1. Attempt all problems. Show all work.
2. The exam is 75 minutes long.
3. All parts carry equal weight.
4. The exam is open book, open notes.
5. Laptops, calculators are allowed.
6. No communication about the exam is allowed with anybody except me.

P1. Let $p$ and $q$ be two propositions. The “exclusive or” of $p$ and $q$, denoted by $p \oplus q$, is the proposition “$p$ or $q$ but not both”. In other words, $p \oplus q$ is true if exactly one of $p, q$ is true and false otherwise.

(a) Show that $(p \oplus q) \lor (p \oplus \neg q)$ is a tautology, that is, this compound proposition is always true.

(b) Use truth tables to prove or disprove that $\oplus$ satisfies the associate law, that is, $(p \oplus q) \oplus r$ is logically equivalent to $p \oplus (q \oplus r)$.

P2. State the negation of each of the following statements if the domain consists of all real numbers. For each, state which is false, the given statement or its negation. Justify your answers.
(a) $\exists x (x^3 = -1)$
(b) $\forall x (2x > x)$

P3. Prove that if $n$ is an integer and $3n + 2$ is even, then $n$ is even, using
(a) a proof by contraposition,
(b) a proof by contradiction.

P4. Let $A$, $B$ and $C$ be three sets such that
$$A \cap B \cap C = \emptyset, \quad A \cap B \neq \emptyset, \quad B \cap C \neq \emptyset, \quad A \cap C \neq \emptyset.$$ 
(a) Draw the Venn Diagram showing these three sets.
(b) Draw the Venn diagram of the set of all elements that belong to exactly one of the three sets \(A, B\) and \(C\).

P5. (a) Compute

\[
\sum_{j=100}^{200} (j + j^3).
\]

(b) Show that

\[
\sum_{i=1}^{n} \sum_{j=1}^{i} i(2j - 1) = \left( \frac{n(n+1)}{2} \right)^2.
\]

**Hint:** The formulas in Table 2 on page 157 may be useful.