



Lecture 2-4

- **Notations**
- **Systems of linear equations**
- **Derivatives:**
 - One variable
 - Two or more variables
- **Optimization:**
 - Unconstrained
 - Constrained



Notations

- **Constants/Parameters:**
 - a, b, c, d, e
- **Variables:**
 - x, y, z, u, v, w
- **Functions:**
 - $f(x)$, $g(y)$, $z(v)$
- **Differentiation:**
 - $f'(x)$, df/dx , dy/dx , $\partial y/\partial x$, f_x



Systems of linear equations

- **Method of deletion/subtraction:**

$$x + y = 2 \quad (1)$$

$$x + 3y = 6 \quad (2)$$

$$(2)-(1): 2y=4, \text{ so } y=2 \text{ and from (1) } x=0$$

- **Method of substitution:**

$$2x + 3y = 10 \quad (3)$$

$$3x + 2y = 10 \quad (4)$$

$$(3): x=5-1.5y, \text{ into (4): } y=2, \text{ from (3) } x=2$$

Derivatives

- **Definition:** rate of changes, e.g. slope
- **One variable:**
 - $f(x)=a$, $f'=0$; $f(x)=ax$, $f'=a$; $f(x)=\ln x$, $f'=1/x$
 - $f(x)=x^a$, $f'=ax^{a-1}$, where $a \in \mathbb{R}$
- **Two variables:**
 - Take partials with respect to one variable only: treat the other as a constant
- **Functions:**
 - $(f \pm g)' = f' \pm g'$, $(fg)' = f'g + g'f$, $(f/g)' = (f'g - g'f)/g^2$
- **Examples:**
 - $f = \ln(ax)$, $f' = 1/x$; $X = L^a K^{1-a}$, $\partial X / \partial L = aL^{a-1} K^{1-a}$



Optimization

- **Unconstrained:**
 - Choose x to max $f(x)$: $f'=0$ & $f''<0$ at x^*
 - If $f(x)=x-x^2$, then the solution is $x^*=1/2$
 - Choose (x,y) to max $f(x,y)$: $f_x=f_y=0$
- **Constrained:**
 - Choose x to max/min $f(x)$ s.t. $g(x)=0$
 - Substitute the constraint $g(x)$ into the objective function, $f(x)$
 - If $f(x)=x^2$, $g(x)=x-2$, then $x^*=2$
 - Choose x,y to max $f(x,y)$ s.t. $g(x,y)=0$
 - If $f=x^2+y^2$, $g=x+y-1$, then $x^*=y^*=1/2$