- Second Fundamental Theorem of Calculus
- Mean Value Theorem for Integrals

Second Fundamental Theorem of Calculus

Let f be continuous (hence integrable) on $[a, b]$, and let F be the antiderivative of f on $[a, b]$. Then

$$\int_a^b f(x)dx = F(x)\big|_a^b = F(b) - F(a)$$

Evaluate the following integrals.

1. $\displaystyle\int_v^w k\,dx$

2. $\displaystyle\int_a^b 2x\,dx$

3. $\displaystyle\int_1^4 \frac{1}{w^2}\,dw$

4. $\displaystyle\int_0^1 (x^2+1)^{10}(2x)\,dx$

5. $\int_0^{\pi/2} \cos^2 x \sin x \, dx$

6. $\int_1^x t^3 \, dt$

7. Find $D_x \int_0^x \sin x \, dx$ in two different ways.

8. Express the following as definite integrals. (This time you have to recognize the interval.)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^2 \frac{3}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\sin\left(\frac{\pi i}{n}\right)\right] \frac{\pi}{n}$$

Mean Value Theorem for Integrals

If f is continuous on $[a, b]$, there is a number c between a and b such that

$$\int_a^b f(t)dt = f(c)(b-a)$$

Geometrically this means that there is value c in the interval of integration such that the rectangle with base $(b-a)$ and height $f(c)$ has the same area as the area under the curve of $f(x)$ from a to b. $f(c)$ is called the mean value. (Finding this value from a graph of f is not as easy and finding the corresponding value guaranteed by the MVT for Derivatives.) However, finding the value algebraically is fairly simple.

Find the value of c guaranteed by the MVT for Integrals for the following functions.

1. $f(x) = x^2; [-1, 1]$

2. $f(x) = |x|; [-1, 3]$

Review of MVT for Derivatives

Now that you have learned the MVT for both derivatives and integrals (MVTD and MVTI), you will have to be very careful to differentiate between the two and their geometric meaning.

The MVTD states that for a differentiable (Note: differentiable is not required for MVTI), there exists a value c in the interior of the interval such that the tangent line at c is parallel to the line connecting the functional points at the endpoints of the interval.

Determine the value guaranteed by the MVTD for $f(x) = c_1x^2 + c_2x + c_3$ on $[a, b]$.