Homework 13
Solutions

This is an $M/M/1$ queue with $\lambda = 1$ per hour, $\mu = 20/24 = 5/6$ per hour, $K = 10$. The machine is off whenever the warehouse is full. The long run fraction of the time the machine is off is given by $p_{10}(10) = .1926$.

2. Computational Exercise 6.16.
This is an $M/M/K/K$ queue with $\lambda = 60$, $\mu = .75$, $K = 75$. The fraction of cars turned away is given by $p_{75}(75) = .1256$.

3. Computational Exercise 6.35.
Let $T$ be a typical service time. We are given

\[ P(T = 2) = .5, \quad P(T = 3) = .2, \quad P(T = 5) = .3. \]

Hence $\tau = 2 \cdot .5 + 3 \cdot .2 + 5 \cdot .3 = 3.1$ minutes and $s^2 = 4 \cdot .5 + 9 \cdot .2 + 25 \cdot .3 = 11.3$ minutes$^2$. Using Equation 6.39 with $\lambda = 18/60 = \text{per minute}$, we get $L = 8.1943$.

Consider the queue in front of server 1. It is a $G/M/1$ queue with iid $\text{Erl}(2,10)$ inter arrival times, and $\text{Exp}(6)$ service times. The traffic intensity is $\rho = (10/2)/6 = 5/6 < 1$. The functional equation, using the results of Example 6.14, becomes

\[ u = \left( \frac{10}{10 + 6(1 - u)} \right)^2, \]

with the required solution $\alpha = .7822$. Using Equation 6.42 we get $L_1 = 3.8259$. Similarly $L_2 = 3.8259$.

This is a $G/M/1$ queue with constant inter arrival times with mean 1 hour, exponential service times with mean 24/30 hours. Hence $\lambda = 1$, $\mu = 30/24 = 1.25$. Hence the traffic intensity is $\rho = \lambda/\mu = .8$. The functional equation is $u = e^{-1.25(1-u)}$, with the solution given by $u = \alpha = .6286$. Using Equation 6.42 we get $L = 2.1540$. 

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