Rehabilitation and Juvenile Justice Policy: A Mathematical Modeling Approach

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Abstract

Youth in the juvenile justice system have diverse needs and problems. Several potentially effective behavioral, rehabilitative interventions have been developed, and evaluations of those programs have been conducted. Little is known, however, about how those programs might be integrated into a juvenile justice system, and a range of system-level questions remain unanswered. Using the tools of operations research, this article presents a mathematical model of juvenile justice involvement and examines the impact on the dynamics of offending when effective mental health services are introduced. Our analyses reveal that such programs generate a modest reduction in juvenile justice involvement in a community. Our findings suggest that optimal policy is one where incarceration is used as little as possible. Low-risk offenders are best returned to their communities; high-risk offenders are detained only when space in mental health services is available.

Keywords: operations research, juvenile justice, systems modeling, behavioral health

1. INTRODUCTION

Over the last three decades, rates of juvenile offending have fallen, and arrest rates
have dropped even more markedly since the early 1990s [1]. However, the organization and functioning of the juvenile justice system remains the subject of much debate and considerable change. Since changes introduced in 1985, the system has increasingly stressed criminalization and control of juvenile offenders.

To some extent, the tide may be shifting back to rehabilitative approaches. This trend is driven by evidence that punitive approaches have little impact on subsequent delinquency (e.g., [2]). Furthermore, research is emerging to show that juvenile justice involvement may have a variety of negative effects on involved youth. For example, the system may expose youth to other delinquent youth [3, 4] and may stunt their emotional development. At an age when most youth are learning to develop decision-making skills, confinement explicitly offers delinquent youth few choices [5, 6]. As a result, juvenile justice involvement may leave youth more at-risk than they were when they entered.

Emphasis on rehabilitation also has been fueled by new research on the complex needs of youth in the system. These youth have been exposed to a range of trauma [7], including sexual abuse [8, 9]. In a much-cited epidemiologic study of psychiatric disorders in youth in juvenile incarceration in Chicago, Teplin and colleagues find that two thirds of males and nearly three quarters of females met diagnostic criteria for one or more psychiatric disorders (other than conduct disorder) [10]. The majority of these youth suffered from multiple disorders [11]. These problems reflect a substantial need for mental health treatment among those in the juvenile justice system, but that system has little capacity for providing such treatment. Researchers at Columbia report that roughly 3.6% of the 1.9 million youth arrested with substance abuse and addiction problems receive any form of substance abuse treatment [12]. A national survey of directors of 141 juvenile institutional and community corrections facilities revealed that the number of youth attending substance abuse or mental health treatment on any given day was very low [13].

A range of programs and interventions target these problems. Among the more popular are juvenile drug courts. As of 2003, more than 400 communities were operating or planning a juvenile drug court [14, 15]. These courts integrate substance abuse and treatment with the overall goals of the court process. They monitor progress through drug testing and coordinate services by offering case management [14]. Research on adult drug courts suggests that those courts have high rates of treatment retention; low recidivism and drug use rates while participants are being treated; and reduced recidivism in the year following treatment [16]. At this time, however, only preliminary data are available on juvenile drug courts [17].

Other research is addressing the nature of effective treatment itself. For instance, Henggeler and colleagues have developed and promoted Multisystemic Therapy (MST) as a means of reducing antisocial behavior among youth involved in the juvenile justice system. It is implemented by master’s level therapists and targets factors at each level; it may include efforts to improve parent-child communication, parenting skills, relations with peers, performance at school, and social networks [18]. A series of randomized trials suggest that the program reduces long-term delinquency, but a recent meta-analysis questions program impact [19]. Another suggests that the program has moderate effects but only on family outcomes [20].
MST and drug courts are only two of several interventions and innovations being developed and evaluated. (Others include intensive aftercare [21] and behavioral therapy, [22].) What these programs all share is the hope that increasing treatment resources in the short term will generate longer-term cost savings, either in the juvenile justice system or elsewhere in publicly funded systems (such as spending on mental health services by the Medicaid program). A handful of studies have attempted to assess this question. Sheidow and colleagues, for example, consider whether MST generates future reductions in Medicaid expenditures on mental health services [23]. Foster and colleagues consider whether improved mental health services can reduce subsequent juvenile justice involvement [24]

These evaluations, however, leave key policy questions unanswered. For example, these studies generally begin with a cohort of children and youth identified for the study and assign half to the putatively better, more expensive treatment and then follow them over time. Left unanswered are a range of system-level questions. For example, given that the system likely lacks the resources to send all youth to these services, how much of the system’s resources should be devoted to those services? Which youth should be sent? What will the overall impact on crime in the community be?

In order to provide some insights into these questions, this article employs an operations research approach. A substantial literature uses these tools for the analysis of various questions related to crime and justice. (For reviews of this literature see [25, 26].) Much of this work empirically models criminal careers or forecasts various “performance measures” under policy regimes. Other work uses simulation to conduct what-if analysis (e.g. JUSSIM model [27, 28]). Our work has much in common with these analyses, but reflects a different tradition within operations research. In particular, our work uses mathematics to identify the “optimal” decision or policy and thus an upper bound on what can possibly be achieved under a given set of conditions. Earlier analyses can yield comparisons of alternative policies but cannot produce the optimal policy. The approach here quickly identifies the optimal policy and how it relates to a range of considerations. The disadvantage is that the tools of optimization become intractable if models exceed a reasonable level of complexity.

So, while the tools of operations research (and methods inspired by them in a general way) have been used by criminal justice researchers, few articles address optimization. In his review, Blumstein [26]identifies only two examples of optimization in criminal justice research: Tragler et al. [29] and Blumstein and Nagin [30]. More specifically, Tragler et al. [29] model resource allocation decisions for the control of drug trafficking and consider minimization of “costs” related to drug use and control.

The second article, Blumstein and Nagin [30], is closest to our work. Like our formulation, the authors consider the relationship between a choice or control variable and an objective. Decisions about the former reflect a constraint. In Blumstein and Nagin [30], a policy maker choses an optimal proportion of convicted who are incarcerated and its link to the overall crime rate. That choice is governed by a constraint—the overall level of incarceration. While our study differs in the details, our work and that of Blumstein and Nagin [30] share a general philosophy of using
mathematical optimization for the purpose of exploration and getting insights on effective policies rather than obtaining the optimal policy, which might not have much practical value. In that light, we see our work as building on the foundation laid by their article.

As with any model, our model incorporates key assumptions. The goal is to provide a model simple enough that an optimal solution is tractable without ignoring key features of the processes being modeled. Some of our assumptions are incorporated in other research on offending. For example, like models of the incapacitation effect, our model assumes the intensity of offending does not vary with age (across the ages modeled) [31]. However, our model relaxes some assumptions found in other models. For example, the model employed here does begin to recognize the heterogeneity of offenders. Furthermore, it also recognizes that incarceration actually may leave offenders more likely to offend.

One advantage of a mathematical approach is that—unlike much empirical work—this approach requires that assumptions be made explicit. In doing so, we allow the reader to assess the relative advantages and disadvantages of the approach. Our approach is much like that used in health policy. Models of key aspects of health and health care service use and delivery also make simplifying assumptions. For example, a range of disease processes are modeled as Markov processes [32-34]. That process makes strong assumptions—e.g., that the current health state reflects only the state in the immediately preceding period. For example, a Markov model of depression assumes that an individual’s likelihood of depression depends only on their health last period and not preceding periods [35]. As typically implemented, these models also assume that the probabilities governing the movement among states do not vary across individuals. Both assumptions are clearly false, but those employing the model believe that the resulting model still captures the essence of the phenomena.

The model presented below includes a generic, effective rehabilitative mental health service that represents the juvenile drug courts or multisystemic therapy or some other intervention or service. We assume that treatment or intervention is effective—otherwise, questions about how to integrate such services into a juvenile justice system seems premature. We believe that there is considerable evidence that a range of approaches are effective. Our task here—and one which has received little if any attention—is how to incorporate these services into an existing juvenile justice system. To address these questions posed above, this article lays out a broader model of juvenile justice functioning. We solve the model for an optimal use of rehabilitation services and populate the parameters of that model using data from a large longitudinal study. We then calculate the optimal policy under alternative budget levels and examine the sensitivity of optimal policy to alternative values of key parameters.

2. METHOD
2.1 Conceptual Model
The model here represents the experiences of a single hypothetical individual in a population and assumes that this individual will remain in this population forever.
Obviously, this assumption is false, but it provides a way to represent the steady state of a dynamic system. Even though a single individual eventually will either die or desist from crime and delinquency (and thus leave the system), new offenders will replace the “old” offenders in time. Thus, our model aims to capture the effects of policy decisions over the long-run holding constant the budget and rehabilitation capacity.

**States of Delinquency:** We model youth in the community as being in one of two states. The first state is “low risk” - these are individuals who can either commit a serious or non-serious offense with certain probabilities. The second state is “high risk” -these are individuals who have recently committed serious crimes, such as interpersonal violence, or have recent involvement with the juvenile justice system. Our goal here is to capture a key feature of juvenile offending in the “real world”—a small percentage of offenders account for high percentage of offending [36, 37].

**Objective function:** The objective is to minimize a weighted average of time that the hypothetical individual spends in the low-risk and high-risk states with the weight for the high-risk state larger than the weight for the low-risk state. One can think of these individuals in the high-risk state as the high-risk youth who are responsible for the bulk of serious offenses in society.

**Movements among states:** In any community, arresting all offenders is impossible. Therefore a key parameter of our model is the probability that an offender is arrested (\( \alpha \)). When arrested, an offender can either be incarcerated or returned to the community. While less common in the adult system, the latter is quite common in the juvenile court system, where courts and judges retain a great deal of discretion [38]. Our model adds a third option, rehabilitative, mental health services. As discussed above, our analyses will determine how these services affect the functioning of the overall system over the long run.

Both rehabilitation and incarceration can affect individuals differently, depending on their history of offending. In order to capture these differing effects, the model includes two states each for rehabilitation (“rehabilitation, entering because of a serious offense” (RS) and “rehabilitation, entering because of a non-serious offense” (RN)) and incarceration (“incarceration, entering because of a serious offense” (IS) and “incarceration, entering because of a non-serious offense” (IN)). The key distinction between these states involves what happens to the youth when he/she leaves the incarceration or rehabilitation facility. These states are described in more detail below.

Our model involves six states identified above as \([L,H,RN,RS,IN,IS]\) as defined in Table 1. The individual spends some time in each state (the sojourn time) and then moves to another state or returns back to the same state according to certain probabilities. The movement of the individual within this system is also given in Figure 1.
Table 1: States of the SMP Model

<table>
<thead>
<tr>
<th>State</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Incarceration, most recently committed a serious crime</td>
<td>IS</td>
</tr>
<tr>
<td>2 Rehab, most recently committed a serious crime</td>
<td>RS</td>
</tr>
<tr>
<td>3 Incarceration, most recently committed a non-serious crime</td>
<td>IN</td>
</tr>
<tr>
<td>4 Rehab, most recently committed a non-serious crime</td>
<td>RN</td>
</tr>
<tr>
<td>5 Low-risk state in the community</td>
<td>L</td>
</tr>
<tr>
<td>6 High-risk state in the community</td>
<td>H</td>
</tr>
</tbody>
</table>

Figure 1: Movement of the Individual from State to State

The model described above is largely conceptual. To understand its functioning, one needs to specify mathematical functions that describe key features of the model. For example, one needs to specify a process governing the risk of offending; such a process could involve the risk of offending rising or falling (or remaining constant) over time. The next section provides this detail.

2.2 Mathematical Representation of Model

We model the movements of the hypothetical individual among different states as a semi-Markov Process (SMP) (see for example [39] for more on semi-Markov processes).\(^1\) We describe the mean duration and transition probabilities for the six

\(^1\)If a stochastic process is modeled as a semi-Markov process, the implicit assumption is that the Markov property (independence of the future evolution from the past given the present state) only holds at times when the system makes a transition, and as a result the evolution of the system state from one transition to the other can be
states as follows:

**Low-risk state** \((L)\): While in the low-risk state, the individual can commit either of two types of crimes: serious or non-serious. The probability that the individual does not commit any crime at all within one year of his transition to the “low-risk state” is denoted by \(P_0\). If the individual does not commit a crime, we assume that he transitions back to the “low-risk” state. We use \(P_N\) to denote the probability that the individual commits a crime within the one-year period and the first crime he commits is a non-serious crime. Similarly, we use \(P_S\) to denote the probability that the individual commits a crime within the one-year period and the first crime he/she commits is a serious crime. Note that \(P_0 + P_N + P_S = 1\).

If the individual commits a crime, he/she is arrested with probability \(\alpha\). If he/she is not arrested, he/she returns to the low-risk state (transition back to the same state). If he/she is arrested for a serious crime, he/she is sent to rehab (state RS) with probability \(x_{RS}\) or to incarceration (state IS) with probability \(x_{IS}\), or sent back to the community (to the high-risk state H) with probability \(x_{HS}\). If he is not caught after committing a serious crime, he moves to state \(H\). On the other hand, if he commits a non-serious crime and he is caught, he is sent to rehabilitation (state \(RN\)) with probability \(x_{RN}\), sent to incarceration (state \(IN\)) with probability \(x_{IN}\), or sent back to the community to the low-risk state \(L\) (transition back to the same state) with probability \(x_{LN}\). Finally, we use \(\mu_L\) to denote the mean time spent in the low-risk state \(L\) at each visit.

**High-risk state** \((H)\): While in the high-risk state, the individual either moves to the low-risk state or commits a serious crime. We use \(q_L\) to denote the probability that the individual transitions to the low-risk state before committing a serious crime and \(q_S\) to denote the probability that the individual commits a serious crime before making transition to the low-risk state. (Note that \(q_L + q_S = 1\).) If the individual commits a crime, he is caught with probability \(\alpha\). If he is not caught, he goes back to the high-risk state. If he is caught, he is sent to rehab (state RS) with probability \(x_{RS}\),

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described with a transition probability matrix. In our model, transition times do not necessarily correspond to times when the individual changes states. In order to be able to estimate the model parameters with the available data, we chose events that trigger transitions differently from state to state. (More details are provided within the text.) Although assuming Markov property at these transition times can be regarded as a limitation, our formulation is still quite flexible in the sense that we do not require the time that the individual spends in each state to have any particular form of probability distribution. For example, a standard continuous-time Markov chain would have required assuming exponential distributions, which would imply that Markov property holds at all times.
sent to incarceration (state IS) with probability $x_{IS}$, or sent back to the community (state H) with probability $x_{HS}$. Finally, we use $\mu_H$ to denote the mean time the individual spends in state H every time he makes a transition into it.

**Rehabilitation for those most recently having committed a non-serious crime (RN):** In rehabilitation, the individual who has most recently committed a non-serious crime spends a random amount of time, with mean $\mu_{RN}$ and once the treatment is over, he returns back to the community. With probability $V_{NL}$, he returns back to the low-risk group and with probability $V_{NH}$, he returns back to the high-risk group.

**Rehab, most recently committed a serious crime (RS):** The description of this state is the same as RN except that the mean time spent is $\mu_{RS}$ and the probabilities of going to states L and H are $V_{SL}$ and $V_{SH}$, respectively.

**Incarceration, most recently committed a non-serious crime (IN):** As with rehabilitation, once in incarceration, the individual who has most recently committed a non-serious crime spends a random amount of time with mean $\mu_{IN}$ and once his sentence is over, he returns back to the community. With probability $o_{NL}$, he returns back to the low-risk group and with probability $o_{NH}$, he returns back to the high-risk group.

**Incarceration, most recently committed a serious crime (IS):** The description of this state is the same as IN except that the mean time spent is $\mu_{IS}$, and the probabilities of going to states L and H are $o_{SL}$ and $o_{SH}$, respectively.

The probabilities $x_{RS}, x_{IS}, x_{HS}, x_{RN}, x_{IN}, x_{LN}$ are the decision variables. In our model they represent the probabilities for a single individual, but they also can be viewed as percentages. For example, $x_{RS}$ can be interpreted to be the percentage of arrested juveniles that are sent to rehabilitation among those who committed a serious crime.

### 2.3 Optimal Solution to the Model

As noted, our objective is to minimize a weighted average of the time the individual spends in the high- and low-risk states under a budget constraint by choosing the probabilities $x = (x_{RS}, x_{IS}, x_{HS}, x_{RN}, x_{IN}, x_{LN})$. Based on the descriptions of the states given above, transition probabilities that describe the movement of the individual from one transition to the next are given in Table 2.
Table 2: Transition Probability Matrix for the Discrete-Time Markov Chain

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>H</th>
<th>RN</th>
<th>RS</th>
<th>IN</th>
<th>IS</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>p_0 + p_R (1 - \alpha) + \alpha p_N x_{LN}</td>
<td>p_S (1 - \alpha) + \alpha p_S x_{HS}</td>
<td>p_N \alpha x_{RN}</td>
<td>p_S \alpha x_{RS}</td>
<td>p_H \alpha x_{IN}</td>
<td>p_S \alpha x_{IS}</td>
</tr>
<tr>
<td>H</td>
<td>q_L</td>
<td>q_S (1 - \alpha) + \alpha q_S x_{HS}</td>
<td>0</td>
<td>q_S \alpha x_{RS}</td>
<td>0</td>
<td>q_S \alpha x_{IS}</td>
</tr>
<tr>
<td>RN</td>
<td>\nu_{NL}</td>
<td>\nu_{NH}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RS</td>
<td>\nu_{SL}</td>
<td>\nu_{SH}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>IN</td>
<td>\omega_{NL}</td>
<td>\omega_{NH}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>IS</td>
<td>\omega_{SL}</td>
<td>\omega_{SH}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Let $F_i$ denote the long-run fraction of time that the individual spends in state $i$ where $i \in S$. Then standard SMP theory (see, e.g., [39]) reveals that

$$F_i(x) = \frac{\pi_i(x)\mu_i}{\theta(x)}$$

where $\theta(x) = \sum_j \pi_j(x)\mu_j$, $i \in S$ and $\pi_i(x)$ solve the following system of equations:

$$\pi_{IS} = \pi_L p_S \alpha x_{IS} + \pi_H q_S \alpha x_{IS}$$
$$\pi_{RS} = \pi_L p_S \alpha x_{RS} + \pi_H q_S \alpha x_{RS}$$
$$\pi_{IN} = \pi_L p_N \alpha x_{IN}$$
$$\pi_{RN} = \pi_L p_S \alpha x_{RN}$$
$$\pi_L = \pi_{R} o_{SL} + \pi_{RS} v_{SL} + \pi_{IN} o_{NL} + \pi_{RN} v_{NL} + \pi_L (p_0 + p_N (1 - \alpha) + \alpha p_N x_{LN}) + \pi_H q_L$$
$$\pi_H = \pi_{RS} v_{SH} + \pi_{IN} o_{NH} + \pi_{RN} v_{NH} + \pi_L (p_S (1 - \alpha) + \alpha p_S x_{HS}) + \pi_H (q_S (1 - \alpha) + \alpha q_S x_{HS})$$

Then, letting $\pi_L = 1$ and solving for other $\pi_i, i \in S$, we can determine $F_i(x)$ for $i \in S$. When deciding on the values of $x = (x_{RS}, x_{IN}, x_{HS}, x_{RN}, x_{IN}, x_{LN})$, there is one limitation. That is, they need to be chosen so that the expenditures remain below the budget available. We use $B$ to denote the budget available per person per unit day. Hence, the long-run average expenditure for a single individual should not exceed $B$. Costs are incurred when the individual is caught and sent to either rehabilitation or incarceration, and as long as he stays in rehabilitation or incarceration. We use $c_{RN}$ to denote the per unit day cost of keeping an individual who committed a non-serious crime in rehabilitation. In other words, $c_{RN}$ is the per unit day cost of an individual residing in the state $RN$. Similarly, we use $c_{IN}, c_{RS}$ and $c_{IS}$ to denote the per unit day costs of the individual being in states IN, RS and IS, respectively.
We denote the administrative and operational cost of sending an individual to rehabilitation and to incarceration by $d_R$ and $d_I$, respectively. Then, the long-run average expenditure per unit time $E(x)$ is given by

$$E(x) = c_{RN} F_{RN}(x) + c_{IN} F_{IN}(x) + c_{RS} F_{RS}(x) + c_{IS} F_{IS}(x)$$

$$+ d_R \left( \frac{\pi_{RN}(x)}{\theta(x)} + \frac{\pi_{RS}(x)}{\theta(x)} \right) + d_I \left( \frac{\pi_{IN}(x)}{\theta(x)} + \frac{\pi_{IS}(x)}{\theta(x)} \right).$$

In deriving the expression, we use the fact that $F_i(x)$ is the long-run average time that the individual spends in state $i$ and $\pi_i(x)/\theta(x)$ is the long-run average rate that state $i$ is visited (see, e.g., [39]).

Our objective is to minimize some weighted average of percentage of time that the individual spends in low-risk and high-risk states while requiring that the expenditure does not exceed the available budget. Suppose that $a_L$ and $a_H$ denote the weights corresponding to states $L$ and $H$, respectively. Then, our optimization problem is

$$\min a_L F_L(x) + a_H F_H(x)$$

subject to

$$E(x) \leq B$$

$$x_{RS} + x_{IS} + x_{HS} = 1$$

$$x_{RN} + x_{IN} + x_{LN} = 1$$

$$x_{RS}, x_{IS}, x_{HS}, x_{RN}, x_{IN}, x_{LN} \geq 0.$$

As a solution method, we used complete enumeration by discretizing the decision variables. To be more precise, we evaluated the objective function for different combinations of values for the decision variables by letting them change from 0 to 1 with increments of 0.01 and selected the combination that gives the largest value for the objective function among those that satisfy the constraints.

2.4 Estimates of Key Model Parameters: Data from the Fast Track Project

Key model parameters—such as sojourn times and transition probabilities—can be determined either from data or for a hypothetical community. For our analyses, we rely on data from a large longitudinal study, the Fast Track Project. Fast Track is a 10-year multi-cohort, multi-site longitudinal study of children at risk for emotional and/or behavioral problems [40-44]. The project includes an intervention to prevent long-term problems in children identified in kindergarten as “at risk”. Our focus here is on the comparison group and a random sample of children in those communities.

Three cohorts of children were recruited from participating schools in 1991, 1992 and 1993. Schools within the four sites were selected as high-risk based on crime and poverty statistics for the neighborhoods they served. Within each site, the schools were divided into two sets matched for demographics (size, percentage free or reduced lunch, ethnic composition), and the sets were randomly assigned to
intervention and control conditions.

Using a multiple-gating procedure for each of three annual cohorts, all 9,594 kindergarteners in these 54 schools were screened for classroom conduct problems by teachers. Those children scoring in the top 40 percent within cohort and site were then solicited for the next stage of screening for home behavior problems by the parents, and 91 percent agreed (n=3,274) [45]. The teacher and parent screening scores were then standardized and combined into a sum score. Children were selected for inclusion into the study based on this sum score, moving from the highest score downward until desired sample sizes were reached within sites, cohorts, and conditions.

In addition, the study also collected a random sample of children in control schools within site according to child's gender, race, and decile scores on the teacher report measure. These children were to represent the not-at-risk population in those schools and were assigned to the non-high-risk, normative group. 1199 families participated in the baseline interview (445 in the intervention group, 446 in the control group, and 308 in the normative group), and 884 individuals participated in the study long enough to contribute the key measure of delinquency described below.

Three of the Fast Track sites are urban: Nashville; Tennessee; Durham, North Carolina; and Seattle, Washington. These three sites were 61% African American. The fourth site was in rural Pennsylvania, where 98% of the sample was Caucasian. The vast majority of participants were White or African American.

The key source of data was court records from study communities. These data were collected annually beginning in grade 6 and involved data culled from county court administrative files from which the youth lived (or surrounding counties if applicable). Permission to search juvenile court records was required for youth outcomes as well as administrative orders from local jurisdictions. The court records allowed us to code the date of offenses, adjudication, seriousness of offense and punishment, including time spent in incarceration. We used offense type to code offenses as serious or other. The latter were largely limited to acts of interpersonal violence, such as assault. The review included not only crimes that have been adjudicated but also crimes that have been diverted at the intake level.

The study was fairly effective in providing this information. We were unable to check court records for roughly 11% of the sample. Table 3 provides estimates of the key model parameters. Information on time spent in incarceration was derived from court records; cost from budgetary data (details in [46]). For rehabilitation, we used costs and duration of multisystemic therapy [23].

Our analyses here assume the rehabilitation services are effective. For the less-serious offenders, rehabilitation increases the chance of returning to the low-risk state from 75% to 90%. For serious offenders, rehabilitation raises the chance of returning to the low-risk state from 0% to 60%. These figures bracket the likelihood that an individual in the higher-risk state desists from serious offenses and moves to lower risk state (48%).
Table 3: Estimates of Key Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition ; Interpretation</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_i$</td>
<td>Mean time in each state (6 values; days)</td>
<td>57.19, 120.0, 45.82, 120.0, 279.48, 259.04</td>
</tr>
<tr>
<td>Probability of transition to State L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{SL}$</td>
<td>Serious offender released from incarceration</td>
<td>.00</td>
</tr>
<tr>
<td>$\nu_{SL}$</td>
<td>Serious offender released from rehabilitation</td>
<td>.60</td>
</tr>
<tr>
<td>$\sigma_{NL}$</td>
<td>Non-serious offender released from incarceration</td>
<td>.75</td>
</tr>
<tr>
<td>$\nu_{NL}$</td>
<td>Non-serious offender released from rehabilitation</td>
<td>.90</td>
</tr>
<tr>
<td>$d_R = d_I$</td>
<td>Transition cost; administrative and operational cost of sending and individual to rehabilitation or incarceration</td>
<td>$72</td>
</tr>
<tr>
<td>$c_{RN}, c_{IN}, c_{RS}, C_{IS}$</td>
<td>Per unit cost of time spent in rehabilitation and incarceration</td>
<td>$105.0, $83.33, $105.0, $83.33</td>
</tr>
<tr>
<td>$q_L$</td>
<td>When in high-risk state, probability of moving to low-risk state before committing a crime</td>
<td>.48</td>
</tr>
<tr>
<td>$q_H$</td>
<td>When in high-risk state, probability of committing a crime before moving to low-risk state</td>
<td>.52</td>
</tr>
<tr>
<td>$p_S$</td>
<td>Probability of committing a crime in low-risk state within one year and the first crime being serious</td>
<td>.144</td>
</tr>
<tr>
<td>$p_N$</td>
<td>Probability of committing a crime in low-risk state within one year and the first crime being non-serious</td>
<td>.286</td>
</tr>
<tr>
<td>$p_L$</td>
<td>Probability of committing no crime at all in low-risk state within one year</td>
<td>.570</td>
</tr>
</tbody>
</table>

3. RESULTS
We determine the optimal solution under several scenarios. Some of these scenarios include rehabilitation services as an alternative to incarcerations while others do not. For each scenario, we examine the optimal solution at alternative levels of juvenile justice expenditures: $100, $300, $600, $1000, and unlimited. The “unlimited case” is provided as a benchmark. For each model, we calculate the value of the minimand; in our baseline analyses, we place all weight on the high-risk state. As a result, we minimize the percentage of time each individual spends in the high-risk state. We determine the optimal value of the choice parameters and the amount of the budget expenditure that they imply. For our initial comparison of models, the probability of being arrested for a crime is set to 0.50. Later, we change this value for our sensitivity analyses.
3.1 The Baseline Model
Table 4 describes the performance of our basic model in the absence of rehabilitation services. The table suggests that the proportion of time a youth spends in the serious-offense state varies little if at all with changes in public policy--between 21% and 22%. The lower bound is reached once the budget reaches around $910 per youth; surprisingly, further increases in the budget produce no further reduction in this percentage.

One can also see that the amount of the budget expended never exceeds $910 per youth. Above that point, money is left unspent. The last column of Table 4 indicates that none of the less serious offenders are sent to incarceration. All are returned to the community, to the low risk state. The table also indicates that when the budget level is high, all the serious offenders are incarcerated. Increasing the budget does have a large impact on the disposition of serious offenders. At a very low budget ($100 per youth), only 10% of youth who commit serious crimes are sent to incarceration. As the budget increases, so does this percentage. One can see that this percentage reaches 100% near a budget of $910. As noted above, further increases in the budget are left unused--all serious offenders are detained, and sending the less serious offenders to incarceration is counter-productive.

Introducing Rehabilitation into the System: Table 5 presents the optimal solution for a model that includes rehabilitation services. Again, the probability of being arrested for an offense committed is constrained to 50%; the table provides the optimal solution for alternative budget levels. At low budget levels, 6% of serious-crime committers are sent to rehabilitation, and the remaining offenders are returned to the community. Increasing the budget increases the former to 42% at a budget of $600. However, as the budget increases still further, eventually all youth who commit serious crimes are sent to rehabilitation. All less serious offenders are returned to the community. It is interesting to note that once the option of rehabilitative services is available, no one is sent to incarceration.

Comparing the percentage of time spent in the high-risk state in Tables 4 and 5 does reveal that for any given budget, we do improve the objective function value by adding the rehabilitative services. In terms of overall system functioning, one can see that the percentage of time the youth spends in the serious offense state is reduced: at very low budget levels, the effect is tiny (21.70% v. 21.06%). Of course, this finding is sensible: the rehabilitation services are used so infrequently that the effect on the community is virtually nil. At higher budget levels the effect is greater (21.25% v. 14.21% at budget of $1000). Further increases in the budget have no effect - once all serious crime-committers are sent to rehabilitation, further budget increases have no impact.

Sensitivity Analysis: Higher Arrest Rates: Another key parameter of the model is the likelihood of "getting caught" - how likely are those offending to be arrested? We see in Table 6 that even for other levels of $\alpha$, it is still suboptimal to send low-risk non-serious offenders for incarceration or rehabilitative services. Again, as before, none of the high risk offenders are sent to incarceration when rehabilitation centers are present. As the available budget increases, more and more high-risk and low-risk serious offenders are sent to rehabilitation centers.
3.2 Sensitivity Analysis: A Modified Objective Function

A key feature of our model is the objective function. Recall that our objective function is

\[ a_L F_L(x) + a_H F_H(x) \]

To this point, we have assumed that society’s objective is to minimize the percentage of time youth spend in the high-risk state: we have set \( a_L = 0 \) and \( a_H = 1 \). Table 7 displays sensitivity analyses that calculate the optimal solution at alternative values of \( a_L \) for two different budget levels, $1,000 and infinite. When budget is $1,000, as the relative importance of the high-risk state decreases (\( a_L \) grows), the optimal policy extends rehabilitation services to the lower-risk individuals. The optimal policy does not change significantly as \( a_L \) increases from 0 to 0.9: the policies implied by the model are robust with respect to the penalties of being in low-risk and high-risk states.

Similar observations can be made for the case with infinite budget except when \( a_L \) changes from 0.1 to 0.4: in this range, the optimal policy involves extending rehabilitation to the less serious offenders. The optimal policy remains unchanged as long as \( a_L \) stays above 0.4 or below 0.1.

Table 4: Performance of the system for \( \alpha = 0.5 \) and without rehabilitation

<table>
<thead>
<tr>
<th>B</th>
<th>% time in state H</th>
<th>Expenditure</th>
<th>% Serious Offenders Released to Rehab</th>
<th>% Serious Offenders Released to Incarceration</th>
<th>% Non-Serious Offenders Released to Community</th>
<th>% Serious Offenders Released to Community</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>100</td>
<td>22%</td>
<td>$ 93</td>
<td>10%</td>
<td>90%</td>
<td>100%</td>
</tr>
<tr>
<td>$</td>
<td>300</td>
<td>22%</td>
<td>$ 296</td>
<td>32%</td>
<td>68%</td>
<td>100%</td>
</tr>
<tr>
<td>$</td>
<td>600</td>
<td>21%</td>
<td>$ 596</td>
<td>65%</td>
<td>35%</td>
<td>100%</td>
</tr>
<tr>
<td>$</td>
<td>1,000</td>
<td>21%</td>
<td>$ 910</td>
<td>Available</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>$</td>
<td>\infty</td>
<td>21%</td>
<td>$ 910</td>
<td>100%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 5: Performance of the system for \( \alpha = 0.5 \) when rehab treatment is available

<table>
<thead>
<tr>
<th>B</th>
<th>% time in state H</th>
<th>Expenditure</th>
<th>% Serious Offenders Released to Rehab</th>
<th>% Serious Offenders Released to Incarceration</th>
<th>% Non-Serious Offenders Released to Community</th>
<th>% Serious Offenders Released to Community</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>100</td>
<td>21%</td>
<td>$ 91</td>
<td>6%</td>
<td>0%</td>
<td>94%</td>
</tr>
<tr>
<td>$</td>
<td>300</td>
<td>20%</td>
<td>$ 295</td>
<td>20%</td>
<td>0%</td>
<td>80%</td>
</tr>
<tr>
<td>$</td>
<td>600</td>
<td>17%</td>
<td>$ 597</td>
<td>42%</td>
<td>1%</td>
<td>57%</td>
</tr>
<tr>
<td>$</td>
<td>1,000</td>
<td>14%</td>
<td>$ 993</td>
<td>76%</td>
<td>0%</td>
<td>24%</td>
</tr>
<tr>
<td></td>
<td>\infty</td>
<td>12%</td>
<td>$ 1,246</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B</th>
<th>% time in state H</th>
<th>Expenditure</th>
<th>% Serious Offenders Released to Rehab</th>
<th>% Serious Offenders Released to Incarceration</th>
<th>% Non-Serious Offenders Released to Community</th>
<th>% Serious Offenders Released to Community</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>100</td>
<td>21%</td>
<td>$ 91</td>
<td>6%</td>
<td>0%</td>
<td>94%</td>
</tr>
<tr>
<td>$</td>
<td>300</td>
<td>20%</td>
<td>$ 295</td>
<td>20%</td>
<td>0%</td>
<td>80%</td>
</tr>
<tr>
<td>$</td>
<td>600</td>
<td>17%</td>
<td>$ 597</td>
<td>42%</td>
<td>1%</td>
<td>57%</td>
</tr>
<tr>
<td>$</td>
<td>1,000</td>
<td>14%</td>
<td>$ 993</td>
<td>76%</td>
<td>0%</td>
<td>24%</td>
</tr>
<tr>
<td></td>
<td>\infty</td>
<td>12%</td>
<td>$ 1,246</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>
Table 6: Performance of the system at alternative levels of $\alpha$, the probability of being arrested for an offense

<table>
<thead>
<tr>
<th>$B$</th>
<th>$\alpha$</th>
<th>% time in state H</th>
<th>Expenditure (X)</th>
<th>% Serious Offenders Released to</th>
<th>% Non-Serious Offenders Released to Community</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rehab (X&lt;sub&gt;rs&lt;/sub&gt;)</td>
<td>Incarceration (X&lt;sub&gt;is&lt;/sub&gt;)</td>
</tr>
<tr>
<td>$100$</td>
<td>25%</td>
<td>21%</td>
<td>$99$</td>
<td>13%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>75%</td>
<td>21%</td>
<td>$91$</td>
<td>4%</td>
<td>0%</td>
</tr>
<tr>
<td>$1,000$</td>
<td>25%</td>
<td>17%</td>
<td>$689$</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>75%</td>
<td>14%</td>
<td>$998$</td>
<td>51%</td>
<td>0%</td>
</tr>
<tr>
<td>$2,000$</td>
<td>25%</td>
<td>17%</td>
<td>$689$</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>75%</td>
<td>9%</td>
<td>$1,704$</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 7. Performance of the system for weighting levels of low-risk state in objective function, $a_{L}$

<table>
<thead>
<tr>
<th>$a_{L}$</th>
<th>Expenditure (X)</th>
<th>% Serious Offenders Released to Community (X&lt;sub&gt;rs&lt;/sub&gt;)</th>
<th>% Non-Serious Offenders Released to Community (X&lt;sub&gt;cs&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>$993$</td>
<td>76%</td>
<td>24%</td>
</tr>
<tr>
<td>0.10</td>
<td>$993$</td>
<td>76%</td>
<td>0%</td>
</tr>
<tr>
<td>0.40</td>
<td>$993$</td>
<td>76%</td>
<td>0%</td>
</tr>
<tr>
<td>0.70</td>
<td>$993$</td>
<td>76%</td>
<td>0%</td>
</tr>
<tr>
<td>0.90</td>
<td>$997$</td>
<td>75%</td>
<td>0%</td>
</tr>
<tr>
<td>1.00</td>
<td>$1,000$</td>
<td>57%</td>
<td>0%</td>
</tr>
</tbody>
</table>

4. DISCUSSION

The model presented above highlights the possibilities of using operations research to improve public policy toward children and youth. Models that incorporate these tools are quite common in other areas of health policy. Such models have shed new insight on a range of topics, such as the timing and allocation of liver transplants [47, 48] and the treatment and prevention of HIV [49-51]. That potential has yet to be realized in research on the criminal justice system [52] and especially the juvenile justice system. These tools also could be used to examine other child-serving systems, such as child welfare, and the interactions among child-serving systems. Children and youth are involved in multiple systems and find their way through these systems, often without regard to how the systems interact (or not).

We determine the optimal solution for a set of parameters derived from simple analyses of longitudinal data. The resulting parameter estimates are illustrative but not
definitive. The data on which the parameter estimates depend are for high-risk communities—the sort of communities where an intervention like MST might be most appropriate. However, one could examine the policy using different parameters, including lower-risk communities or individuals. The same model would apply unless the underlying dynamics of offending differed.

Of course, one could estimate the optimal solution under alternative sets of parameter estimates. Our sensitivity analyses include such calculations, but others are possible and likely informative. One area for additional work involves the parameters representing the effectiveness of rehabilitation. Our estimates reflect the fairly strong assumption that the program is effective. Whether programs like MST can generate such large effects in the real-world is still largely unknown. Our exercise here was to examine the system-level implications of an effective intervention. It did not seem very useful to examine the behavior of such a system unless rehabilitation was indeed effective. While the model solutions based on these parameters were not definitive, they did seem sensible and offered some reassurance that the underlying parameter estimates were plausible.

Our findings suggest that optimal policy is one where incarceration is used as little as possible. Low-risk offenders are best returned to their communities; high-risk offenders are detained only when space in rehabilitative services are available. Policy makers should consider other programs and interventions that affect other aspects of the model. For example, in this model, the rate at which serious offenders move back to the lower-risk state is determined outside the model. Perhaps a community-based intervention might influence that rate and prove more effective in reducing overall delinquency.

4.1 Limitations

As with any model, our approach here relies on simplifying assumptions. Most prominent is that we model the experience of a single representative individual. Such a model captures some key elements of offending (e.g., heterogeneity of offenders) but omits others. For example, our model does not capture contagion effects where individuals influence each other, nor “replacement effects”. The latter describes the phenomena where the “supply” is not perfectly inelastic; new offenders may replace an offender who is arrested [53]. Our approach is quite atomistic—individuals make decisions independently.

Some of the assumptions that the model incorporates have been used in prior research. As noted, like models of the incapacitation effect, our model assumes the intensity of offending does not vary with age (across the ages modeled) [31]. In effect, we are modeling the system’s performance averaged over many individuals and ages. One could argue that this assumption is more troublesome in a study of juvenile offending, but there is a clear age profile in adult offending as well. We view these assumptions as highlighting areas for future research, which can examine the sensitivity of our findings to these assumptions.

Simulation might provide an alternative that would allow offending rates to vary with age and would relax some of the model’s other assumptions. Such an approach would permit us to model a more complex system that considers a population of
potential offenders and their interactions with each other. However, our objective in this article has been to compare possible outcomes under various scenarios, which necessitate the use of a relatively simple mathematical model that allows numerical optimization. Despite its obvious appeal, simulation is not an efficient optimization tool and therefore is impractical for answering some of the questions posed here. Mathematical models help us generate insights on the problems much more quickly, which can then be verified using elaborate simulation models. Such models might reveal the robustness of the findings we offer here.

Finally, another limitation of the analyses involves the data used to estimate model parameters. Of course, these data capture the experiences of only a limited number of youth in four communities, perhaps inaccurately. The conclusions suggested by our model might not apply to other communities or be invalid altogether if the data are poor quality. In that case, the model presented still has use as a target for future analyses and a framework for organizing further data collection.

4.2 Future research
Future research could better embed the model presented here in a community context. Further analyses of the current model offer the potential for the implications of model parameters (such as arrest rates) that vary across communities. Such a model would incorporate other characteristics of communities, such as opportunities for crime that vary across communities. Such a model also could incorporate other aspects of public policy, such as the presence of community-level interventions. A broader model also would incorporate arrest rate as determined within the system; indeed, a key policy issue not addressed here is the tradeoff between devoting more resources to policing (and presumably increasing the arrest rate) and the disposition of those arrested.

A future model might allow for changes in the budget and rehabilitation capacity relative to population size. The results of this article would apply as a population grew but only if the size of the budget and rehabilitation capacity—relative to the population—remained constant. A future model might allow these constraints to be policy choices as well.

Future research will incorporate new estimates of model parameters as they appear in the literature. The estimates of model parameters included here largely depend on a single study and speculation. Clearly, as better evidence emerges on the effectiveness of rehabilitative services—and their costs—the analyses presented here could be performed under alternative assumptions. Indeed, the model may be extended to capture the tradeoff between alternative forms of rehabilitative services.

An expanded model may further differentiate among offenders. A first effort in that regard would be to separate drug-related offenses. In a broader model, one form of rehabilitative services might be appropriate treatment for those offenders; another form might be better suited to other offenders. A future model might allow for additional heterogeneity, distinguishing not just levels of offenses but levels of risk and gender. In such a model, the parameters of the model may depend on individual characteristics that are observed before an individual commits an offense. It also may allow the criminal justice system to respond differently to different types of offenders. For example, the system appears to treat offenders of different races differently even
when their offense is the same [54].

While our parameter estimates were calculated for males, the same model might very well apply to females. As the proportion of juvenile offenders who are female increases, future work might determine the impact of policy choices for this population as well. As females increase as a percentage of those arrested, gender differences in both offending and the policy response will be increasingly important.

References


[44] Conduct Problems Prevention Research Group, "Fast Track Randomized Controlled Trial to Prevent Externalizing Psychiatric Disorders: Findings From


