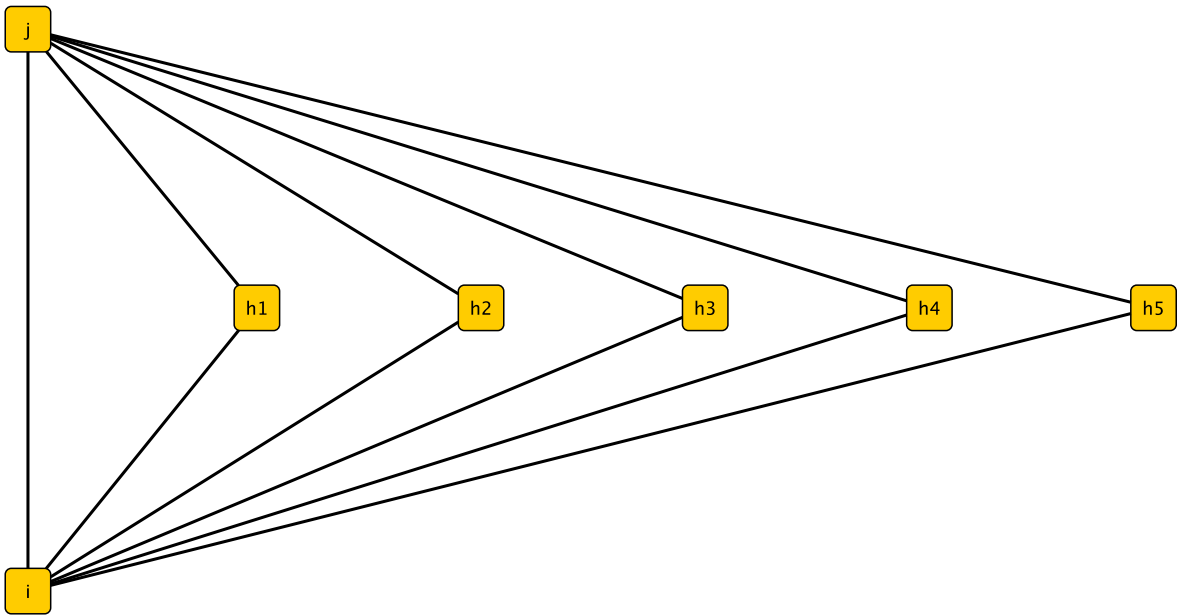
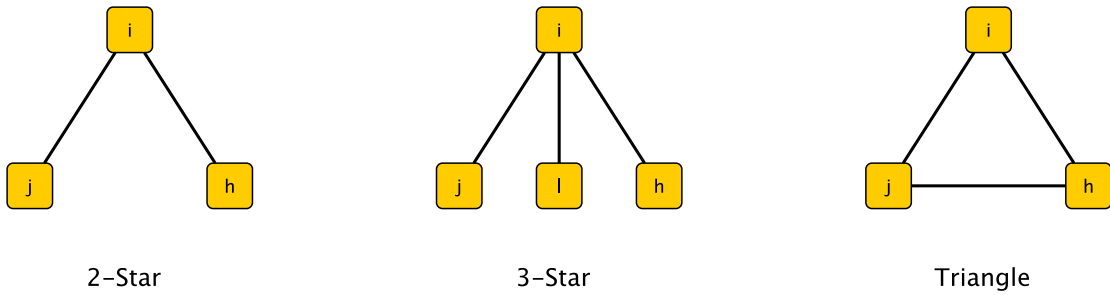


Figure 4: The network of defense pacts in 2003.



A special case of the  $k$ -triangle: the 5-triangle.  
 Geometric weights to the number of partners shared by  $i$  and  $j$  are added to form the GWESP statistic.

Figure 5: From top left: a 2-star, a 3-star, and a triangle. Below, a  $k$ -triangle where  $k = 5$  (Snijders et al. 2006, 118). In this figure, squares represent nodes in the network and the lines between them represent links.

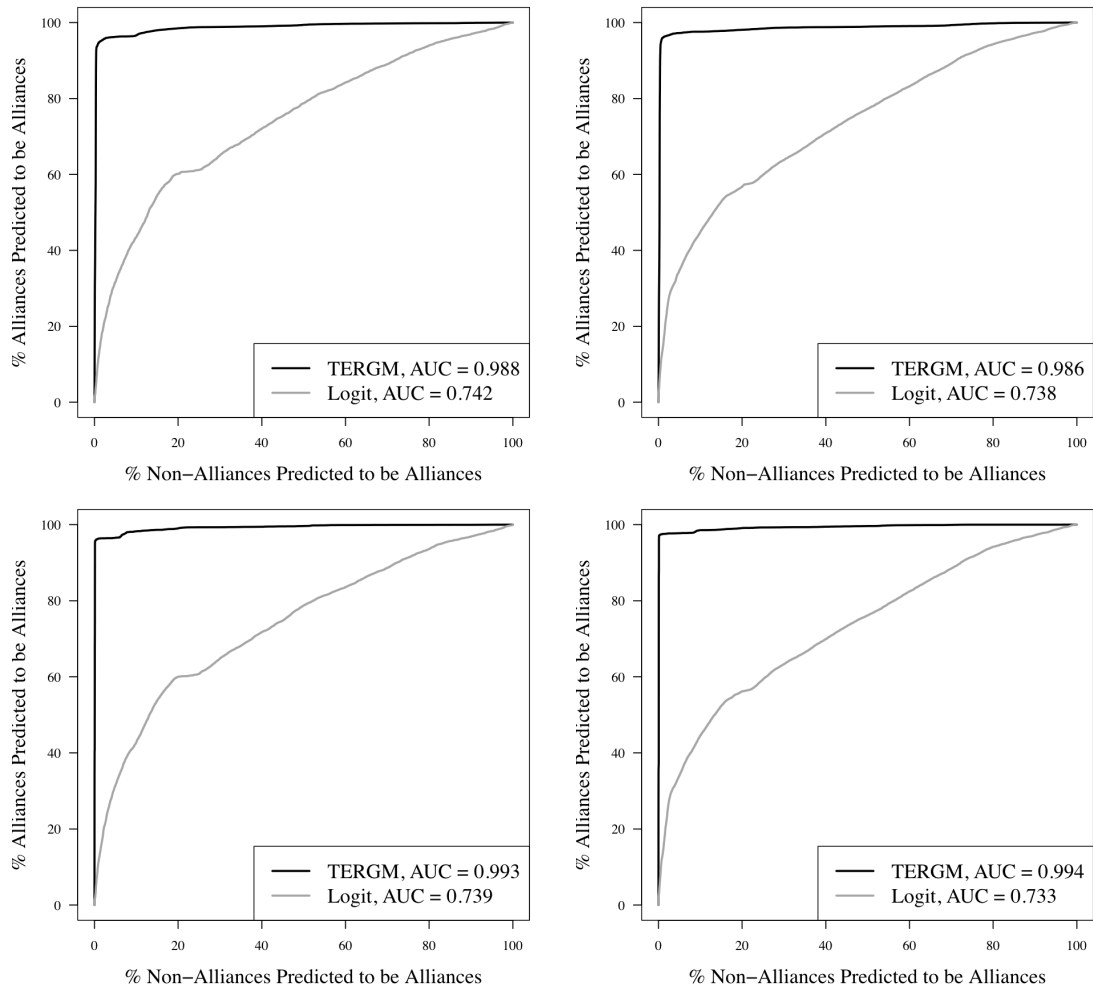


Figure 6: ROC plots and areas under the curve (AUC) for the four specifications we considered across logistic regression and ERGM. The top-left cell shows the specification with links coded as any alliance (offensive, defensive, neutrality, or consultation) on the period from 1816 to 2000. The top-right cell shows the same specification adding trade as a covariate for the period 1955 to 2000. The bottom-left cell shows the specification for only defense pacts from 1816-2000 and the bottom-right cell shows the same specification adding trade as a covariate for the period 1955-2000.

<b>Variable</b>	<b>Level</b>	<b>Measure</b>	<b>Expected Effect</b>
Major Power	State	Indicator	+
Threat	State	Number of disputes in previous 10 years	+
Joint Democracy	Dyad	Both states have democracy score > 6	+
Pol. Similarity	Dyad	Absolute difference in Polity scores	-
Contiguity	Dyad	Indicator	+
Distance	Dyad	Distance between capitals	-
Hist. Conflict	Dyad	Conflict in dyad in previous 10 years	-
Common Enemy	Dyad	Conflict with (same) third state in 10 years	+
Trade	Dyad	Total dyadic trade	+
2-stars	Network	Number of 2-star formations	+
GWESP	Network	Weighted shared partner count on alliance links	+

Table 1: This table summarizes the variables and network effects we include in our model, the level at which they are measured, a summary of how they are measured, and the direction of the effect we expect them to have on the alliance network.

	All Alliances				Defense Pacts Only			
	Logit 1	TERGM 1	Logit 2	TERGM 2	Logit 3	TERGM 3	Logit 4	TERGM 4
links	-3.1867*	-8.3280*	-2.7870*	-9.8282	-3.2605	-9.6099	-2.8740	-11.6411
	(0.0122)	(0.2223)	(0.0145)	(0.2951)	(0.0127)	(0.2321)	(0.0151)	(0.1780)
Two-Stars		0.0675*		0.0921*		0.0846*		0.1138*
		(0.0046)		(0.0059)		(0.0054)		(0.0070)
GWESP		2.0028*		2.2399*		2.3780*		2.7177*
		(0.0434)		(0.0523)		(0.0477)		(0.0263)
Hist. Conflict	-0.1864*	0.3929*	0.0849*	0.0946	-0.2936*	-0.1849	0.0053	-1.1385*
	(0.0290)	(0.0921)	(0.0367)	(0.0606)	(0.0308)	(0.1800)	(0.0379)	(0.1060)
Com. Enemy	1.2991*	0.7563*	1.0792*	0.2696*	1.3291*	0.6410*	1.1039*	0.3160*
	(0.0158)	(0.0862)	(0.0208)	(0.0718)	(0.0164)	(0.1102)	(0.0215)	(0.0984)
Threat	0.0006	-0.0080*	-0.0088*	0.0047*	0.0014*	-0.0028	-0.0079*	0.0122*
	(0.0004)	(0.0025)	(0.0006)	(0.0021)	(0.0004)	(0.0030)	(0.0006)	(0.0021)
Contiguity	1.8959*	2.0662*	2.2645*	1.6833*	1.8555*	2.7088*	2.2203*	2.7952*
	(0.0158)	(0.0960)	(0.0203)	(0.0890)	(0.0165)	(0.1066)	(0.0207)	(0.1166)
Joint Dem.	1.1443*	-0.3545*	0.9254*	-0.4464*	1.1277*	-0.2365*	0.8958*	-0.1897*
	(0.0140)	(0.0768)	(0.0158)	(0.0856)	(0.0147)	(0.0579)	(0.0165)	(0.0658)
Pol. Similarity	-0.0015*	-0.0021*	-0.0027*	-0.0028*	-0.0010*	-0.0025*	-0.0021*	-0.0036*
	(0.0003)	(0.0009)	(0.0004)	(0.0010)	(0.0003)	(0.0011)	(0.0004)	(0.0013)
Major Power	0.1350*	-0.2045	0.6625*	-1.5877*	-0.0315	0.7240*	0.4850*	-0.3120*
	(0.0160)	(0.1433)	(0.0238)	(0.1156)	(0.0172)	(0.1271)	(0.0253)	(0.1117)
Distance	-0.00017*	-0.00003*	-0.00007*	-0.00003	-0.00007*	-0.00001	-0.00007*	0.00000
	(0.00000)	(0.00001)	(0.00000)	(0.00002)	(0.00000)	(0.00001)	(0.00000)	(0.00001)
Trade			0.00006*	0.00012*			0.00005*	0.00010*
			(0.00000)	(0.00001)			(0.00000)	(0.00001)
Network-Years	1816-2000		1955-2000		1816-2000		1955-2000	
Dyad-Years	645,431		421,061		645,431		421,061	

Table 2: Maximum Pseudolikelihood estimates reported. Standard errors are based on 1,000 network-year bootstrap iterations. Asterisks indicate that the coefficient is statistically significant at or beyond the traditional 0.05 level.