

# Corrections to the Second Edition of Modeling and Analysis of Stochastic Systems

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## Chapter 2

1. Page 11, Equation (2.5): insert “,” after “)”.
2. Page 15, line 15 from below: the entry  $P_{0,1}$  should be  $2p_d(1 - p_d)$ , and  $P_{0,2}$  should be  $(1 - p_d)^2$ .
3. Page 16, line 5: replace “ $X_n$ ” by “ $Z_n$ ”.
4. Page 16, line 7: replace “ $Z_n$ ” by “ $Z_k$ ”.
5. Page 25, line 10: replace “exiting” by “existing”.
6. Page 30, line 3 from below: replace “serves” by “serve”.
7. Page 38, line 13 from below: replace “ $D = XAX^{-1}$ ” by “ $A = XDX^{-1}$ ”.
8. Page 39, line 12 from below: replace “ $x_i$ ” by “ $|x_i|$ ”.
9. Page 46, line 6 of Exercise 2.20: replace “ $i$ ” by “ $k$ ”.

## Chapter 3

1. Page 56, line 7 from below: replace “between between” by “between”.
2. Page 62, line 2 of Example 3.7: replace “form” by “from”.
3. Page 68, line 2: replace “ $v_1$ ” by “ $v_i$ ”.
4. Page 75, line 4 and 5: replace “ $\phi'(1)$ ” by “ $\phi'_i(1)$ ”.
5. Page 70, Line 5 from below: insert “is” before “tossed”.
6. Page 74, line 15: replace
$$E(z^T | X_0 = i, X_1 = 0) = 1.$$
by
$$E(z^T | X_0 = i, X_1 = 0) = z.$$
7. page 75, Equation 3.31: replace  $\phi^{(k)}$  by  $\phi^{(k)}(1)$ .
8. Page 90, line 2 of Definition 4.4: insert “ if ” after “closed”.

## Chapter 4

1. Page 91, Figures 4.1: The label on the arc (1,2) should be  $1 - \alpha$  and on the arc (2,1) should be  $1 - \beta$ .
2. Page 91, Figures 4.2: The label on the arc (2,2) should be  $1 - \beta$ .
3. Page 91, line 2 from below: replace “ $C_1 = \{1\}$ ” by “ $C_1 = \{0\}$ ”.
4. Page 92, line 8 from below: replace the second “ $n$ ” by “ $m$ ”.
5. Page 92, Figure 4.3: the labels on the nodes should be  $0, 1, 2, i - 1, i, i + 1, N - 1$  and  $N$ .
6. Page 93, Example 4.8: Replace

$$P \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

by

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

7. Page 94, line 9: replace  $\frac{1}{(n+1)(n+2)}$  by  $\frac{1}{n(n+1)}$ .
8. Page 94, line 13: replace  $\frac{1}{(n+1)(n+2)}$  by  $\frac{1}{n(n+1)}$ .
9. Page 94, replace line 15 by

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

10. Page 94: replace lines 17, 18, 19 by

$$\begin{aligned} \tilde{m}_0 &= \sum_{n=0}^{\infty} n \mathbb{P}(\tilde{T}_0 = n | X_0 = 0) \\ &= \sum_{n=1}^{\infty} \frac{n}{n(n+1)} \\ &= \sum_{n=0}^{\infty} \frac{1}{n+1} = \infty. \end{aligned}$$

11. Page 97, Theorem 4.7 (ii): replace “null transient” by “null recurrent”.
12. Page 98, line 12 from below: replace “finite” by “a communicating class”.
13. Page 99, line 1: Replace “The” by “Then”.
14. Page 99, line 2: delete “visits”.
15. Page 102, line 5 from below: replace “=” by “ $\leq$ ”.
16. Page 103, Equations 4.16 and 4.17: replace “ $\nu(i)$ ” by “ $d(i)$ ”.
17. Page 103, line 3 from below: replace “ $y_i(r+1)$ ” by “ $y_i^{(r+1)}$ ”.
18. Page 104, line 3: replace “ $\leq$ ” by “=”.
19. Page 104, line 5: replace “ $y_i^{(r+1)}$ ” by “ $y_i^{(n+1)}$ ”.
20. Page 104, line 14 from below: insert “positive” before “recurrent”.
21. Page 105, line 4 of Theorem 4.12: replace “ $>$ ” by “ $<$ ”

22. Page 107, Equation 4.23: replace “ $n = 1$ ” in the sum by “ $n = 0$ ”.
23. Page 107, last line: replace “ $n \geq 0$ ” by “ $k \geq 0$ ”.
24. Page 109, line 13: replace “ $\mathbb{P}(\tilde{T}_j = m)$ ” by “ $\mathbb{P}(\tilde{T}_j = m | X_0 = j)$ ”.
25. Page 109, line 14: replace “ $\tilde{u}_j(m)$ ” by “ $u_m$ ”.
26. Page 110, line 7: replace “ $\nu_n$ ” by “ $nu_n$ ”.
27. Page 113, line 5: insert “(by using the bounded convergence theorem)” after “get”.
28. Page 114, Equation 4.31: replace “ $p_{jj}^{(nd+r)}$ ” by “ $p_{ij}^{(nd+r)}$ ”.
29. Page 114, Equation 4.32: replace “ $M_{ij}(n+1)$ ” by “ $M_{ij}^{(n+1)}$ ”.
30. Page 116: last line: replace “two” by “three”.
31. Page 117, line 5 from below: insert “is” after “This”.
32. Page 118: line 5: replace “analytical examples” by “analytical solutions”.
33. Page 118, line 4 in Example 4.21: replace “positive recurrent” by “irreducible”.
34. Page 119, the line above Example 4.22: replace “is” by “has”.
35. Page 119, line 6 in Example 4.22: replace “recurrent” by “irreducible”.
36. Page 120, line 14: replace “positive recurrent” by “irreducible”.
37. Page 122, line 10: insert “unique” before “solution”.
38. Page 122, line 11: replace “an” by “and”.
39. Page 123, line 2 of Theorem 4.22: insert “non-negative” after “smallest”.
40. Page 124, line 8 from below: replace “ $\alpha_i(r)$ ” by “ $u_i(r)$ ”.
41. Page 124, line 6 from below: replace

$$\sum_{m=N+1}^{\infty} u_m - \alpha_i(r) \leq \epsilon/2$$

by

$$u_i(r) - \sum_{m=N+1}^{\infty} u_m \leq \epsilon/2$$

42. Page 124, last line: replace “the help of several examples” by “an example”.
43. Page 127, last line: replace “ $d(1 - p_u)$ ” by “ $d\alpha(1 - p_u)$ ”.
44. Page 128, line 4: replace “ $d(1 - p_u)$ ” by “ $d\alpha(1 - p_u)$ ”.
45. Page 124, line 9: replace “decision making” by “decisions”.
46. Page 129, line 14 from below: replace “ $\pi_2$ ” by “ $\pi_B$ ”.
47. page 129, line 6 from below: replace “DMTC” by “DTMC”.
48. Page 134, line 4 of exercise 4.14: replace “ $p_{o1}$ ” by “ $p_{01}$ ”.
49. Page 140, line 6: replace “east” by “least”.

50. Page 141, line 12: insert “)” after “ $X_{n+1}$ ”.
51. Page 142, line 8 from below: insert “recurrent” after “positive”.

## Chapter 5

1. Page 147, line 8 from below: insert “,” after “ $e^{-\lambda x}$ ”.
2. Page 150, line 14: replace “random  $t$ ” by “random  $T$ ”.
3. Page 151, line 7 from below: replace “ $exp$ ” by “exp”.
4. Page 152, line 2 of Theorem 5.5: replace “ $Erl$ ” by “Erl”. Page 153, line 2 from above, and line 3 from below: replace “ $X + 2$ ” by “ $X_2$ ”.
5. Page 152, line 4 from below: replace “ $n!$ ” by “ $(n - 1)!$ ”.
6. page 154, line 2 of Theorem 5.7: replace “ $G$ ” by “ $\mathbf{G}$ ”.
7. Page 157, line 3 from below: delete “a”.
8. Page 158, line 5 of Theorem 5.11: replace “ $k_n - 1$ ” in the superscript by “ $k_{n-1}$ ”.
9. Page 160: replace “ $h^i$ ” in the second sum by “ $h^{i-1}$ ”.
10. Page 163, lines 2,3 and 4 of the proof of Theorem 5.14: replace “ $(t_i + dt_i)$ ” by “ $(t_i, t_i + dt_i)$ ”.
11. Page 163, Page 163, line 5 of the proof of Theorem 5.14: replace “ $(t_i - t_{i-1} + dt_i)$ ” by “ $(t_i - t_{i-1}, t_i - t_{i-1} + dt_i)$ ”.
12. Page 167: figure 5.5: The label on the y-axis on the bottom graph should be “ $N(t) = N_1(t) + N_2(t)$ ”.
13. Page 167, line 11 from below: replace “call” by “class”.
14. Page 168, line 10: replace “ $Z_n$ ” by “ $Z_1$ ”.
15. Page 168, line 16: replace “in the” by “in this”.
16. Page 169, line 8 from below: replace “ $R_n = i$ ” by “ $R_n = k$ ”.
17. Page 169, line 9 from below: replace “iid be” by “iid”.
18. Page 172, line 4 from below: insert “has” after “ $0\}$ ”.
19. Page 173, line 3: replace “ $G\cdot)$ ” by “ $G(\cdot)$ ”.
20. Page 177, Theorem 5.21: Replace
 
$$\frac{\Lambda^n}{n!}$$
 on the right hand side by
 
$$\frac{\Lambda^{n-1}}{(n-1)!}$$
21. Page 177, last line: replace “call” by “called”
22. Page 179, line 4: replace “ $(sZ_1$ ” in the middle term by “ $s(Z_1$ ”.
23. Page 182, line 3: replace “lifetime” by “lifetimes”.
24. Page 184, lines 6 and 7: replace “minute” by ‘hour” (two instances).
25. Page 184, line 9: insert “a” after “is”.

26. Page 187, line 9 from below: replace “.” by “...”.

## Chapter 6

1. Page 205: line 3 of Theorem 6.4: replace “respect  $t$ ” by “respect to  $t$ ”.
2. Page 209, line 15 from below: replace “had” by “hand”.
3. Page 210, line 3 in Theorem 6.7: replace “ $M$ ” by “ $N$ ”.
4. Page 210, line 2 from below: replace “ $m$ ” by “ $N$ ”.
5. Page 211, line 4 of Example 6.18: replace the (0,0)th entry of  $X^{-1}$  by “ $\frac{\mu}{\lambda+\mu}$ ”.
6. Page 214, line 2 above Equation 6.27: delete “.” before “Now”.
7. Page 216, last line: replace “computed by Equation 6.28” by “defined in Theorem 6.10”
8. Page 227, line 2: remove all four large parentheses.
9. Page 230, line 14 from below: replace “ $t \geq 0$ ” by “ $t \geq Y_1$ ”.
10. Page 230, line 12 from below: replace “ $t \geq 0$ ” by “ $t \geq Y_1$ ”.
11. Page 230, line 10 from below: replace “ $n \geq 0$ ” by “ $n \geq 1$ ”.
12. Page 233, line 1: replace “ $p_i = 1/(i + 2)$ ” by “ $p_i = (i + 1)/(i + 2)$ ”.
13. Page 237, last line: replace “ $n \rightarrow \infty$ ” by “ $t \rightarrow \infty$ ”.
14. Page 242, line 5: insert “ death parameters ” before “ $\mu_i$ ”.
15. Page 244, line 2: replace “ $p_{i,n+1}$ ” by “ $p_{1,n+1}$ ”.
16. Page 256, line 12: replace “DTMC” by “CTMC”.

## Chapter 7

1. Page 279, line 7 from below: replace “sections” by “section”.
2. Page 280, line 8 from below: replace “7.31” by “7.1”.
3. Page 282, line 8: insert “are” after “there”.
4. Page 282, line 12 from below: replace “ $i + 1$ ” by “ $i$ ” and “ $i + 2$ ” by “ $i + 1$ ”.
5. Page 283, line 3: replace “Once” by “One”.
6. Page 284, line 6 from below: replace “ $j$ ” by “ $j$ ”.
7. Page 285, line 3 from below: replace “ $p_{kj}$ ” by “ $b_{kj}$ ”.
8. Page 286, line 5: replace “213 on page 6.25” by “6.25 on page 213”.
9. Page 286, line 16 from below: delete “we have,”.
10. Page 288, line 2 from below: replace “for a” by “for an”.
11. Page 290, line 11: replace “for an” by “for a”.
12. Page 290, line 14: replace “Little Law” by “Little’s Law”.

13. Page 291, line 17: replace " $1_{S_n \leq t < D_n}$ " by " $1_{\{S_n \leq t < D_n\}}$ ".
14. Page 296, line 5 from below: replace " $n\mu$ " by " $n\mu,$ ".
15. Page 297, line 9 in Section 7.3.5: replace " $(N - n)$ " by " $\min(N - n, s)$ ".
16. Page 297, line 11 in Section 7.3.5: replace " $\min(n, s)$ " by " $n$ ".
17. Page 301, line 10: insert "given" after "is".
18. Page 303, Figure 7.4: the return arc label should be " $1 - \alpha$ ".
19. page 303, line below Figure 7.4: insert "(" before " $\lambda$ ".
20. Page 309, lines 2 and 4 above Example 7.11: replace " $G(K)$ " by " $G_N(K)$ ".
21. Page 311, Figure 7.7: the label on the arc from CPU to Printer should be " $\alpha$ " and that on the arc from CPU to the Disc Drive should be " $1 - \alpha$ ".
22. Page 315, line 7: insert "a" after "in".
23. Page 316, line 5 from below: replace "the  $n$ " by "the  $n$ th".
24. Page 319: line 1 of Example 7.15: replace "service" by "inter-arrival".
25. Page 321, Figure 7.8: The caption in the lower box should read

Single Server  
No Waiting Space

26. Page 325, line 3 of Exercise 7.1: replace "essentially" by "an".
27. Page 328, line 3 of exercise 7.3: replace "little's law" by "Little's Law".
28. Page 331, line 3 of exercise 7.29: insert "is" after "server".
29. Page 331, Figure 7.9: the label on the horizontal arrow out of the last node should be " $1 - p$ ".
30. Page 332, Figure 7.9: the labels on the horizontal arrows should be " $1 - p_1$ ", " $1 - p_2$ ", " $1 - p_{N-1}$ ", and " $1 - p_N$ ".
31. Page 333, part 2 of exercise 7.41: replace " $1 \leq j \leq K$ " by " $1 \leq i \leq N$ ".
32. Page 336, line 1 of Exercise 7.55: replace "7.12" by "7.13".
33. Page 336, line 2 of Exercise 7.56: replace "7.12" by "7.13".

## Chapter 8

1. Page 342, line 6: delete "0,".
2. Page 342, line 12: replace " $k_n$ " by " $k_2$ ".
3. Page 342, line 11 from below : replace the first " $k_n$ " by " $k_2$ ".
4. Page 342, line 5 from below: replace " $S_{k_n+1} \leq t_n + X_1$ " by " $S_{k_n+1} - X_1 \leq t_n$ ".
5. Page 343, line 11: replace " $G(0-) = 1$ " by " $G(0-) = 0$ ".
6. Page 344, line 3 from below: replace "which" by "whose probability is".
7. Page 347, line 6 from below: replace "infy" by "infinity".

8. Page 348, equation 8.12: replace “ $= \leq x$ ” by “ $\leq x =$ ”.
9. Page 352, line 12 from below: replace “ $S_n^* \leq S_n$ ” by “ $S_n \leq S_n^*$ ”.
10. Page 355, line 2: replace “ $\tilde{p}_k$ ” by “ $\tilde{p}_r$ ”.
11. Page 355, line 2: replace “ $\tilde{G}^k$ ” by “ $\tilde{G}^r$ ”.
12. Page 370, line 7: insert “in” before “the”.
13. Page 370, line 9 from below: replace “ $G$ ” by “ $F$ ”.
14. Page 373, Figure 8.7: replace the label “ $Z(t)$ ” by “ $X(t)$ ” on the vertical axis.
15. Page 375, the lower part of Figure 8.8: replace the label “ $Z(t)$ ” by “ $X(t)$ ” on the vertical axis.
16. Page 377, last line: insert “ $+P(U_1 > t)$ ” before the “,”
17. Page 378, line 3: insert “ $+ \lim_{t \rightarrow \infty} P(U_1 > t)$ ” before the “.”
18. Page 378, line 15 from below: replace “chapter” by “section”.
19. Page 378, line 6 from below: insert “ $= G_{ii}(\infty)$ ” after “ $p_{ii}$ ”.
20. Page 380, line 14 from below: replace “ $S_1$ ” by “ $Y_1$ ”.
21. Page 395, Figure 8.10: replace “ $Z(t)$ ” by “ $X(t)$ ” as the label for the vertical axis.
22. Page 400: replace “Compute” by “Derive” in the first line of Exercises 8.26, 8.27, 8.28 and 8.29.
23. Page 401, Exercise 8.39: replace “An” by “A” in parts 2 and 3.
24. Page 401, last line replace “ $E(\min(U + D, t))$ ” by “ $E(\min(U + D, t))$ ”.
25. Page 404, line 12 of Exercise 8.51: replace “ $\phi_k(s)$ ” by “ $\mu/(s + \mu)$ ”.

## Chapter 9

1. Page 413, last line: replace “min” by “sup”.
2. Page 418, line 4: replace “ $\leq$ ” by “ $=$ ”.
3. Page 424, line 12 from below: replace “implicity” by “implicitly”.
4. Page 425, lines 5 and 6 in Theorem 9.15: replace “ $j \in S$ ” by “ $j \in I$ ”.
5. Page 426, Equation 9.35: replace “ $X(t) = k$ ” by “ $Z(t) = k$ ”.
6. Page 427, line 3: replace “ $\tau_{rr}$ ” by “ $\tau_r$ ”.
7. Page 427, line 11 from below: replace “ $P(X(t) = j)$ ” by “ $P(X(t) = j | X(0) = i)$ ”.
8. Page 428, lines 7 and 8: On each line insert “to” after “ $S_{n+1}$ ”.
9. Page 430, line 7: replace “ $s$ ” by “ $u$ ”.
10. Page 430, line 15: replace “ $\alpha_k$ ” by “ $\alpha_{kj}$ ”.
11. Page 432, line 3: replace the right hand side by “ $1 - e^{-2\mu x}$ ,”
12. Page 432, line 14: replace “ $\tilde{A}(\lambda)$ ” by “ $\tilde{A}(\mu)$ ”.
13. Page 433, line 8: insert “death parameters” before “ $\mu$ ”.

14. Page 433, Theorem 9.20: replace “process” by “death”.
15. Page 435, line 14 from below: insert “ be ” after “ $Z(t)$ ”.
16. Page 436, lines 14 and 15: replace “ $1_{Z(t)=j}$ ” by “ $1_{\{Z(t)=j\}}$ ”.
17. Page 441, line 3: replace “(1, 2)” by “( $i = 1, 2$ )”.

## Chapter 10

1. Page 446, line 14: delete “part 1 of”.
2. Page 446, line 5 from below: insert “finite dimensional ” after “the ”.
3. Page 447, lines 9 and 11: replace “]” by “] $^T$ ”.
4. Page 447, line 15: insert “ = ” before “0”.
5. Page 453, lines 10 and 11: replace “ $a$ ” by “0”.
6. Page 462: the four lines above Theorem 10.17 should not be in italic.
7. Page 465, line 1: replace “ $B(u)$ ” by “ $X(u)$ ”.
8. Page 466, line 6: the short “|” should be longer.
9. page 467, line 13: the long “|” should be shorter.
10. Page 471, line :16: replace “is the” by “is”.
11. Page 472, line 3 of Definition 10.9: replace “ $X(u)$ ” by “ $B(u)$ ”.
12. Page 473, line 3: replace “ $X(u)$ ” by “ $B(u)$ ”.
13. Page 483, line 17: delete “and” after “American”.
14. Page 483: line below Equation 10.69: replace “Theorem 10.27” by “Theorem 10.26”.
15. Page 485, line 1: replace “ $b(t)$ ” by “ $b(t)$ ”.

## Appendices

1. Page 491, last line: replace “ $P(\phi) = 1$ ” by “ $P(\phi) = 0$ ”.
2. Page 494, line 6 from below: replace “function” by “functions”.
3. Page 497, line 4 from below: delete “for”.
4. Page 499, line 11: replace “differential” by “continuous”.
5. Page 501, line 6 from below: replace
 
$$\mu = \begin{bmatrix} \mu 1 \\ \mu 2 \end{bmatrix}$$
 by
 
$$\mu = [\mu 1 \ \mu 2]$$
6. Page 501, line 3 from below: replace “ $(a - \mu 2)$ ” by “ $(a - \mu 2)^\top$ ”.
7. Page 503, line 6: replace “in” by “are given in”.
8. Page 505, line 6: replace “in” by “are given in”.
9. Page 513, line 6 from below: insert “.” after “ $k \geq 0$ ”.