Solutions to HW7

Section 5.3

16. 
\[ C(10, 1) + C(10, 3) + C(10, 5) + C(10, 7) + C(10, 9) \]
\[ = 2C(10, 1) + 2C(10, 3) + C(10, 5) \]
\[ = 512 \]

18. (a) \(2^8\)
   (b) \(C(8, 3) = 56\)
   (c) 
   \[ C(8, 3) + C(8, 4) + C(8, 5) + C(8, 6) + C(8, 7) + C(8, 8) \]
   \[ = 2C(8, 3) + C(8, 4) + C(8, 2) + C(8, 1) + C(8, 0) \]
   \[ = 112 + 70 + 28 + 8 + 1 \]
   \[ = 219 \]
   (d) \(C(8, 4) = 70\)

Section 5.4

28. (a) Suppose that there are \(n\) items in one set and \(n\) items in a second set. Then the total number of ways to pick 2 elements from the union of these sets is \(C(2n, 2)\). Another equivalent way is to pick \(k\) elements from the first set and then \(2 - k\) elements from the second set, where \(k\) is an integer with \(0 \leq k \leq 2\). This can be done in \(C(n, k)C(n, 2 - k)\) ways, using the product rule. Hence, the total number of ways to pick 2 elements from the union also equals

\[ C(2n, 2) = C(n, 0)C(n, 2) + C(n, 1)C(n, 1) + C(n, 2)C(n, 0) = 2C(n, 2) + n^2. \]

This proves the statement.

(b) \(RHS = 2n(n - 1)/2 + n^2 = 2n^2 - n = 2n(2n - 1)/2 = C(2n, 2) = LHS\)
Section 5.5

15. (a) Let’s assume there are 5 buckets, each named $x_1$, $x_2, ..., x_5$ in which 21 balls are placed indistinguishably. First ball is put in $x_1$, then 20 more are dropped into all five remaining boxes, $x_1$ to $x_5$, with repetition allowed. Thus the number of possible solutions is “20-combination of 5 elements”. That is

$$C(24, 20) = C(24, 4) = 10626$$

(b) let’s place 2 balls in every bucket. Thus, only 11 balls remain. These 11 balls can be placed into the buckets $x_1$, $x_2, ..., x_5$ indistinguishably. So the number of possible solutions is “11-combination of 5 elements”. That is $C(15, 11) = C(15, 4) = 1365$

32. Take ”AAA” as one letter and then there are 6 position for letters. First we pick two positions for letter ”R” and there are $C(6, 2)$ ways. Then, we put the other 4 letters into the rest of 4 positions and there are $P(4, 4)$ ways. So, there are $C(6, 2)P(4, 4) = 360$ different strings.