Solutions to HW9

Section 6.3

6. Let $E$ be the event that a soccer player takes steroids, and $F$ be the event that a soccer player tests positive. So we have

$$P(F|E) = 0.98,$$
$$P(F|\bar{E}) = 0.12,$$
$$P(E) = 0.05,$$
$$P(\bar{E}) = 0.95,$$

$$P(E|F) = \frac{P(F|E)P(E)}{P(F|E)P(E) + P(F|\bar{E})P(\bar{E})} = 0.3.$$ 

16. a) Let $E_1$ be the event that Ramesh drives car, $E_2$ be the event that Ramesh takes bus, $E_3$ be the event that Ramesh rides bicycle, and $F$ be the event that Ramesh is late. So we have

$$P(F|E_1) = 0.5, P(F|E_2) = 0.2, P(F|E_3) = 0.05,$$
$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}.$$ Thus

$$P(E_1|F) = \frac{P(F|E_1)P(E_1)}{P(F|E_1)P(E_1) + P(F|E_2)P(E_2) + P(F|E_3)P(E_3)} = 0.67.$$ 

b) Because $P(E_1) = 0.3, P(E_2) = 0.1, P(E_3) = 0.6,$

$$P(E_1|F) = \frac{P(F|E_1)P(E_1)}{P(F|E_1)P(E_1) + P(F|E_2)P(E_2) + P(F|E_3)P(E_3)} = 0.75.$$ 

Section 6.4

6. The probability of that the ticket will win is $\frac{1}{C(50,6)}$, so the expected value is

$$(10^7 - 1) \times \frac{1}{C(50,6)} + (-1) \times \frac{C(50,6) - 1}{C(50,6)} = -0.37.$$ 

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10. Let $x$ be the number of times we flip. We stop when we have two tails or we have flipped 6 times, so we can discuss like follows:

When $2 \leq x \leq 5$, the last outcome must be tail and there is another tail before it. So we have

\[
P(x = 2) = \frac{C(1, 1)}{2^2} = \frac{1}{4},
\]

\[
P(x = 3) = \frac{C(2, 1)}{2^3} = \frac{1}{4},
\]

\[
P(x = 4) = \frac{C(3, 1)}{2^4} = \frac{3}{16},
\]

\[
P(x = 5) = \frac{C(4, 1)}{2^5} = \frac{1}{8}.
\]

When $x = 6$, there will be three kinds of cases.

Case 1: All are heads.

Case 1: One tail in the six outcomes of flipping.

Case 1: The sixth outcome is tail and another tail is before it.

So

\[
P(x = 6) = \frac{1 + 6 + 5}{2^6} = \frac{3}{16}.
\]

Thus

\[
E(x) = \sum_{i=2}^{6} iP(x = i) = 3.75,
\]

i.e. the expected number of times we flip the coin is 3.75.

42. Let $x$ be the number of balls in the first bin. We do $m$ independent Bernoulli trials and $p = \frac{1}{n}$.

\[
P(x = r) = C_m^r \left(\frac{1}{n}\right)^r \left(\frac{n-1}{n}\right)^{m-r}, 0 \leq r \leq m.
\]

So

\[
E(x) = m \times \frac{1}{n} = \frac{m}{n}.
\]