Math 522 - Spring 09
Homework set 9 due March 26, 2009

1. Let $R \subset \mathbb{R}^2$ be a rectangle and suppose $B \subset \text{interior}(R)$ is a figure. Let $f : B \to \mathbb{R}$ be bounded and define $\tilde{f} : R \to \mathbb{R}$ by $\begin{cases} \tilde{f} = f \text{ on } B \\ \tilde{f} = 0 \text{ on } R \setminus B \end{cases}$. Show that $\tilde{f}$ is integrable on $R \Leftrightarrow f$ is integrable on B, and the two integrals are equal. (A similar statement is true in $\mathbb{R}^n$)

2. Suppose $g : [a,b] \to \mathbb{R}$ and $h : [a,b] \to \mathbb{R}$ are continuous functions such that $g(x) \le h(x)$ for all x and let $F = \{(x,y) : a \le x \le b,\ g(x) \le y \le h(x)\}$. By the result of an earlier HW problem we know F is a figure. Suppose $f : F \to \mathbb{R}$ is bounded and integrable on F. Suppose also that for each x, $f(x,y)$ is integrable on $\{y : g(x) \le y \le h(x)\}$. Let $\phi(x) := \int_{g(x)}^{h(x)} f(x,y)dy$. Show that ϕ is integrable on $[a,b]$ and that

$$\int_F f dV_2 = \int_a^b \phi(x)dx. \tag{1}$$

(A similar statement is true for higher dimensional integrals.)

3. Let C be a closed cube in $\mathbb{R}^n$ and suppose $f : C \to \mathbb{R}$ is bounded. Suppose also that f is continuous at all but finitely many points of interior(C). Show that f is integrable on C. (Clearly, more general domains and sets of discontinuities - e.g., a finite union of line segments, when $n > 1$ - can be allowed.)

From the text:

p. 209 #1 (note; the author's hint, given below Theorem 8.9, is not quite right), and #7.

Read: Theorem 8.12 and its proof.