Solutions to HW1

Section 1.6

6. Suppose $x$ and $y$ are two arbitrary odd numbers. There exists integer $n$ and $m$ such that $x = 2n + 1$, $y = 2m + 1$. So we have

$$xy = (2n + 1)(2m + 1) = 2(2mn + n + m) + 1.$$ 

$2(2mn + n + m)$ is an even number. Thus $xy$ is odd.

12. Let $r$ be a nonzero rational number and $x$ be an irrational number. Suppose $rx$ is rational. Then there exists four nonzero integers $a, b, p, q$, such that

$$r = \frac{p}{q}, \quad rx = \frac{a}{b}.$$ 

So

$$x = \frac{aq}{bp}.$$ 

This implies that $x$ is rational. We have a contradiction. Thus $rx$ must be an irrational number.

Section 2.1

8. (a) T
(b) T
(c) F
(d) T
(e) T
(f) T
(g) No answer. The set on the right hand side does not make sense since it has the same element repeated twice, and by definition, we do not repeat elements in a set.
28.

\[ A \times B \times C = \{ (a, x, 0), (b, x, 0), (c, x, 0), (a, x, 1), (b, x, 1), (c, x, 1), (a, y, 0), (b, y, 0), (c, y, 0), (a, y, 1), (b, y, 1), (c, y, 1) \} \]

\[ C \times B \times A = \{ (0, x, a), (0, x, b), (0, x, c), (1, x, a), (1, x, b), (1, x, c), (0, y, a), (0, y, b), (0, y, c), (1, y, a), (1, y, b), (1, y, c) \} \]

\[ C \times A \times B = \{ (0, a, x), (0, b, x), (0, c, x), (1, a, x), (1, b, x), (1, c, x), (0, a, y), (0, b, y), (0, c, y), (1, a, y), (1, b, y), (1, c, y) \} \]

\[ B \times B \times B = \{ (x, x, x), (x, x, y), (x, y, x), (y, x, x), (x, y, y), (y, x, y), (y, y, x), (y, y, y) \} \]