What, Exactly, is a Paradox?

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Quine (1966) offered his classic characterization of the notion of paradox, a taxonomy for paradoxical arguments, and some vocabulary for discussing them. In this paper I shall generalize Quine's taxonomy and defend a simpler characterization. The simpler characterization will have the virtue or the flaw (as might be) of making paradox a matter of degree.

1. QUINE'S VIEW

For Quine, a paradox is an apparently successful argument having as its conclusion a statement or proposition that seems obviously false or absurd. That conclusion he calls "the proposition of" the paradox in question. What is paradoxical is of course that if the argument is indeed successful as it seems to be, its conclusion must be true. On this view, to *resolve* the paradox is (1) to show either that (and why) despite appearances the conclusion is true after all, or that the argument is fallacious, and (2) if the former, to explain away the deceptive appearances.

Quine divides paradoxes into three groups. A "veridical" paradox is one whose "proposition" or conclusion is in fact true despite its air of absurdity. We decide that a paradox is veridical when we look carefully at the argument and it convinces us, i.e., it manages to show us how it is that the conclusion is true after all and appearances to the contrary were misleading. Quine's two main examples of this are the puzzle of Frederic in

The Pirates of Penzance (who has reached the age of twenty-one after passing only five birthdays), and the Barber Paradox, which Quine considers simply a sound proof that there can be no such barber as is described.[1]

A "falsidical" paradox is one whose "proposition" or conclusion is indeed obviously false or self-contradictory, but which contains a fallacy that is detectably responsible for delivering the absurd conclusion. We decide that a paradox is falsidical when we look carefully at the argument and spot the fallacy. Quine's leading example here is De Morgan's trick argument for the proposition that $2 = 1$.[2]

Oddly, Quine does not mention a third related category, the obverse of a veridical paradox: the argument in question could have an obviously false or self-contradictory conclusion, yet rest on no error of reasoning however subtle—so long as it has a premise that looks for all the world true until we let the argument itself show us that and how the premise is false after all. E.g., the Barber is classified as veridical because its conclusion is the truth that there is no barber who shaves all and only those who do not shave themselves; but turn it on its head, so that its conclusion is rather the absurdity that there is a barber who both does and does not shave her-/himself. There is still no fallacy, but only the innocent-seeming premise that there *is* a barber who shaves all and only those who do not shave themselves.[3] We might call this sort of paradox, for want of better, "premise-flawed."

Finally (returning to Quine), an "antinomy" is an *intractable* paradox, one that we cannot see how to resolve in either of the foregoing two ways: The argument does not succeed in convincing us that its conclusion is true-despite-appearances (often because the conclusion is overtly contradictory or otherwise incoherent), yet we can find no fallacious move in the argument; nor is there a premise that is shown false-despite-appearances. Antinomies, Quine says, “bring on the crises in thought” (p. 5); they show the need of drastic revision in our customary ways of looking at things.

2. OBJECTION

My main problem with Quine’s taxonomy is its heavy dependence on the current state of one’s knowledge and on one’s ability to figure things out. Let me explain.

All a valid deductive argument shows, just in virtue of its validity, is that a certain set of propositions is internally inconsistent. If all we know about an argument

P1
P2
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.
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———
∴ C

is that it is valid, we do not thereby have even the slightest reason to believe that the conclusion C is true, even though doubtless the person who constructed the argument was reasoning, from P1, P2,... taken as premises, to C. For unless we are independently moved to accept those premises, their jointly implying C is of little interest; and even if we do already accept them, seeing that they imply C may make us reconsider one or more of them, rather than inclining us any the more strongly to endorse C as well.[4] In this sense, an argument has no intrinsic direction; its direction has been imparted to it rhetorically by a speaker who has recruited it for a particular dialectical purpose. Intrinsically the argument is just the inconsistent set {P1, P2,..., ~C} .

The relevance of that (I hope uncontroversial) point for Quine is that, faced with an apparently inconsistent set of plausible statements, anyone may choose which of the statements' denials to single out as "the proposition" or "conclusion" of the corresponding paradoxical argument, either arbitrarily or on a ground of some sort. (Remember that a paradox in Quine's sense has an already appalling conclusion, so there is not initially any epistemic reason, as opposed to expository reasons, why the conclusion appears as such rather than being negated and taken as a premise.) And given a paradoxical argument, two theorists might well disagree on whether the paradox is veridical, because they may disagree as to which component propositions are more plausible than which; in particular, one theorist may find the argument veridical while the other finds the "conclusion"'s denial more plausible than one of the "premises."

In Quine's examples of veridical and falsidical arguments, the comparison of plausibility is sufficiently obvious and uncontroversial that in fact no one would dispute his judgments about them. But we should bear in mind that that is because we all think roughly alike on issues of Leap Year and birthdays and barbers and arithmetic; a person who for whatever reason had different background beliefs and very different interests might resist Quine's judgments. In short, to classify a paradox as veridical is to assume that one's own preferred way of resolving the paradox is the correct way.

3. MY VIEW

More generally, then, a Paradox (I mark my own proposed usage with the capital letter) is an inconsistent set of propositions, each of which is very plausible.[5] And to resolve a Paradox is to decide on some principled grounds which of the propositions to abandon.

One might then think of saying that when that decision is comparatively easy, we could call the Paradox either veridical or falsidical, depending on which of the component propositions have been designated as "premises" and which has been denied by way of "conclusion," though when the choice of culprit is difficult or controversial, we call the Paradox an antinomy, as Quine does. But that translation of Quine's terminology into mine would not be accurate, for in regard to my notion of Paradox, Quine's "veridical" and "falsidical" are not natural opposites. A Paradox would count as veridical in the proposed new sense just in case (a) one of its members turns out to be clearly less plausible than the others and (b) the Paradox is already (for whatever reason) cast in the form of a deductive argument having the culprit's denial as its conclusion. But there would be no such thing as a falsidical Paradox, for a falsidical argument in Quine's sense is fallacious, and the apparent inconsistency corresponding to it is not real. In my terms, then, a falsidical paradox in Quine's sense is only an apparent Paradox.[6]

But on my view the "veridical"/"falsidical" distinction really loses its point. Rather, there are inconsistent sets containing identifiable culprits—call those "tractable" Paradoxes—and there are the antinomies. It is better just to drop Quine's terms.

There is a further complication. I have defined "Paradox" in terms of actual inconsistency, viz., as a set of propositions that is in fact inconsistent. But (a) a set can be inconsistent without anyone's knowing that it is or even being able to know that it is; and more importantly, (b) we may have reason to think that a set is inconsistent when actually it is not. Case (b), Quine's falsidical again, is not uncommon; we have some plausible premises and we use what seems to be a sound principle of reasoning to deduce our absurdity, but in fact the absurdity does not follow and the fault is in the principle of reasoning rather than in any of the premises. (De Morgan divides through by a number that is covertly equal to 0; elsewhere (Lycan (1992), (2001)) I have argued that when one is reasoning in English rather than in a truth-functional calculus, reliance on the rule Modus Ponens can lead from perfectly acceptable premises to contradictory conclusions and so must be rejected.)

Now, to circumvent this complication and keep the terminology neat, let us dispense with every even faintly dubious principle of reasoning. That is, let us require that for a Paradox to be worthy of its capital "P," it must be formulated truth-functionally, with every initially non-truth-functional principle of reasoning replaced by that principle's corresponding material conditional inserted as a member of the inconsistent set that constitutes the Paradox in question; thus we shall make every possibly controversial inference principle explicit. And, owing to the semantical completeness of the propositional calculus, any Paradox is provably as well as model-theoretically inconsistent.[7]

It must be conceded that in at least three ways, even the propositional calculus is "controversial." First, there is the question of relevant implication. Relevance logicians[8] brand the truth-functional calculus as libertine, charging that some of the inferences it sanctions (notably Disjunctive Syllogism) introduce informational irrelevancies and are therefore not ones that English speakers would or should make. But however we feel about this as a complaint against the analysis of English or any other natural language in truth-functional terms, it does not apply to my idea of reconstructing paradoxes into Paradoxes; I am here using the propositional calculus and its libertine notion of validity only as a tool for strict truth-preservation in inferential moves between propositions that are already formulated in the truth-functional idiom. Disjunctive Syllogism may not be a valid principle in the logic of English (any more than are Antecedent-Strengthening and Modus Ponens), but it is valid trivially and by definition for the tilde and the vel.

Second, paraconsistent systems have been offered as rivals to standard contradiction-shunning logic,[9] and in particular dialetheists led by Graham Priest (1987, 1995) have maintained that there are actually true contradictions. Indeed Priest cites some familiar paradoxes as examples; there is nothing to "resolve," because the contradictions simply are true. But this view is no opponent of my characterization of a Paradox. It merely takes a refreshing attitude toward Paradoxes once they are identified.

Third, Quine, Putnam and others have suggested that even elementary logic may be brought into question by exotic scientific developments such as in quantum mechanics. Once again we must distinguish between logical laws intended as representing the logic of English and the theorems of a formal system whose truth-theoretic semantics has been officially and stipulatively assigned. My notion of Paradox is tied to the latter, and has no implications regarding the former. But if(!) I understand quantum logicians correctly, they mean to impugn even standard propositional logic understood as formulated in terms of the traditional truth-defined connectives. Depending on one's view of analyticity,[10] and once we have distinguished the matter of epistemic revisability from the metaphysical issue of truth by virtue of stipulated meaning, this may not make sense. But even if it does and we are thereby forced to admit a notion of falsidical Paradox after all, at least my format will make the point of contention as explicit as anything could.

4. TWO IMPRESSIONISTIC OBJECTIONS

Two objections have been made to me; oddly (but fortunately) they oppose each other. The first[11] is that most or at least many people think of paradoxes as *arguments*, hence, contrary to my conception, as being intrinsically directional, from premises to conclusion.

It may be that many people do so; and that way of thinking is well represented by Quine's model. But I have pointed out that arguments themselves are not intrinsically directional, save by prior commitments of their proponents. To repeat, the speaker who has deployed the argument has chosen to present one or more propositions as its premises and infer another proposition as its conclusion, but that is rhetoric; we can equally argue from the "conclusion"'s negation to that of one of the "premises," and nothing about the argument itself tells us which should be preferred.

The second objection is that paradigmatic paradoxes are single sentences or statements such as the Liar, hence not inconsistent sets of propositions (and, n.b., not arguments either).[12]

Whether or not the Liar is paradigmatic, it is a single sentence rather than an inconsistent set, and it does suggest mild readjustment of our formula. I said that a Paradox is an inconsistent set of propositions "each of which is very plausible," but on its face the Liar cannot be so described. It can be expressed as an inconsistent set, {"(L) is true," "(L) is false," "(L) is either true or false but not both"}, but none of those three propositions has great intuitive appeal. What is true is at best that each of the propositions has a strong argument in its defense.[13]

Those three arguments have premises, such as "What (L) says is that (L) is false" and "If S says that P then S is true iff P," so a Paradox as I originally defined it would feature those undefended premises, not the lemmas as above. So, either we can say that the Paradox is that more complicated set, which is not very natural (also, we would get different versions depending on exactly how the ultimate premises were formulated), or we can liberalize the definition by replacing "each of which is very plausible" by "each of which either is very plausible in its own right or has a seemingly conclusive argument for it."

5. DENOUEMENT

If we stick by my notion of Paradox, we will regard the distinction between "tractable" Paradoxes and antinomies as theory-infected and somewhat presumptuous, but above all a distinction of degree. There are actually two matters of degree involved: the disparity in plausibility between a putative culprit proposition and the other, more plausible propositions in the set, and the average degree of plausibility all around. The higher both of these degrees go, the more readily we will see a Paradox as *in*tractable; we get a real antinomy when the first is near zero and the second is still high. If the second is low, we have only a mild Paradox, and I am loath to call it antinomic, but only an array of competing theories.[14]

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Notes

[1] "[Though] we are mildly surprised at being able to exclude the barber on purely logical grounds by reducing him to absurdity[,]...[e]ven this surprise ebbs as we review the argument; and anyway we had never positively believed in such a barber" (p. 12).

[2] Sainsbury (1988) roughly follows Quine: "This is what I understand by a paradox: an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises" (p. 1). But he thereupon agrees with me that paradox comes in degrees.

[3] Obviously there is a tradeoff here: Any falsidical paradox in Quine's sense can be turned into one of this new type by adding a plausible but false bridge premise that will make the argument valid.

[4] For very fruitful exposition and development of this point, see Harman (1987).

[5] This requires a small qualification, and will receive it in the next section.

[6] Strictly, Quine's text leaves it unclear whether the "fallacy" in a falsidical argument must always be an illicit inferential move. If the "fallacy" can be simply a false assumption taken as a premise, then falsidical Paradoxes can after all be represented in the suggested terminology: A Paradox will count as falsidical just in case (a) one of its members turns out to be clearly less plausible than the others and (b) the Paradox is already (for whatever reason) cast in the form of a deductive argument having the culprit as a premise.

[7] Someone might urge that we liberalize the present requirement by allowing standard first-order quantificational inferences to stand as they are; I would not resist that suggestion violently, but I prefer to err on the side of conservatism.

[8] Anderson and Belnap (1975), Routley et al. (1982).

[9] E.g., Arruda (1979), Priest et al. (1989).

[10] See Lycan (1994), Chs. 11 and 12.

[11] From Keith Simmons.

[12] This was put to me by Doug Kelly. Along the same lines, it may be said that the axioms of naïve set theory form an inconsistent set, but that naïve set theory cannot be called a "paradox." But I would reply that the axioms do constitute a paradox, for they lead ineluctably to Russell's. Despite being individually nearly undeniable on any intuitive reading of "class" or "collection," they cannot all be maintained, and the matter demands resolution in the form of denying one of the axioms and motivating that denial.

[13] You might think that since the argument for the first sentence and the argument for the second are obvious and impeccable, what we have is simply a refutation of the third: Barring Dialetheism, the Liar just show that (L) lacks truth-value and is a not very surprising exception to Bivalence. But as Simmons points out, what (L) says is that (L) is false, hence that (L) has truth-value; so if (L) does lack truth-value (L) is false; so (L) is true….

The Strengthened Liar is even more straightforwardly couched as a truth-functionally inconsistent set, {"(L) is true," "(L) is not true"}.

[14] Thanks very much to Keith Simmons and to Doug Kelly for useful discussion.