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Labor Market Institutions and Business Cycles

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Abstract

Although there has been a significant progress in explaining the impacts of labor market institutions on macroeconomic performance, one aspect of this relationship still remains unexplored: their impacts on business cycles. We investigate these impacts by considering firing costs, unemployment benefits and workers' bargaining power within a DSGE framework. We derive the impulse responses under alternative parameterizations for these institutions and find that the firing costs do not have an impact on the responses. We show that higher unemployment benefits makes key macroeconomic variables less responsive to disturbances where higher bargaining power makes output more responsive and inflation less responsive to macroeconomic shocks.

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1 Introduction

Although there has been a significant progress in explaining the impacts of labor market institutions (LMI) on macroeconomic performance, one aspect of this relationship still remains unexplored: their impacts on business cycles. These institutions might have an impact on business cycles due to a number of reasons. First, Calmfors and Driffill (1988) and Rumler and Scharler (2009) claim that the characteristics of the wage bargaining process influence the responses of macroeconomic variables to economic shocks. The behaviors of the bargaining agents will influence the macroeconomic outcomes to the degree that they internalize the macroeconomic consequences of their actions. Second, these institutions play a very important role in determining the worker and job flows at the business cycle frequency. Therefore, LMI might influence the responses of the macroeconomic variables to the disturbances.[1] Third, Veracierto (2008) discusses that some of these institutions are introduced by the policymakers to reduce the magnitude of economic downturns. Finally, Abbritti and Weber (2008) argue that the economies that are characterized by high levels of LMI will experience smoother and more prolonged cycle compared to the economies characterized by low levels of LMI; because high levels of LMI amplifies the adjustment via prices and restrict the responses of real variables.

The main goal of this study is to investigate the impacts of LMI on business cycles; however, we have to take into account the fact that there are various institutions employed in different countries.[2] In the literature these institutions are collected under three main categories - employment protection regulations (e.g. firing costs, severance payments, lifetime employment), unemployment insurance benefits (e.g. duration of benefits, replacement ratio) and collective bargaining (e.g. coverage of collective bargaining, density, rules of bargaining, coordination between unions). In this paper, we have representatives of all of these categories. In particular, we look at the

[1] For some examples see Bentolila and Bertola (1990), Bertola (1990), Garibaldi (1998), Blanchard and Portugal (2001) and Pries and Rogerson (2005).

[2] For a list of these institutions, see Freeman (2008), page 3.

impacts of firing costs measured by severance payments, unemployment benefits and bargaining power of workers on business cycles.

In the literature there are a few theoretical studies that analyze the impacts of LMI on business cycles to different extents. Alvarez and Veracierto (2000) build a Lucas-Prescott equilibrium search model and investigate the role played by the labor market policies in explaining the differences in employment across economies. Joseph, Pierrard and Sneessens (2003) focus on the impacts of real wage rigidities as well as employment protection and unemployment benefits within a variation of an RBC model. Pries and Rogerson (2005) develop a matching model to account for the differences in the worker turnovers in the US and in the Euro Area. Veracierto (2008) employs an RBC model to analyze the effects of firing costs on cyclical fluctuations. All of these studies ignore inflation dynamics and nominal disturbances and some of them does not incorporate labor market frictions in their analyses. Bowdler and Nunziata (2007), Zanetti (2007) and Campolmi and Faia (2009) incorporate these features to their frameworks and show that labor market frictions play an important role in explaining the impacts of LMI on business cycles. Furthermore, Campolmi and Faia (2009) find that alternative institutions will have a role in explaining the differentials in inflation volatilities across economies.

This study contributes to this literature by building a dynamic stochastic general equilibrium search and match model by taking these findings into account. In particular, we build a variation of Mortensen and Pissarides model.[3] We extend the conventional MP model with staggered wage contracts and nominal price rigidities.

The closest studies to this one in terms of main question of interest and methodology are Zanetti (2007) and Abbritti and Weber (2008). The framework of this study differs from Zanetti (2007) in two ways. First, that study considers only nominal price rigidity, however in this study real wage rigidity is incorporated into the model. In the conventional model the wages are determined by period-by-period Nash bargaining between firms and workers. This results in a high volatility

[3] Due to Pissarides (1990), Mortensen and Pissarides (1994, 1999) and Pissarides (2000).

in real wage. To improve the empirical performance of the model Shimer (2005) and Hall (2005) include an *ad hoc* real wage stickiness to the conventional model. They find that sticky wages assumption improves the empirical performance of the model. Moreover, although beyond the scope of this study, our model is able to test the arguments of Abbritti and Weber (2008) and Joseph et al. (2003). The former study claims that real wage rigidities and labor market rigidities might have opposite effects on business cycle fluctuations. The latter study argue that downward rigidities, rather than LMI, may play a dominant role in explaining the cyclical properties of an economy. Secondly, we consider three institutions where Zanetti (2007) considers only firing costs and unemployment benefits. The difference between this study and Abbritti and Weber (2008) is that they do not focus on the institutions per se. They look at the impacts of real wage rigidities and labor market frictions on the business cycles and measure the frictions with the steady state values of unemployment and job-finding rate. They do not identify the institutions where in this study we clearly model these institutions.

This study also contributes to the literature, pioneered by Trigari (2006) and Gertler and Trigari (2009), which aims to replicate the observed dynamics of unemployment and inflation by incorporating search and match labor market frictions into the standard New Keynesian framework.[4] This line of research has been criticized by not being able to capture the business cycle facts due to exogenous separation rate[5] . This study extends the baseline framework of Gertler and Trigari (2009) with endogenous separations.

To identify the impacts of LMI on business cycles we derive the impulse responses of key macroeconomic variables to a monetary policy shock under alternative levels of institutions. The results of this study are as follows. A change in the firing cost does not have a significant impact on the impulse responses of the variables. An increase in the unemployment benefits makes the output, employment, unemployment and inflation less responsive to the macroeconomic disturbances. In

[4] Some other examples of this literature are, Walsh (2005), Krause and Lubik (2007), Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008).

[5] See Ramey (2008) for a detailed discussion of exogenous versus endogenous separations.

the mean time, an increase in the bargaining power increase the responses of the real side of the economy where it reduces the response of inflation to a shock.

In the next section we lay out the main framework of the paper. Third section calibrates the model. In the fourth section, we present the log-linearized version of the key equations of the model to derive the impulse responses under alternative levels of institutions. Final section concludes the paper.

2 The Model

The agents in the economy are: households, intermediate goods producers, retailers, a final good producer and a monetary authority. Households maximize their lifetime utility subject to a budget constraint. The intermediate firms operate in a perfectly competitive market and face with labor market frictions. They hire workers through a search and matching process and set their wages according to Calvo-type price setting framework. Retailers operate in a monopolistically competitive market and set their prices as in Calvo (1983). They transform one unit of intermediate good to a unit of retial good and sell it to the final good producer. The final good producer collects all goods produced by retailers, combines them via a Dixit-Stiglitz aggregator and produces the consumption good. Finally, the monetary authority employs a Taylor-type monetary policy rule and uses the nominal interest rates as the instrument.

2.1 The household

Households in the economy are distributed on a unit interval and are assumed to form a large extended family. This family is simply referred as the household.[6] At any point in time some members of the family are employed. Rest of the members are unemployed and search for a job. The household enjoys consumption and maximizes her lifetime utility subject to the budget constraint. Formally, the problem of the household can be written as:

$$\begin{aligned} \max \quad & E_t \sum_{t=0}^{\infty} \beta^t \log(c_t) \\ \text{s.t.} \quad & c_t + k_{t+1} + \frac{B_t}{P_t r_t^n} = w_t n_t + b(1 - n_{t-1}) + \rho_t^{in} n_{t-1} \Upsilon + \\ & (z_t + 1 - \delta) k_t + \Pi_t + T_t + \frac{B_{t-1}}{P_t} \end{aligned} \tag{1}$$

c_t, k_t and B_t are consumption of the final good, saving in terms of capital and per-capita holdings of a nominal one-period bond, respectively. w_t is the aggregate wage rate paid to the working

[6] To avoid the distributional problems among individuals it is assumed that the members of the family combine their incomes and make the decision of consumption. This is a common assumption in the literature.

members of the family and b is the unemployment benefits received by the unemployed members. Υ denotes the amount of payment to a worker who is involuntarily separated[7] in the beginning of the period and δ stands for the depreciation rate of the capital. r_t^n represents the nominal return from a one-period bond and P_t is the aggregate price level. The household rents the capital she owns to the firms with a rental rate z_t. Then the total income of the household at time t is the sum of wage income of employed members, $w_t n_t$, unemployment benefits received by unemployed members, $b(1-n_{t-1})$, total payments to the workers that are separated involuntarily in the beginning of the period, $\rho_t^{in} n_{t-1}\Upsilon$, total rent income from capital remaining after depreciation, $(z_t+1-\delta)\,k_t$, the profits distributed by the firms, Π_t, total transfers from the government, T_t, and real return from bonds that the household bought in the previous period, $\frac{B_{t-1}}{P_t}$. Assuming that λ_t shows the marginal utility of consumption at date t, the first order conditions of the problem above can be written as:

$$B_t : \lambda_t = \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} r_t^n \tag{2}$$

$$k_{t+1} : \lambda_t = \beta E_t \lambda_{t+1} \left[z_{t+1} + 1 - \delta \right] \tag{3}$$

The real interest rate is defined in percentage terms: $1 + r_t = E_t \frac{P_t}{P_{t+1}} r_t^n$.

2.2 Matching and production in intermediate market

In the intermediate market a similar framework in Kilinc (2008) is employed. The firms and workers meet on a matching market where firms post vacancies and search for workers from an unemployment pool and workers search for jobs. The matching function depends on the vacancies posted by the firms and the number of unemployed workers. It is increasing in both of its arguments and represents constant returns to scale.

$$m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma} \tag{4}$$

[7] The involuntary separations are further discussed below.

m_t, u_t and v_t denote the matches within period t, the number of unemployed workers and the number of vacancies posted at time t, respectively. σ_m measures the efficiency of the matching process and $\sigma \in (0,1)$ is the elasticity of the matching with respect to u_t. The aggregate number of vacancies posted at time t and total employment in the economy are given by $v_t = \int_0^1 v_t(i)\, di$ and $n_t = \int_0^1 n_t(i)\, di$. The number of unemployed workers at time t will be the difference between the unity and the number of workers employed in the beginning of the period.

$$u_t = 1 - n_{t-1} \tag{5}$$

Let q_t denote the probability an open vacancy is going to be matched with a searching worker and s_t denote the probability a worker searching for a job is going to be matched with an open vacancy; then, it is convenient to define

$$q_t = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma} \qquad \text{and} \qquad s_t = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma} \tag{6}$$

θ_t measures the labor market tightness and is defined as the ratio of the number of open vacancies to the number of job-seeking workers, $\theta_t = \frac{v_t}{u_t}$.

The intermediate goods producers are distributed on the unit interval. They have access to a Cobb-Douglas type production function. Intermediate firm i rents capital from households, $k_t(i)$, and hires workers, $n_t(i)$. The total production of the firm, $y_t(i)$, is given by

$$y_t(i) = k_t(i)^{\sigma}\, n_t(i)^{1-\sigma} \tag{7}$$

In the literature it is common to define the hiring rate, $x_t(i)$, as the ratio of new hires to the existing workforce.

$$x_t(i) = \frac{q_t v_t(i)}{n_{t-1}(i)} \tag{8}$$

The workforce of the firm at time t is the sum of the surviving workers from the previous period and current hires. We assume that new workers immediately go to work. Therefore, the total workforce at time t will be

$$n_t(i) = (1 - \rho_t)\, n_{t-1}(i) + q_t v_t(i) \tag{9}$$

The separation can occur due to reasons such as migration, death, retirement,...etc, or due to exogenous shocks which results in involuntary separations. The rate of former type of separations is assumed to be constant and denoted by ρ_x. The latter separation rate is represented by ρ_t^{in}. Therefore, the total separations rate is given by

$$\rho_t = \rho_x + \rho_t^{in} \tag{10}$$

We also assume that involuntary separations follow an AR(1) process.

$$\rho_t^{in} = \rho_c^{in} + \rho^\rho \rho_{t-1}^{in} + \varepsilon_t^\rho \quad \text{with} \quad \varepsilon_t^\rho \sim NIID\left(0, \sigma_\rho^2\right)$$

Given the evolution of the workforce and the hiring rate, the value of firm i at time t can be described as:

$$\begin{aligned} F_t(i) \;\; = \;\; & p_t^w y_t(i) - w_t(i) n_t(i) - \frac{\kappa}{2} x_t(i)^2 n_{t-1}(i) - \rho_t^{in} \Upsilon n_{t-1}(i) - z_t k_t(i) + \\ & \beta E_t \Lambda_{t,t+1} F_{t+1}(i) \end{aligned} \tag{11}$$

The firm's discount factor is denoted by $E_t \Lambda_{t,t+1} = E_t \frac{\lambda_{t+1}}{\lambda_t}$, relative price of the good that the firm produces is given by p_t^w and the ongoing wage at the firm is represented by $w_t(i)$. p_t^w is also the marginal cost of the retailer who buys this intermediate good, because the intermediate goods producers operate in a perfectly competitive market. It is assumed that only the workers that are separated due to exogenous shocks are eligible to receive severance payments. The firm also bears labor adjustment cost, which is paid in terms of the final good and is a function of the firm's hiring rate, $\frac{\kappa}{2} x_t(i)^2$.

The wage rate at each intermediate firm is different because at any period only a fraction of the firms are allowed to bargain their wages with their existing workforces. If a firm is allowed to bargain its wage, it will go through the bargaining process over the new wage. If it is not allowed to bargain, the previous period's wage will prevail.

At each period the firm maximizes its value with respect to vacancies and capital stock given its existing employment stock, probability of filling a vacancy, the rental rate and current and expected path of the real wage. The first order conditions are:

$$v_t(i) : \kappa x_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t \Lambda_{t,t+1} \left[\left(1-\rho_{t+1}\right) \kappa x_{t+1}(i) + \frac{\kappa}{2} x_{t+1}(i)^2 - \rho_{t+1}^{in} \Upsilon \right] \quad (12)$$

$$k_t(i) : z_t = \alpha p_t^w \frac{y_t(i)}{k_t(i)} \quad (13)$$

Here, $f_{nt}(i) = (1-\alpha)\frac{y_t(i)}{n_t(i)}$, is the marginal product of labor. The firm chooses the employment level by choosing the hiring rate. The equation 12 represents the hiring decision of the firm. Equation 13 gives the capital decision. Due to the assumptions that the capital market is perfectly competitive and capital is free to move, the output/capital ratios are equal across the firms. Therefore, we can drop the indices in equation 13 and say $z_t = \alpha p_t^w \frac{y_t}{k_t}$.

It is convenient to derive the marginal contribution of a worker to the firm's value as:

$$\frac{\partial F_t(i)}{\partial n_t(i)} = J_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t \Lambda_{t,t+1} \frac{\partial F_{t+1}(i)}{\partial n_t(i)} = \kappa x_t(i) \quad (14)$$

This value will be used in the bargaining process. By using the hiring decision, equation 14 can be revised as:

$$J_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t \Lambda_{t,t+1} \left[\frac{\kappa}{2} x_{t+1}(i)^2 - \rho_{t+1}^{in} \Upsilon \right] + \beta E_t \Lambda_{t,t+1} \left(1-\rho_{t+1}\right) J_{t+1}(i) \quad (15)$$

Before defining the bargaining process, we will discuss the value of each state to the workers. To derive the value of being unemployed, U_t, first the average value of all vacancies posted in that period, $V_{x,t}$, should be derived. Since the unemployed worker does not know which firm she is going to be matched with, she considers $V_{x,t}$ during her job search. This value is given by

$$V_{x,t} = \int_0^1 V_t(i) \frac{x_t(i) n_{t-1}(i)}{x_t n_{t-1}} di = \frac{1}{v_{t-1}} \int_0^1 V_t(i) v_{t-1}(i) \, di \quad (16)$$

In the current period the unemployed worker receives unemployment benefits, b. In the next period the unemployed worker will find a successful match with probability s_{t+1} and with probability $1 - s_{t+1}$ the worker will be unemployed again.

$$U_t = b + \beta E_t \Lambda_{t,t+1} \left[s_{t+1} V_{x,t+1} + (1-s_{t+1}) U_{t+1} \right] \quad (17)$$

The unemployment benefits are calculated as a fraction, $\bar{b}$, of the worker's marginal contribution to the production and firm's savings from labor adjustment cost.

The value of being employed at firm i at time t, $V_t(i)$, is the sum of the wage that the worker is receiving from the firm at the current period and expected discounted return in the next period. At time $t+1$ the worker will either work for the same firm or be separated from the firm. The worker will be eligible for unemployment benefits once she is separated. Moreover, if the worker is involuntarily separated from the firm, the worker will receive a severance payment. Therefore,

$$V_t(i) = w_t(i) + \beta E_t \Lambda_{t,t+1}\left[(1-\rho_t) V_{t+1}(i) + \rho_{t+1}^{in}(\Upsilon + U_{t+1}) + \rho_x U_{t+1}\right] \tag{18}$$

In the wage setting process a Calvo-type price-setting framework is employed. It is assumed that the probability that a firm is allowed to renegotiate its wage at each period follows a Poisson process. At any point in time an intermediate firm will change its wage with probability $1-\varphi_w$. Therefore, the average duration of the wage at a firm is $\frac{1}{1-\varphi_w}$. The probability that a firm can re-negotiate its wage is independent from its negotiation history.

In the bargaining process, the total surplus generated from the contract is distributed according to the bargaining powers of each party. This process is summarized by:

$$\max \qquad H_t(r)^{\eta} J_t(r)^{1-\eta} \tag{19}$$

$H_t(r)$ stands for the surplus of an average negotiating worker once she accepts a job and $J_t(r)$ denotes the value of adding another worker to the firm. Since only a fraction of the firms are allowed to bargain at time t, the firms that are bargaining and that are not bargaining have to be distinguished. This distinction is made by indexing the bargaining ones with r. The surplus of the worker from accepting a job with firm i, $H_t(i)$, and the average surplus of a worker hired at time t, $H_{x,t}$ are defined as:

$$H_t(i) = V_t(i) - U_t \tag{20}$$

$$H_{x,t} = V_{x,t} - U_t \tag{21}$$

Plugging the values of being employed and unemployed to the worker and rearranging the terms result in:

$$H_t(i) = w_t(i) - b - \beta E_t \Lambda_{t,t+1}\left[s_{t+1}H_{x,t+1} - \rho^{in}_{t+1}\Upsilon\right] + \beta E_t \Lambda_{t,t+1}\left(1-\rho_{t+1}\right) H_{t+1}(i) \tag{22}$$

The uncertainty about the time of the next negotiation period results in horizon effect, which is discussed by Gertler and Trigari (2009) in details. Since both the firm and the worker do not know when they will have a chance to renegotiate, they consider the future path of the wage during the bargaining. However, the worker considers only his tenure at the firm where the firm has to think about its future workforce as well as its existing workforce. In other words, the firm has a longer horizon than the worker. This effect is observed in the difference between the firm's cumulative discount factor, $\Sigma_t(r)$, and the worker's cumulative discount factor, Δ_t, which are given by

$$\Sigma_t(r) = E_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \tag{23}$$

$$\Delta_t = E_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \Lambda_{t,t+s}\left(1-\rho_{t+s}\right) \tag{24}$$

The discount factors are similar except the fact that the firm's discount factor depends on the employment of the firm at time $t+s$ relative to time t, where the worker's discount factor depends on the expected survival rate. On the average $\frac{n_{t+s}}{n_t}(r) > \left(1-\rho_{t+s}\right)$, which implies that the firm places relatively more weight on the future than does the worker.

Given these discount factors, the expected wage revenue of a bargaining worker and expected wage payment of a firm to its workers can be written as:[8]

$$\begin{aligned} W^w_t(r) &= \Delta_t w^*_t + \beta(1-\varphi_w) E_t \Lambda_{t,t+1}\left(1-\rho_{t+1}\right)\Delta_{t+1} w^*_{t+1} \\ &\quad + \beta^2 (1-\varphi_w) E_t \Lambda_{t,t+2}\left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)\Delta_{t+2} w^*_{t+2} + \ldots \end{aligned} \tag{25}$$

$$W^f_t(r) = \Sigma_t(r) w^*_t + (1-\varphi_w) E_t \sum_{s=1}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r)\, \Sigma_{t+s}(r)\, w^*_{t+s} \tag{26}$$

As the appendix proves, the first order condition of the bargaining problem is:

$$\eta \Delta_t J_t(r) = (1-\eta)\Sigma_t(r) H_t(r) \Rightarrow \chi_t(r) J_t(r) = (1-\chi_t(r)) H_t(r) \tag{27}$$

[8] The appendix provides the details.

$\chi_t(r)$ denotes the relative weight in the sharing rule and is derived as $\chi_t(r) = \frac{\eta}{\eta + (1-\eta)\frac{\Sigma_t(r)}{\Delta_t}}$.

Due to constant returns assumption, each negotiating firm settles on the same wage rate, w_t^*. In other words, the firm bargains with the marginal worker instead of bargaining with its whole existing workforce. Letting $w_t^o(r)$ stand for the target wage, the following equation defines the optimum bargained wage.

$$\Sigma_t(r) w_t^* = w_t^o(r) + \beta\varphi_w E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) \Sigma_{t+1}(r) w_{t+1}^* \tag{28}$$

where

$$\begin{aligned} w_t^o(r) &= \chi_t(r)\left[p_t^w f_{nt}(r) - \beta E_t \Lambda_{t,t+1}\left(\frac{\kappa}{2}x_{t+1}(r)^2 + \rho_{t+1}^{in}\Upsilon\right)\right] \\ &\quad + [1-\chi_t(r)]\left[b + \beta E_t \Lambda_{t,t+1}\left(s_{t+1}H_{x,t+1} - \rho_{t+1}^{in}\Upsilon + \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)}\kappa x_{t+1}(r)^2\right)\right] \end{aligned} \tag{29}$$

According to the equation above, the target wage is a convex combination of the contribution of a worker to the match and his foregone benefits from being unemployed.[9] The average wage in the economy can be driven by taking the average wage of all employed workers.

$$w_t = \int_0^1 w_t(i)\frac{n_t(i)}{n_t}di \tag{30}$$

Using the law of large numbers and with the fact that only a fraction of the firms can change their wages, the equation above can be rewritten as:

$$w_t = (1-\varphi_w) w_t^* + \varphi_w \int_0^1 w_{t-1}(i)\frac{n_t(i)}{n_t}di \tag{31}$$

2.3 Retailers, final good market and monetary authority

The retailers are distributed on the unit interval. The only function of the retailers is to buy intermediate goods, differentiate them with a technology that transforms one unit of intermediate good at no cost, and re-sell it to the final good producer. Each retailer has a monopolistic power

[9] The target wage in Gertler and Trigari (2009) represents the wage that would arise under period-by-period Nash bargaining, modified to allow for the horizon effect. In our case the target wage is also adjusted for endogenous separations. We represent the bargained wage in this form to make it comparable with that of Gertler and Trigari (2009).

on the good it produces and at any point in time only a fraction of the firms are allowed to reset their prices. Since the intermediate goods market is perfectly competitive, marginal cost of buying an intermediate good is its relative price. The final good producer uses a Dixit-Stiglitz aggregator and produces the final good that is directly sold to the consumers. The monetary authority uses the short-term nominal interest rates as the policy instrument and employs a Taylor-type rule.

The total output produced in the economy can be written as:

$$y_t = \left[\int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{32}$$

where the aggregate output, y_t, is described in terms of each retailer's output, y_{jt}, and the firm's own price elasticity of substitution across differentiated retail goods, ε. The monopolistic power of the retailers and Calvo-type price setting results in a nominal price rigidity in the economy. The aggregate price level then is the aggregate of the nominal sale price of each retailer's product. Formally,

$$P_t = \left[\int_0^1 P_{jt}^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \tag{33}$$

Solving for the demand for each retail good results in

$$y_{jt} = \left[\frac{P_{jt}}{P_t} \right]^{-\varepsilon} y_t \tag{34}$$

At any point in time each retailer can reset its price with a probability $1 - \varphi_p$. This probability is independent from the firm's price setting history. The aggregate price index in the economy is defined as:

$$P_t = \left[\left(1 - \varphi_p\right) \left(P_t^*\right)^{1-\varepsilon} + \varphi_p \left(P_{t-1}\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{35}$$

where P_t^* is the optimal price for the firms that are able to reset their prices. The firm sets its price for the expected number of periods that it will not be able to reset its price. Formally, P_t^* is the solution to the following problem:

$$\max \quad E_t \sum_{s=0}^{\infty} \left(\beta \varphi_p\right)^s \left[\frac{P_{jt}^*}{P_t} - p_{t+s}^w \right]^{-\varepsilon} y_{j,t+s} \tag{36}$$

subject to the demand for that good. The optimal price level for the firm that is able to change its price is:

$$P_{jt}^{*} = \frac{\varepsilon}{\varepsilon - 1} E_t \frac{\sum_{s=0}^{\infty} \left(\beta \varphi_p\right)^s P_{t+s}^{\varepsilon} p_{t+s}^{w} y_{t+s}}{\sum_{s=0}^{\infty} \left(\beta \varphi_p\right)^s P_{t+s}^{\varepsilon - 1} y_{t+s}} \tag{37}$$

Finally, the model is completed with the monetary policy rule and market clearing condition.

$$r_t^n = \beta^{-(1-\rho_m)} \left(r_{t-1}^n\right)^{\rho_m} E_t \left(\pi_{t+1}\right)^{\gamma_\pi (1-\rho_m)} \left(y_t^z\right)^{\gamma_y (1-\rho_m)} e^{\varepsilon_t^m} \tag{38}$$

According to the equation 38, the degree of interest rate smoothing is captured by the parameter ρ_m. The monetary authority responds to the output gap, denoted by y_t^z, and the expected inflation. The corresponding response coefficients of the output gap and expected inflation are represented by γ_y and γ_π, respectively. The monetary policy shock is represented by ε_t^m, which is an *i.i.d.* process.

The market clearing condition for the final good is given by

$$y_t = c_t + k_{t+1} - (1-\delta) k_t + \frac{\kappa}{2} x_t^2 n_t \tag{39}$$

The equation 39 states that the consumption of the household, c_t, investment on capital, $k_{t+1} - (1-\delta)k_t$, and aggregate labor adjustment costs of the intermediate firms, $\frac{\kappa}{2} x_t^2 n_t$, sum up to the amount of final good produced in the economy.

3 Calibration

To analyze the impacts of LMI, first the model is calibrated and then the steady state of the model with zero inflation is derived. The parameters are calibrated for the US economy. Table 1 collects the values that are assigned to the parameters of the model and implied steady state values of some key variables. The length of a period is assumed to be a quarter.

We start with setting the discount factor, β, to 0.99. This results in an approximately 4 percent annual real rate of interest. δ is set to 0.025; therefore, annual depreciation rate of the capital is around 10 percent. These values are consistent with Christiano, Eichenbaum and Evans (2005), Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008). In the literature, the steady state capital's share of income, α, is found to be between 0.3 and 0.36. The midpoint of these values is chosen and it is set to 0.33. The parameter that measures the elasticity of matches to unemployment, σ, is calibrated as 0.5. This value is within the range of the lowest and highest values assigned to this parameter in the literature, 0.4 and 0.72.[10]

The separation rate at the steady state has two components; ρ_x and ρ^{in}. According to Hall (1995) and Davis et al. (1996) the average separation rate is between 0.08 and 0.1. Moreover, Gertler and Trigari (2009) argues that the average duration of a job in the US is two and a half years. To match this fact, they set monthly separation rate to 0.035. In this study we set the total separation rate to ten percent per quarter. To do that we set $\rho_x = 0.09$ and $\rho^{in} = 0.01$. To get the involuntary separations we set the other parameters determining that value accordingly.

To be able to calibrate κ, first we need to determine the steady state value of job finding rate, s. In the literature the values vary between 0.6 and 0.95. The studies that use a similar model to the one employed here set $s = 0.95$. Therefore, we use the same value.

In order to match the average duration of a vacancy, the efficiency parameter in the matching function, σ_m is calibrated as 0.925. With this calibration the probability a vacancy will be filled at

[10] See Blanchard and Diamond (1989) and Shimer (2005) for the lowest and highest values, respectively.

the steady state, q, is 0.9. The duration of an average vacancy is reported to be under one month by Blanchard and Diamond (1989), which requires q to be 1. However, the distinction between the removal time of the advertisement and actual date that the vacancy is filled is pointed out by van Ours and Ridder (1992). In that study it is reported that in the first two weeks after the vacancy is posted, seventy five percent of the vacancies are filled. However, it takes around forty five days to choose the best applicant from the pool.

The probability that an intermediate firm is allowed to reset its wage at any point in time is assumed to be $1 - \varphi_w = 1 - 0.718$. In other words, a firm will be able to renegotiate its wage on average once in every three quarters.

The probability that a retailer is not allowed to change its price is set as $\varphi_p = 0.6$. This value implies that an average firm will keep its price unchanged for around 2.5 quarters. This is consistent with the literature and close to the empirical findings.[11] Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008) assume that markup rate on marginal cost is around 11 percent. This requires setting $\varepsilon = 10$. While calibrating the parameters of the nominal interest rate rule Clarida, Gali and Gertler (2000) are followed and it is assumed that $\rho_m = 0.9$, $\gamma_\pi = 1.5$ and $\gamma_y = 0.5$.

Finally, we calibrate the parameters that represent LMI. To save space we discuss only the benchmark calibration of these parameters. In the impulse response analysis we consider alternative values and examine their impacts on responses of variables. In the benchmark case we set $fr = 0.3$, $\bar{b} = 0.5$ and $\eta = 0.5$. The first parameter represents the fraction of the average wage paid to the separated worker as severance payment, $\Upsilon = fr * w$. In the benchmark case, the separated worker will receive 30 percent of the average wage in the economy. The total unemployment benefits paid to a worker is $b = \bar{b}\left(p^w f_n + \beta \frac{\kappa}{2} x^2\right)$, which is a fraction of the sum of her possible contribution to the production and firm's saving from labor adjustment cost. Given this

[11] See Gali and Gertler (1999), Bils and Klenow (2002), Sbordone (2002) and Christiano, Eichenbaum and Evans (2005)

calibration the unemployment benefits will be 50 percent of this sum. Finally, setting $\eta = 0.5$ results in an equal bargaining power to each bargaining agents.

Given this calibration, the steady state employment rate is 90 percent and unemployment rate is 10 percent. The unemployment rate is much higher than the average unemployment rate of the US in the last decades. However, in this model the workers do not have an option of leaving the labor force and in the literature there is evidence that shows that the flow from unemployed to employment and out-of-labor-force to employment are almost the same. Therefore, setting unemployment to 10 percent is acceptable in this framework.

4 Log-Linearization and Impulse Responses

To identify the impacts of the LMI on the impulse responses of variables to a monetary policy shock, the key equations of the model are log-linearized around the steady state.[12] A variable with a hat denotes percentage deviation from its steady state value and a variable without a hat and without a time subscript denotes the steady state value of the regarding variable. We will start log-linearization with the aggregate wage index.

$$\widehat{w}_t = (1-\varphi_w)\,\widehat{w}_t^* + \varphi_w \widehat{w}_{t-1} \tag{40}$$

The variations in the aggregate wage will be weighted sum of the variations in the previous period's aggregate wage and in the bargained wage. The weights are determined by the frequency of the wage bargaining in the intermediate market. The variations in the bargained wage will be given by

$$\widehat{\Sigma}_t + \widehat{w}_t^* = \Sigma^{-1}\widehat{w}_t^o(r) + \rho\beta\varphi_w E_t\left[\widehat{x}_{t+1} - \widehat{\rho}_{t+1}\right] + \beta\varphi_w E_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{\Sigma}_{t+1} + \widehat{w}_{t+1}^*\right] \tag{41}$$

The percentage deviation of the target wage from its steady state is

$$\begin{aligned}
\widehat{w}_t^o(r) \;=\;& \chi p^w f_n w^{-1}\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] + \left[\left(s+\frac{\rho}{2}\right)\beta\chi\kappa x - \beta\rho^{in}\Upsilon\right] w^{-1}E_t\widehat{\Lambda}_{t,t+1} \\
&+\beta s\chi\kappa x w^{-1}E_t\left[\widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right] + \left[p^w f_n - b - \beta\frac{\kappa}{2}x^2 - (s+\rho)\,\beta H\right]\chi w^{-1}\widehat{\chi}_t(r) \\
&+\beta\frac{\chi}{(1-\chi)}\kappa x^2 w^{-1}E_t\widehat{\chi}_{t+1}(r) + \beta\chi\kappa x^2 w^{-1}E_t\widehat{x}_{t+1}(r) - \beta\rho^{in}\Upsilon w^{-1}E_t\widehat{\rho}_{t+1}^{in}
\end{aligned} \tag{42}$$

According to the equation 42, the variations in the target wage will be affected by the variations in the marginal worker's contribution to the firm value and the worker's foregone benefit from unemployment. These impacts are captured by the first three terms. Except the differences in the coefficients in front of these terms, these are the factors that affect the variations in the target wage in the case of conventional period-by-period bargaining framework. Once the sticky wages assumption is introduced, the horizon effect takes place. As discussed in Gertler and Trigari (2009) there are two dimensions of the horizon effect. First, in the baseline calibration, although

[12] The full log-linearized model is presented in the appendix.

$\eta = 0.5$, the effective bargaining power, χ, is 0.45. In other words, the pure bargaining power will be lessened. Second, the variations in $\chi_t(r)$ will lead to variations in the target wage. This effect is captured by the fourth term in the equation 42. The rest of the terms are the results of endogenous separation and severance payment assumptions.

The variations in the separation rate have two effects on the target wage. First effect is captured by the last term in the equation 42. The second effect operates through the discount factor of the bargaining firm which in turn affects the expected variations in the effective bargaining power of the worker, $\widehat{\chi}_{t+1}(r)$.

$$\widehat{\Sigma}_t(r) = \beta\varphi_w E_t\left[\widehat{\Lambda}_{t,t+1} + \rho\left(\widehat{x}_{t+1}(r) - \widehat{\rho}_{t+1}\right) + \widehat{\Sigma}_{t+1}(r)\right] \tag{43}$$

$$E_t\widehat{\chi}_{t+1}(r) = -(1-\chi)E_t\left(\widehat{\Sigma}_{t+1}(r) - \widehat{\Delta}_{t+1}\right) \tag{44}$$

Although we do not investigate the impacts of spillover impacts, we revise the equation 42 as in Gertler and Trigari (2009). As derived in the appendix, the variations in the target wage is

$$\begin{aligned}\widehat{w}_t^o(r) \quad = \quad & \varphi_{f_n}\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] + \varphi_\chi\widehat{\chi}_t + \varphi_\Lambda E_t\widehat{\Lambda}_{t,t+1} + \frac{\varphi_x + \varphi_s}{(1-\chi)}E_t\widehat{\chi}_{t+1} + [\varphi_x + \varphi_s]E_t\widehat{x}_{t+1} \\ & + \varphi_s E_t\widehat{s}_{t+1} - \beta\rho^{in}\Upsilon w^{-1}E_t\widehat{\rho}_{t+1}^{in} + \tau_1\left(\widehat{w}_t - \widehat{w}_t^*\right) + \tau_2 E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)\end{aligned} \tag{45}$$

where τ_2 measures the direct spillover effect and τ_1 measures the indirect spillover effect. The market wage has a direct spillover effect on the bargained wage because the worker's outside option depends on the wage that she can expect to earn elsewhere. If the expected wage exceeds the expected bargained wage, the outside option of the worker is good and the worker will move to employment in the next period. This induces a spillover effect on the bargained wage. The indirect spillover effect stems from the difference between the hiring rate of the bargaining firm and the average hiring rate. The appendix proves that there is a positive relationship between this difference and the gap between average market wage and contract wage. Formally,

$$\widehat{x}_{t+1}(r) = \widehat{x}_{t+1} + w\Sigma\epsilon\varphi_w\left(\widehat{w}_t - \widehat{w}_t^*\right) \tag{46}$$

To save space rest of the log-linearized versions of the equations are presented in the appendix.

To analyze the impacts of LMI on the business cycles, we derive the impulse responses of key macroeconomic variables to a monetary policy shock under alternative levels of institutions. However, we do not report the results for the firing costs because a change in the firing cost does not have an impact on the responses of macroeconomic variables. The firing cost appears in the equations for the hiring rate and the bargained wage. However, since the expected variation in the involuntary separation is zero, the firing cost does not have an impact on the target wage. Moreover, its impact on the hiring rate decreases to insignificant levels due to the same reason.

The literature does not provide a conclusive answer for the impacts of firing costs on the business cycles. Although Rumler and Scharler (2009), Zanetti (2007) and Veracierto (2008) report a negative relationship, Fonseca et al. (2007) and Joseph et al. (2009) find almost no relationship. When we compare our results with those of Zanetti (2007), we see that the real wage rigidities play an important role. The period-by period bargaining makes the real wage very sensitive to the severance payments. However, if only a fraction of the firms are allowed to bargain at each period, the impacts of a change in the severance payments on the responses decrease to insignificant levels.

Figure 1 plots impulse responses of the nominal interest rate, inflation, output, consumption, employment, unemployment, hiring rate and the real wage to a one standard deviation monetary policy shock under alternative levels of unemployment benefits. According to the figure, the most significant impact of a change in unemployment benefits is on the responses of real wage and inflation. A change in the benefits has a negative impact on the target wage, therefore on the aggregate wage. This impact is carried onto inflation through marginal cost. This leads to the conclusion that as the unemployment benefits increase, the response of inflation decreases. These results are supported by the findings of Campolmi and Faia (2007) and Zanetti (2007), where they find a negative impact of unemployment benefits on inflation volatility. The responses of output, employment and unemployment decreases with an increase in the unemployment benefits but thess changes are not very significant.

Figure 2 shows the responses of key variables to the monetary policy shock under alternative values for worker's bargaining power. The figure reveals the fact that the impacts of a change in the worker's bargaining power has opposite effects on the responses of the real side of the economy with the impacts of a change in the unemployment benefits. Compared to the influence of a change in unemployment benefits, the bargaining power has more impact on responses of output, employment and unemployment. The figure proves that the higher the bargaining power the more sensitive the output, consumption and employment to the monetary policy shock. In the literature, the impacts of bargaining power on the business cycles are ambiguous. Rumler and Scharler (2009) find a limited impact where Campolmi and Faia (2007) report a positive impact. However, Fonseca et al. (2007) claim that the response of employment is larger for lower bargaining power. The main reason for this ambiguity might be the different aspects of collective bargaining. As discussed in Rumler and Scharler (2009) these different aspects might have different impacts. Although they find that strong unionization has a significantly positive impact on output volatility, the decisions given by the union and coordination between the unions might reduce the impacts of disturbances on the economy. When we look at the change in the reponse of inflation to the monetary policy shock, we see that an increase in the bargaining power of the workers has a negative impact.

5 Conclusion

In this study we investigate the impacts of alternative labor market institutions on the business cycles. We consider three institutions which represent the main categorization of a variety of labor market rules and regulations. We use firing cost to represent employment protection institutions, unemployment benefits for unemployment insurance institutions and worker's bargaining power for collective bargaining institutions.

To be able to identify the impacts of these institutions on the business cycles we employ a dynamic stochastic general equilibrium search and match model. We derive the impulse responses of key macroeconomic variables to a monetary policy shock under alternative parameterization of labor market institutions. We find that a change in the firing cost does not have a significant impact on the impulse responses of the variables. As we increase the unemployment benefits all of the variables, including output and inflation, become less responsive to the monetary policy shock. In the mean time, an increase in the worker's bargaining power reduces the response of inflation to the same shock and makes the real side of the economy more responsive to the shocks.

This study can be extended in several directions. First, this study analyzes the impacts of LMI qualitatively. Quantitative analysis is required to measure the exact impacts of institutions on the responses of macroeconomic variables. Second, this study proves that the main impacts of changes in the institutions are on the inflation. This finding can be taken to the next step and the optimal monetary policy can be driven under alternative institution levels.

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6 Tables and Figures

Table 1: Benchmark Calibration

Parameter	**Description**	**Value**
β	discount factor	0.99
δ	capital depreciation rate	0.025
α	share of capital income	0.33
ρ	job separation rate	0.1
σ	elasticity of matches to unemployment	0.5
s	job finding rate at steady-state	0.95
η	bargaining power	0.5
φ_w	infrequency of wage renegotiation	0.718
$\bar{b}$	unemployment insurance	0.5
φ_p	infrequency of price setting	0.6
ε	elasticity of substitution	10
ρ_m	interest rate smoothing	0.9
γ_π	response coefficient of expected inflation	1.5
γ_y	response coefficient of output gap	0.5
u	unemployment rate at the steady state	0.10
n	employment rate at the steady state	0.90

Figure 1

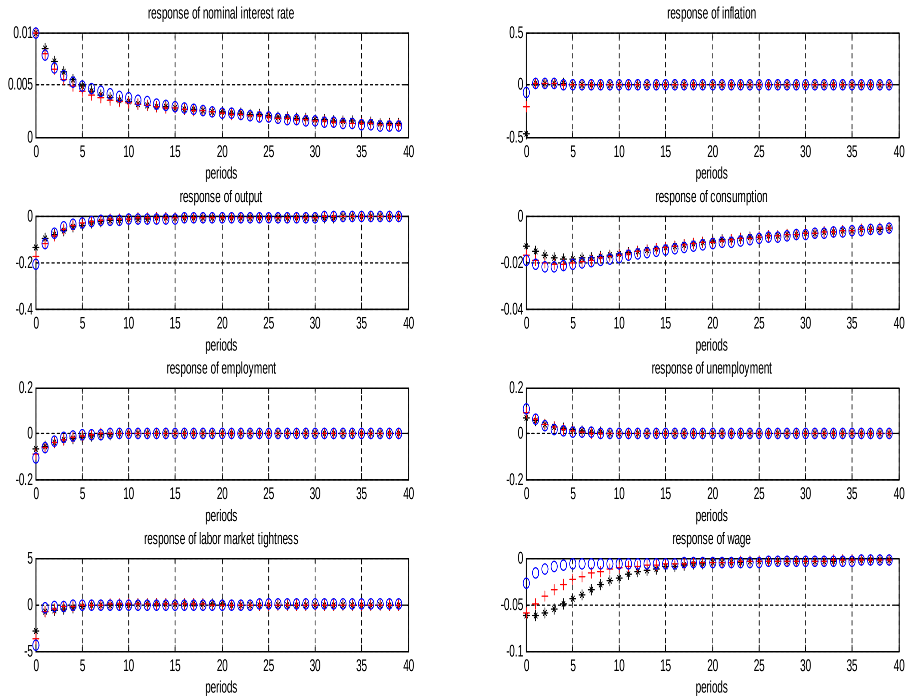

Figure shows the impulse responses of key macroeconomic variables to a monetary policy shock under alternative levels of unemployment benefits levels. Black line with stars on it is for $\bar{b} = 0.1$, red line with plus signs on it is for $\bar{b} = 0.5$, blue line with zeros on is for $\bar{b} = 0.72$.

Figure 2

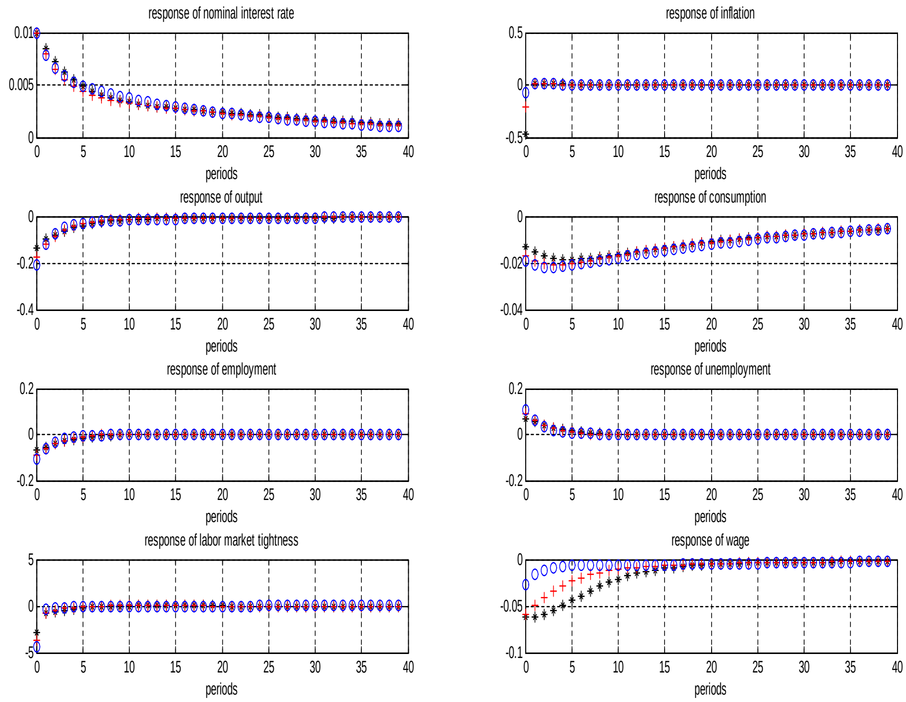

Figure shows the impulse responses of key macroeconomic variables to a monetary policy shock under alternative levels of worker's bargaining power. Black line with stars on it is for $\eta = 0.2$, red line with plus signs on it is for $\eta = 0.5$, blue line with zeros on is for $\eta = 0.8$.

7 Appendix

7.1 Expected wage revenue of a negotiating worker, $W_t^w(r)$

The expected lifetime wage revenues, which depends on whether the relationship will survive or not, of a worker negotiating at time t can be written as

$$W_t^w(r) = w_t(r) + \beta E_t \Lambda_{t,t+1} \left(1-\rho_{t+1}\right) w_{t+1}(r) + \beta^2 E_t \Lambda_{t,t+2} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right) w_{t+2}(r)$$

$$+\beta^3 E_t \Lambda_{t,t+3} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)\left(1-\rho_{t+3}\right) w_{t+3}(r) + ...$$

By using the fact that $w_t = (1-\varphi_w) w_t^* + \varphi_w w_{t-1}$ we can say that $w_t(r) = w_t^*$. Then, in the forthcoming periods

$$E_t w_{t+1}(r) = (1-\varphi_w) E_t w_{t+1}^* + \varphi_w w_t^*$$

$$\Rightarrow$$

$$E_t w_{t+2}(r) = (1-\varphi_w) E_t w_{t+2}^* + \varphi_w w_{t+1}^* = \varphi_w^2 w_t^* + \varphi_w (1-\varphi_w) E_t w_{t+1}^* + (1-\varphi_w) E_t w_{t+2}^*$$

$$\Rightarrow$$

$$E_t w_{t+3}(r) = (1-\varphi_w) E_t w_{t+3}^* + \varphi_w w_{t+2}^*$$

$$\Rightarrow$$

$$E_t w_{t+3}(r) = \varphi_w^3 w_t^* + \varphi_w^2 (1-\varphi_w) E_t w_{t+1}^* + \varphi_w (1-\varphi_w) E_t w_{t+2}^* + (1-\varphi_w) E_t w_{t+3}^*$$

and so on. Therefore,

$$W_t^w(r) = w_t^* + \beta E_t \Lambda_{t,t+1} \left(1-\rho_{t+1}\right) [\varphi_w w_t^* + (1-\varphi_w) w_{t+1}^*]$$

$$+\beta^2 E_t \Lambda_{t,t+2} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right) [\varphi_w^2 w_t^* + \varphi_w (1-\varphi_w) w_{t+1}^* + (1-\varphi_w) w_{t+2}^*]$$

$$+\beta^3 E_t \Lambda_{t,t+3} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)\left(1-\rho_{t+3}\right) [\varphi_w^3 w_t^* + \varphi_w^2 (1-\varphi_w) w_{t+1}^* + \varphi_w (1-\varphi_w) w_{t+2}^* + (1-\varphi_w) w_{t+3}^*] +$$

...

$$\Rightarrow$$

$$W_t^w(r) = [1 + \beta\varphi_w E_t \Lambda_{t,t+1} \left(1-\rho_{t+1}\right) + (\beta\varphi_w)^2 E_t \Lambda_{t,t+2} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)$$

$$+(\beta\varphi_w)^3 E_t \Lambda_{t,t+3} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)\left(1-\rho_{t+3}\right) + ...] w_t^*$$

$$+[\beta (1-\varphi_w) E_t \Lambda_{t,t+1} \left(1-\rho_{t+1}\right) + \beta^2 \varphi_w (1-\varphi_w) E_t \Lambda_{t,t+2} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)$$

$$+\beta^3 \varphi_w^2 (1-\varphi_w) E_t \Lambda_{t,t+3} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)\left(1-\rho_{t+3}\right) + ...] w_{t+1}^*$$

$+[\beta^2 (1-\varphi_w) E_t\Lambda_{t,t+2} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right) + \beta^3\varphi_w (1-\varphi_w) E_t\Lambda_{t,t+3} \left(1-\rho_{t+1}\right)$

$\left(1-\rho_{t+2}\right)\left(1-\rho_{t+3}\right) + ...]w^*_{t+2} + ...$

Assuming that

$\Delta_t = 1 + \beta\varphi_w E_t\Lambda_{t,t+1}\left(1-\rho_{t+1}\right) + (\beta\varphi_w)^2 E_t\Lambda_{t,t+2}\left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)$

$+(\beta\varphi_w)^3 E_t\Lambda_{t,t+3}\left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)\left(1-\rho_{t+3}\right) + ...$

$\Rightarrow$

$\Delta_t = 1 + \beta\varphi_w E_t\Lambda_{t,t+1}\left(1-\rho_{t+1}\right)\Delta_{t+1}$

Therefore,

$W^w_t (r) = \Delta_t w^*_t + \beta (1-\varphi_w) E_t\Lambda_{t,t+1}\left(1-\rho_{t+1}\right)\Delta_{t+1} w^*_{t+1}$

$+\beta^2 (1-\varphi_w) E_t\Lambda_{t,t+2}\left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)\Delta_{t+2} w^*_{t+2} + ...$

7.2 Expected surplus of the bargaining worker, $H_t(r)$

The average surplus of a worker hired at time t, $H_{x,t}$, is defined as $H_{x,t} = V_{x,t} - U_t$ and the surplus of a worker working at firm i is defined as $H_t(i) = V_t(i) - U_t$. Substituting $V_t(i)$ and U_t will yield

$H_t(i) = w_t(i) - b + \beta E_t\Lambda_{t,t+1}\left[\left(1-\rho_{t+1}\right)V_{t+1}(i) + \rho_{t+1}U_{t+1} + \rho^{in}_{t+1}\Upsilon - s_{t+1}V_{x,t+1} - (1-s_{t+1})U_{t+1}\right]$

$\Rightarrow$

$H_t(r) = w_t(r) - b - \beta E_t\Lambda_{t,t+1}\left[s_{t+1}H_{x,t+1} - \rho^{in}_{t+1}\Upsilon\right] + \beta E_t\Lambda_{t,t+1}\left(1-\rho_{t+1}\right)H_{t+1}(r)$

$\Rightarrow$

$H_{t+1}(r) = w_{t+1}(r) - b - \beta E_t\Lambda_{t+1,t+2}\left[s_{t+2}H_{x,t+2} - \rho^{in}_{t+2}\Upsilon\right] + \beta E_t\Lambda_{t+1,t+2}\left(1-\rho_{t+2}\right)H_{t+2}(r)$

$\Rightarrow$

$H_{t+2}(r) = w_{t+2}(r) - b - \beta E_t\Lambda_{t+2,t+3}\left[s_{t+3}H_{x,t+3} - \rho^{in}_{t+3}\Upsilon\right] + \beta E_t\Lambda_{t+2,t+3}\left(1-\rho_{t+3}\right)H_{t+3}(r)$

$\Rightarrow$

$H_{t+1}(r) = w_{t+1}(r) + \beta E_t\Lambda_{t+1,t+2}\left(1-\rho_{t+2}\right)w_{t+2}(r) - b - \beta E_t\Lambda_{t+1,t+2}\left(1-\rho_{t+2}\right)b$

$-\beta E_t\Lambda_{t+1,t+2}\left[s_{t+2}H_{x,t+2} - \rho^{in}_{t+2}\Upsilon\right] + \beta^2 E_t\Lambda_{t+1,t+3}\left(1-\rho_{t+2}\right)\left[s_{t+3}H_{x,t+3} - \rho^{in}_{t+3}\Upsilon\right]$

$+\beta E_t\Lambda_{t+2,t+3}\left(1-\rho_{t+3}\right)H_{t+3}(r)$

Then,

$H_t(r) = w_t(r) + \beta E_t \Lambda_{t,t+1} \left(1-\rho_{t+1}\right) w_{t+1}(r) + \beta^2 E_t \Lambda_{t,t+2} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right) w_{t+2}(r)$

$-b - \beta E_t \Lambda_{t,t+1} \left(1-\rho_{t+1}\right) b - \beta^2 E_t \Lambda_{t,t+2} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right) b$

$-\beta E_t \Lambda_{t,t+1} \left[s_{t+1} H_{x,t+1} - \rho_{t+1}^{in} \Upsilon\right] - \beta^2 E_t \Lambda_{t,t+2} \left(1-\rho_{t+1}\right) \left[s_{t+2} H_{x,t+2} - \rho_{t+2}^{in} \Upsilon\right]$

$-\beta^3 E_t \Lambda_{t+1,t+3} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right) \left[s_{t+3} H_{x,t+3} - \rho_{t+3}^{in} \Upsilon\right] + \beta E_t \Lambda_{t+2,t+3} \left(1-\rho_{t+3}\right) H_{t+3}(r)$

$\Rightarrow$

$H_t(r) = W_t^w(r) - [1 + \beta E_t \Lambda_{t,t+1} \left(1-\rho_{t+1}\right) + \beta^2 E_t \Lambda_{t,t+2} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right) + ...]b$

$-\beta E_t \Lambda_{t,t+1}[s_{t+1} H_{x,t+1} - \rho_{t+1}^{in} \Upsilon] - \beta^2 E_t \Lambda_{t,t+2} \left(1-\rho_{t+1}\right) [s_{t+2} H_{x,t+2} - \rho_{t+2}^{in} \Upsilon]$

$-\beta^3 E_t \Lambda_{t+1,t+3} \left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right) [s_{t+3} H_{x,t+3} - \rho_{t+3}^{in} \Upsilon] - ...$

7.3 Expected wage payments of a negotiating firm, $W_t^f(r)$

The expected lifetime wage payments of the bargaining firm at time t can be written as

$W_t^f(r) = E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r)\, w_{t+s}(r)$

$\Rightarrow$

$W_t^f(r) = w_t(r) + \beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r)\, w_{t+1}(r) + \beta^2 E_t \Lambda_{t,t+2} \frac{n_{t+2}}{n_t}(r)\, w_{t+2}(r) + ...$

$\Rightarrow$

$W_t^f(r) = w_t^* + \beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) \left[\varphi_w w_t^* + (1-\varphi_w) w_{t+1}^*\right]$

$+\beta^2 E_t \Lambda_{t,t+2} \frac{n_{t+2}}{n_t}(r) \left[\varphi_w^2 w_t^* + \varphi_w (1-\varphi_w) w_{t+1}^* + (1-\varphi_w) w_{t+2}^*\right]$

$+\beta^3 E_t \Lambda_{t,t+3} \frac{n_{t+3}}{n_t}(r) \left[\varphi_w^3 w_t^* + \varphi_w^2 (1-\varphi_w) w_{t+1}^* + \varphi_w (1-\varphi_w) w_{t+2}^* + (1-\varphi_w) w_{t+3}^*\right] + ...$

Then,

$W_t^f(r) = E_t \left[1 + \beta\varphi_w \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) + (\beta\varphi_w)^2 \Lambda_{t,t+2} \frac{n_{t+2}}{n_t}(r) + ...\right] w_t^*$

$+ (1-\varphi_w) E_t \beta \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) \left[1 + \beta\varphi_w \Lambda_{t+1,t+2} \frac{n_{t+2}}{n_{t+1}}(r) + ...\right] w_{t+1}^*$

$+ (1-\varphi_w) E_t \beta^2 \Lambda_{t,t+1} \frac{n_{t+2}}{n_t}(r) \left[1 + \frac{n_{t+3}}{n_{t+2}}(r) \beta\varphi_w \Lambda_{t+1,t+2} + ...\right] w_{t+2}^* + ...$

If the discount factor for the firm is defined as

$\Sigma_t(r) = E_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r)$

$\Rightarrow$

$\Sigma_t(r) = 1 + \beta\varphi_w E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r)\, \Sigma_{t+1}(r)$

Then

$W_t^f(r) = \Sigma_t(r) w_t^* + (1-\varphi_w) E_t \sum_{s=1}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \Sigma_{t+s}(r) w_{t+s}^*$

7.4 Expected surplus of the bargaining firm, $J_t(r)$

Having derived the expected wage revenues of a negotiating worker and expected wage payments of a negotiating firm, we can now derive the explicit functions of surpluses of firms and workers.

$J_t(r) = p_t^w f_{nt}(r) - w_t(r) + \beta E_t \Lambda_{t,t+1} \left[\frac{\kappa}{2} x_{t+1}(r)^2 - \rho_{t+1}^{in} \Upsilon\right] + \beta E_t \Lambda_{t,t+1}\left(1-\rho_{t+1}\right) J_{t+1}(r)$

Using the fact that $n_{t+1}(r) = \left(1-\rho_{t+1}\right) n_t(r) + q_{t+1} v_{t+1}(r) \Rightarrow \left(1-\rho_{t+1}\right) = \frac{n_{t+1}}{n_t}(r) - x_{t+1}(r)$ and $J_{t+1}(r) = \kappa x_{t+1}(r)$, we can rewrite the equation above as

$J_t(r) = p_t^w f_{nt}(r) - w_t(r) - \beta E_t \Lambda_{t,t+1} \left[\frac{\kappa}{2} x_{t+1}(r)^2 + \rho_{t+1}^{in} \Upsilon\right] + \beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) J_{t+1}(r)$

$\Rightarrow$

$J_{t+1}(r) = p_{t+1}^w f_{nt+1}(r) - w_{t+1}(r) - \beta E_t \Lambda_{t+1,t+2} \left[\frac{\kappa}{2} x_{t+2}(r)^2 + \rho_{t+2}^{in} \Upsilon\right] + \beta E_t \Lambda_{t+1,t+2} \frac{n_{t+2}}{n_{t+1}}(r) J_{t+2}(r)$

$\Rightarrow$

$J_{t+2}(r) = p_{t+2}^w f_{nt+2}(r) - w_{t+2}(r) - \beta E_t \Lambda_{t+2,t+3} \left[\frac{\kappa}{2} x_{t+3}(r)^2 + \rho_{t+3}^{in} \Upsilon\right] + \beta E_t \Lambda_{t+2,t+3} \frac{n_{t+3}}{n_{t+2}}(r) J_{t+3}(r)$

and so forth. Therefore,

$J_t(r) = E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \left[p_{t+s}^w f_{nt+s}(r) - \beta \Lambda_{t+s,t+s+1} \left[\frac{\kappa}{2} x_{t+s+1}(r)^2 + \rho_{t+s+1}^{in} \Upsilon\right]\right]$
$-E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) w_{t+s}(r)$

$\Rightarrow$

$J_t(r) = E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \left[p_{t+s}^w f_{nt+s}(r) - \beta \Lambda_{t+s,t+s+1} \left[\frac{\kappa}{2} x_{t+s+1}(r)^2 + \rho_{t+s+1}^{in} \Upsilon\right]\right]$
$-W_t^f(r)$

7.5 Deriving the contract wage

The Nash bargaining is summarized by

$max \qquad H_t(r)^{\eta} J_t(r)^{1-\eta}$

The first order condition will result in

$\eta \Delta_t H_t(r)^{\eta-1} J_t(r)^{1-\eta} = (1-\eta) \Sigma_t(r) H_t(r)^{\eta} J_t(r)^{-\eta}$

$\Rightarrow$

$\eta \Delta_t J_t(r) = (1-\eta)\Sigma_t(r) H_t(r)$

Letting $\chi_t(r) = \frac{\eta\Delta_t}{\eta\Delta_t + (1-\eta)\Sigma_t(r)}$ will provide $\chi_t(r) J_t(r) = (1-\chi_t(r)) H_t(r)$. $H_t(r) = w_t(r) - b - \beta E_t \Lambda_{t,t+1}\left[s_{t+1} H_{x,t+1} - \rho_{t+1}^{in}\Upsilon\right] + \beta E_t \Lambda_{t,t+1}\left(1-\rho_{t+1}\right) H_{t+1}(r)$

$\Rightarrow$

$H_t(r) = w_t(r) - b - \beta E_t \Lambda_{t,t+1}\left[s_{t+1} H_{x,t+1} - \rho_{t+1}^{in}\Upsilon\right] + \beta E_t \Lambda_{t,t+1}\left(\frac{n_{t+1}}{n_t}(r) - x_{t+1}\right) H_{t+1}(r)$

$\Rightarrow$

$H_t(r) = w_t(r) - b - \beta E_t \Lambda_{t,t+1}\left[s_{t+1} H_{x,t+1} - \rho_{t+1}^{in}\Upsilon + \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)}\kappa x_{t+1}^2(r)\right]$

$+\beta E_t \Lambda_{t,t+1}\frac{n_{t+1}}{n_t}(r) H_{t+1}(r)$

Then

$H_{t+1}(r) = w_{t+1}(r) - b - \beta E_t \Lambda_{t+1,t+2}\left[s_{t+2} H_{x,t+2} - \rho_{t+2}^{in}\Upsilon + \frac{\chi_{t+2}(r)}{1-\chi_{t+2}(r)}\kappa x_{t+2}^2(r)\right]$

$+\beta E_t \Lambda_{t+1,t+2}\frac{n_{t+2}}{n_{t+1}}(r) H_{t+2}(r)$

$\Rightarrow$

$H_{t+2}(r) = w_{t+2}(r) - b - \beta E_t \Lambda_{t+2,t+3}\left[s_{t+3} H_{x,t+3} - \rho_{t+3}^{in}\Upsilon + \frac{\chi_{t+3}(r)}{1-\chi_{t+3}(r)}\kappa x_{t+3}^2(r)\right]$

$+\beta E_t \Lambda_{t+2,t+3}\frac{n_{t+3}}{n_{t+2}}(r) H_{t+3}(r)$

$\Rightarrow$

$H_t(r) = W_t^f(r) -$

$E_t \sum_{s=0}^{\infty}\beta^s \Lambda_{t,t+s}\frac{n_{t+s}}{n_t}(r)\left[b + \beta\Lambda_{t+s,t+s+1}\left(s_{t+s+1}H_{x,t+s+1} - \rho_{t+s+1}^{in}\Upsilon + \frac{\chi_{t+s+1}(r)}{1-\chi_{t+s+1}(r)}\kappa x_{t+s+1}(r)^2\right)\right]$

Now we can derive the contract wage by substituting $H_t(r)$ and $J_t(r)$ into the first order condition of the Nash bargaining.

$\chi_t(r) J_t(r) = (1-\chi_t(r)) H_t(r)$

$\Rightarrow$

$\chi_t(r)\{E_t \sum_{s=0}^{\infty}\beta^s \Lambda_{t,t+s}\frac{n_{t+s}}{n_t}(r)[p_{t+s}^w f_{nt+s}(r) - \beta\Lambda_{t+s,t+s+1}(\frac{\kappa}{2}x_{t+s+1}(r)^2 + \rho_{t+s+1}^{in}\Upsilon)] - W_t^f(r)\}$

$=$

$[1-\chi_t(r)]\{W_t^f(r) - E_t \sum_{s=0}^{\infty}\beta^s \Lambda_{t,t+s}\frac{n_{t+s}}{n_t}(r)[b + \beta\Lambda_{t+s,t+s+1}(s_{t+s+1}H_{x,t+s+1} - \rho_{t+s+1}^{in}\Upsilon + \frac{\chi_{t+s+1}(r)}{1-\chi_{t+s+1}(r)}\kappa x_{t+s+1}(r)^2)]\}$

$\Rightarrow$

$W_t^f(r) = \chi_t(r) E_t \sum_{s=0}^{\infty}\beta^s \Lambda_{t,t+s}\frac{n_{t+s}}{n_t}(r)\left[p_{t+s}^w f_{nt+s}(r) - \beta\Lambda_{t+s,t+s+1}\left(\frac{\kappa}{2}x_{t+s+1}(r)^2 + \rho_{t+s+1}^{in}\Upsilon\right)\right] +$

$[1-\chi_t(r)] E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \left[b + \beta\Lambda_{t+s,t+s+1}\left(s_{t+s+1}H_{x,t+s+1} - \rho^{in}_{t+s+1}\Upsilon + \frac{\chi_{t+s+1}(r)}{1-\chi_{t+s+1}(r)}\kappa x_{t+s+1}(r)^2\right)\right]$

$\Rightarrow$

$\Sigma_t(r) w^*_t + (1-\varphi_w) E_t \sum_{s=1}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \Sigma_{t+s}(r) w^*_{t+s}$

$= \chi_t(r) E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \left[p^w_{t+s} f_{nt+s}(r) - \beta\Lambda_{t+s,t+s+1}\left(\frac{\kappa}{2} x_{t+s+1}(r)^2 + \rho^{in}_{t+s+1}\Upsilon\right)\right]$

$+[1-\chi_t(r)] E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \left[b + \beta\Lambda_{t+s,t+s+1}\left(s_{t+s+1}H_{x,t+s+1} - \rho^{in}_{t+s+1}\Upsilon + \frac{\chi_{t+s+1}(r)}{1-\chi_{t+s+1}(r)}\kappa x_{t+s+1}(r)^2\right)\right]$

$\Rightarrow$

$\Sigma_t(r) w^*_t = \chi_t(r) [p^w_t f_{nt}(r) - \beta E_t \Lambda_{t,t+1}(\frac{\kappa}{2} x_{t+1}(r)^2 + \rho^{in}_{t+1}\Upsilon)] + [1-\chi_t(r)] [b + \beta E_t \Lambda_{t,t+1}(s_{t+1}H_{x,t+1} -$

$\rho^{in}_{t+1}\Upsilon + \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)}\kappa x_{t+1}(r)^2)] - (1-\varphi_w)\beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) \Sigma_{t+1}(r) w^*_{t+1} + \beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) \Sigma_{t+1}(r) w^*_{t+1}$

$\Rightarrow$

$\Sigma_t(r) w^*_t = w^o_t(r) + \beta\varphi_w E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) \Sigma_{t+1}(r) w^*_{t+1}$

where

$w^o_t(r) = \chi_t(r) \left[p^w_t f_{nt}(r) - \beta E_t \Lambda_{t,t+1}\left(\frac{\kappa}{2} x_{t+1}(r)^2 + \rho^{in}_{t+1}\Upsilon\right)\right]$

$+[1-\chi_t(r)] \left[b + \beta E_t \Lambda_{t,t+1}\left(s_{t+1}H_{x,t+1} - \rho^{in}_{t+1}\Upsilon + \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)}\kappa x_{t+1}(r)^2\right)\right]$

7.6 Dynamics of the intermediate market

To log-linearize the target wage, we will rewrite the regarding equation such as

$w^o_t(r) = \chi_t(r) \left[p^w_t f_{nt}(r) - \beta E_t \Lambda_{t,t+1}\left(\frac{\kappa}{2} x_{t+1}(r)^2 + \rho^{in}_{t+1}\Upsilon\right)\right]$

$+[1-\chi_t(r)] \left[b + \beta E_t \Lambda_{t,t+1}\left(s_{t+1}H_{x,t+1} - \rho^{in}_{t+1}\Upsilon + \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)}\kappa x_{t+1}(r)^2\right)\right]$

$\Rightarrow$

$w^o_t(r) = \chi_t(r) p^w_t f_{nt}(r) - \beta\chi_t(r) E_t \Lambda_{t,t+1} \frac{\kappa}{2} x_{t+1}(r)^2 - \beta\chi_t(r) E_t \Lambda_{t,t+1} \rho^{in}_{t+1}\Upsilon + b - \chi_t(r) b -$

$[1-\chi_t(r)] \beta E_t \Lambda_{t,t+1} \rho^{in}_{t+1}\Upsilon + \beta E_t \Lambda_{t,t+1} s_{t+1} H_{x,t+1} + \beta E_t \Lambda_{t,t+1} \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)} \kappa x^2_{t+1}(r) - \beta\chi_t(r) E_t \Lambda_{t,t+1} s_{t+1} H_{x,t+1} -$

$\beta\chi_t(r) E_t \Lambda_{t,t+1} \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)} \kappa x^2_{t+1}(r)$

$\Rightarrow$

$w^o_t(r) = \chi_t(r) p^w_t f_{nt}(r) - \beta\chi_t(r) E_t \Lambda_{t,t+1} \frac{\kappa}{2} x_{t+1}(r)^2 - \beta E_t \Lambda_{t,t+1} \rho^{in}_{t+1}\Upsilon + b - \chi_t(r) b$

$+\beta E_t \Lambda_{t,t+1} s_{t+1} H_{x,t+1} + \beta E_t \Lambda_{t,t+1} \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)} \kappa x^2_{t+1}(r)$

$-\beta\chi_t(r) E_t \Lambda_{t,t+1} s_{t+1} H_{x,t+1} - \beta\chi_t(r) E_t \Lambda_{t,t+1} \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)} \kappa x^2_{t+1}(r)$

Log-linearizing the equation above and canceling the constant terms will result in

$$w\widehat{w}_t^o(r) = \chi p^w f_n \left[\widehat{\chi}_t(r) + \widehat{p}_t^w + \widehat{f}_{nt}(r)\right] - \beta\chi\tfrac{\kappa}{2}x^2 E_t\left[\widehat{\chi}_t(r) + \widehat{\Lambda}_{t,t+1} + 2\widehat{x}_{t+1}(r)\right] - \beta\rho^{in}\Upsilon E_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{\rho}_{t+1}^{in}\right] +$$
$$\beta s H_x E_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right] - b\chi\widehat{\chi}_t(r) + \beta\tfrac{\chi}{1-\chi}\kappa x^2 E_t\left[\widehat{\Lambda}_{t,t+1} + \tfrac{1}{1-\chi}\widehat{\chi}_{t+1}(r) + 2\widehat{x}_{t+1}(r)\right]$$
$$-\beta\chi s H_x E_t\left[\widehat{\chi}_t(r) + \widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right] - \beta\tfrac{\chi^2}{1-\chi}\kappa x^2 E_t\left[\widehat{\chi}_t(r) + \widehat{\Lambda}_{t,t+1} + \tfrac{1}{1-\chi}\widehat{\chi}_{t+1}(r) + 2\widehat{x}_{t+1}(r)\right]$$

Here we used the fact that $f(x_t) \simeq f(x) + f'(x)(x_t - x) \simeq f(x)(1 + \eta\widehat{x}_t)$ where $\eta = \frac{\partial f(x)}{\partial x}\frac{x}{f(x)}$.

Simplifying the equation above yields

$$\widehat{w}_t^o(r) = \chi p^w f_n w^{-1}\left[\widehat{p}_t^w + \widehat{f}_{nt}(r)\right] + \left[p^w f_n - \beta\tfrac{\kappa}{2}x^2 - b - \beta s H_x - \beta\tfrac{\chi}{1-\chi}\kappa x^2\right]\chi w^{-1}\widehat{\chi}_t(r)$$
$$+\left[(1-\chi)\beta s H_x - \beta\rho^{in}\Upsilon + \tfrac{\beta\chi\kappa x^2}{2}\right] w^{-1}E_t\widehat{\Lambda}_{t,t+1} + \tfrac{\beta\chi\kappa x^2}{(1-\chi)} w^{-1}E_t\widehat{\chi}_{t+1}(r)$$
$$+\beta\chi\kappa x^2 w^{-1}E_t\widehat{x}_{t+1}(r) - \beta\rho^{in}\Upsilon w^{-1}E_t\widehat{\rho}_{t+1}^{in} + (1-\chi)\beta s H_x w^{-1}E_t\left[\widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right]$$

Due to constant returns $\widehat{f}_{nt}(r) = \widehat{f}_{nt}$ and $\widehat{H}_{x,t} = \widehat{H}_t$. At the steady state $H_x = H = \frac{\chi J}{(1-\chi)}$ and $J = \kappa x \Rightarrow H = \frac{\chi\kappa x}{(1-\chi)}$. Therefore, the equation above can be revised such that

$$\widehat{w}_t^o(r) = \chi p^w f_n w^{-1}\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] + \left[p^w f_n - b - \beta\tfrac{\kappa}{2}x^2 - (s+\rho)\beta H\right]\chi w^{-1}\widehat{\chi}_t(r)$$
$$+\left[\left(s + \tfrac{\rho}{2}\right)\beta\chi\kappa x - \beta\rho^{in}\Upsilon\right] w^{-1}E_t\widehat{\Lambda}_{t,t+1} + \beta\tfrac{\chi}{(1-\chi)}\kappa x^2 w^{-1}E_t\widehat{\chi}_{t+1}(r) + \beta\chi\kappa x^2 w^{-1}E_t\widehat{x}_{t+1}(r)$$
$$-\beta\rho^{in}\Upsilon w^{-1}E_t\widehat{\rho}_{t+1}^{in} + \beta s\chi\kappa x w^{-1}E_t\left[\widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right]$$

To find the percentage deviation of the target wage from its steady state, we will follow the footsteps of Gertler and Trigari (2009) and start with the hiring rate of a bargaining firm. It is given by

$$\kappa x_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t\Lambda_{t,t+1}\left[\left(1-\rho_{t+1}\right)\kappa x_{t+1}(i) + \tfrac{\kappa}{2}x_{t+1}(i)^2 - \rho_{t+1}^{in}\Upsilon\right]$$

$\Rightarrow$

$$\kappa x_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t\Lambda_{t,t+1}\kappa x_{t+1}(i) - \beta E_t\Lambda_{t,t+1}\rho_{t+1}\kappa x_{t+1}(i) + \beta E_t\Lambda_{t,t+1}\tfrac{\kappa}{2}x_{t+1}(i)^2 -$$
$$\beta E_t\Lambda_{t,t+1}\rho_{t+1}^{in}\Upsilon$$

$\Rightarrow$

$$\kappa x\widehat{x}_t(r) = p^w f_n\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] - w\widehat{w}_t(r) + \beta\kappa x E_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{x}_{t+1}(r)\right] - \beta\rho\kappa x E_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{\rho}_{t+1} + \widehat{x}_{t+1}(r)\right]$$
$$+\beta\tfrac{\kappa}{2}x^2 E_t\left[\widehat{\Lambda}_{t,t+1} + 2\widehat{x}_{t+1}(r)\right] - \beta\rho^{in}\Upsilon E_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{\rho}_{t+1}^{in}\right]$$

$\Rightarrow$

$$\widehat{x}_t(r) = p^w f_n\epsilon\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] - w\epsilon\widehat{w}_t(r) + \left(1 - \tfrac{\rho}{2} - \rho^{in}\Upsilon\epsilon\right)\beta E_t\widehat{\Lambda}_{t,t+1} + \beta E_t\widehat{x}_{t+1}(r) - \rho\beta E_t\widehat{\rho}_{t+1} -$$

$\beta\rho^{in}\Upsilon\epsilon E_t\widehat{\rho}^{in}_{t+1}$

where $\epsilon = (\kappa x)^{-1}$. At the steady state $n = (1-\rho)\,n + qv$ and $x = \frac{qv}{n}$, then $x = \rho$. Given the equation above, the difference between the hiring rate of a bargaining firm and the aggregate hiring rate will be given by

$\widehat{x}_t\,(r) - \widehat{x}_t = w\epsilon\,(\widehat{w}_t - \widehat{w}_t\,(r)) + \beta E_t\,(\widehat{x}_{t+1}\,(r) - \widehat{x}_{t+1})$

iterating this equation forward will provide us

$\widehat{x}_t\,(r) - \widehat{x}_t = w\epsilon\,(\widehat{w}_t - \widehat{w}_t\,(r)) + \beta w\epsilon E_t\,(\widehat{w}_{t+1} - \widehat{w}_{t+1}\,(r)) + \beta^2 w\epsilon E_t\,(\widehat{w}_{t+2} - \widehat{w}_{t+2}\,(r))$

$+\beta^3 w\epsilon E_t\,(\widehat{w}_{t+3} - \widehat{w}_{t+3}\,(r)) + ...$

We can simplify this equation by using the real wage index. The wage index is

$w_t = \varphi_w w_{t-1} + (1-\varphi_w)\,w^*_t$

$\Rightarrow$

$\widehat{w}_t = \varphi_w \widehat{w}_{t-1} + (1-\varphi_w)\,\widehat{w}^*_t$

$\Rightarrow$

$E_t\widehat{w}_{t+1} = \varphi_w \widehat{w}_t + (1-\varphi_w)\,E_t\widehat{w}^*_{t+1}$

$\Rightarrow$

$E_t\widehat{w}_{t+2} = \varphi_w \widehat{w}_{t+1} + (1-\varphi_w)\,E_t\widehat{w}^*_{t+2}$

We also know the fact that expected bargained wage for the next period is convex combination of the optimum bargained wage at next period and the optimum wage for this period:

$\widehat{w}_t(r) = \widehat{w}^*_t$

$\Rightarrow$

$E_t\widehat{w}_{t+1}\,(r) = \varphi_w \widehat{w}^*_t + (1-\varphi_w) E_t\widehat{w}^*_{t+1}$

and the aggregate wage at time $t+1$ is given by

$E_t\widehat{w}_{t+1} = \varphi_w \widehat{w}^*_t + (1-\varphi_w) E_t\widehat{w}^*_{t+1}$

$\Rightarrow$

$E_t\widehat{w}_{t+2}\,(r) = \varphi_w \widehat{w}^*_{t+1} + (1-\varphi_w) E_t\widehat{w}^*_{t+2}$

Then it is convenient to write

$E_t\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}\left(r\right)\right)=\varphi_w\left(\widehat{w}_t-\widehat{w}_t^*\right)$

$\Rightarrow$

$E_{t+1}\left(\widehat{w}_{t+2}-\widehat{w}_{t+2}\left(r\right)\right)=\varphi_w\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}^*\right)$

We also know that

$E_t\widehat{w}_t\left(r\right)=\widehat{w}_t^*\Rightarrow E_{t+1}\widehat{w}_{t+1}\left(r\right)=\widehat{w}_{t+1}^*$.

Taking the expectation of both sides will yield

$E_t\widehat{w}_{t+1}\left(r\right)=E_t\widehat{w}_{t+1}^*\Rightarrow \varphi_w\widehat{w}_t^*+(1-\varphi_w)E_t\widehat{w}_{t+1}^*=E_t\widehat{w}_{t+1}^*\Rightarrow \widehat{w}_t^*=E_t\widehat{w}_{t+1}^*$.

Having found this relationship will allow us to rewrite

$E_{t+1}\left(\widehat{w}_{t+2}-\widehat{w}_{t+2}\left(r\right)\right)=\varphi_w\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}^*\right)$

such that

$E_t\left[E_{t+1}\left(\widehat{w}_{t+2}-\widehat{w}_{t+2}\left(r\right)\right)\right]=E_t\left[\varphi_w\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}^*\right)\right]$

$\Rightarrow$

$E_t\left[\widehat{w}_{t+2}-\widehat{w}_{t+2}\left(r\right)\right]=\varphi_w\left(E_t\widehat{w}_{t+1}-E_t\widehat{w}_{t+1}^*\right)$

$\Rightarrow$

$E_t\left[\widehat{w}_{t+2}-\widehat{w}_{t+2}\left(r\right)\right]=\varphi_w(\varphi_w\widehat{w}_t+(1-\varphi_w)E_t\widehat{w}_{t+1}^*-E_t\widehat{w}_{t+1}^*)$

$\Rightarrow$

$E_t\left[\widehat{w}_{t+2}-\widehat{w}_{t+2}\left(r\right)\right]=\varphi_w^2(\widehat{w}_t-E_t\widehat{w}_{t+1}^*)$

$\Rightarrow$

$E_t\left[\widehat{w}_{t+2}-\widehat{w}_{t+2}\left(r\right)\right]=\varphi_w^2(\widehat{w}_t-\widehat{w}_t^*)$

Therefore,

$\widehat{x}_t\left(r\right)-\widehat{x}_t=w\epsilon\left(\widehat{w}_t-\widehat{w}_t\left(r\right)\right)+\beta w\epsilon E_t\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}\left(r\right)\right)+\beta^2 w\epsilon E_t\left(\widehat{w}_{t+2}-\widehat{w}_{t+2}\left(r\right)\right)$

$+\beta^3 w\epsilon E_t\left(\widehat{w}_{t+3}-\widehat{w}_{t+3}\left(r\right)\right)+...$

$\Rightarrow$

$\widehat{x}_t\left(r\right)-\widehat{x}_t=w\epsilon\left(\widehat{w}_t-\widehat{w}_t^*\right)+\beta\varphi_w w\epsilon\left(\widehat{w}_t-\widehat{w}_t^*\right)+(\beta\varphi_w)^2 w\epsilon\left(\widehat{w}_t-\widehat{w}_t^*\right)+(\beta\varphi_w)^3 w\epsilon\left(\widehat{w}_t-\widehat{w}_t^*\right)+...$

$\Rightarrow$

$\widehat{x}_t\left(r\right)=\widehat{x}_t+w\Sigma\epsilon\left(\widehat{w}_t-\widehat{w}_t^*\right)$ where $\Sigma=\frac{1}{1-\beta\varphi_w}$. Then, $\widehat{x}_{t+1}\left(r\right)=\widehat{x}_{t+1}+w\Sigma\epsilon\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}^*\right)$.

Following the same path for the weights in the Nash bargaining first order condition yields

$\Sigma_t(r) = 1 + \beta\varphi_w E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r)\, \Sigma_{t+1}(r)$

$\Rightarrow$

$\widehat{\Sigma}_t(r) = \beta\varphi_w E_t \left[\widehat{\Lambda}_{t,t+1} + \rho\left(\widehat{x}_{t+1}(r) - \widehat{\rho}_{t+1}\right) + \widehat{\Sigma}_{t+1}(r)\right]$

where we used the fact that

$n_t(r) = (1-\rho_t)\, n_{t-1}(r) + q_t v_t \Rightarrow n_{t+1}(r) = \left(1-\rho_{t+1}\right) n_t(r) + q_{t+1} v_{t+1}$

$\Rightarrow$

$\frac{n_{t+1}}{n_t}(r) = 1 - \rho_{t+1} + x_{t+1}(r)$

$\Rightarrow$

$\widehat{n}_{t+1}(r) - \widehat{n}_t(r) = \rho\left(\widehat{x}_{t+1}(r) - \widehat{\rho}_{t+1}\right)$

Therefore

$\widehat{\Sigma}_t(r) - \widehat{\Sigma}_t = \rho\beta\varphi_w E_t\left[\widehat{x}_{t+1}(r) - \widehat{x}_{t+1}\right] + \beta\varphi_w E_t\left[\widehat{\Sigma}_{t+1}(r) - \widehat{\Sigma}_{t+1}\right]$

$\Rightarrow$

$\widehat{\Sigma}_{t+1}(r) - \widehat{\Sigma}_{t+1} = \rho\beta\varphi_w E_t\left[\widehat{x}_{t+2}(r) - \widehat{x}_{t+2}\right] + \beta\varphi_w E_t\left[\widehat{\Sigma}_{t+2}(r) - \widehat{\Sigma}_{t+2}\right]$

We can deduce that

$\widehat{\Sigma}_t(r) - \widehat{\Sigma}_t = \rho\beta\varphi_w E_t\left[\left(\widehat{x}_{t+1}(r) - \widehat{x}_{t+1}\right) + \beta\varphi_w\left(\widehat{x}_{t+2}(r) - \widehat{x}_{t+2}\right) + \left(\beta\varphi_w\right)^2\left(\widehat{x}_{t+3}(r) - \widehat{x}_{t+3}\right) + ...\right]$

$\Rightarrow$

$\widehat{\Sigma}_t(r) - \widehat{\Sigma}_t = \rho w \Sigma \epsilon \Psi\left(\widehat{w}_t - \widehat{w}_t^*\right)$

where $\Psi = \frac{\beta\varphi_w^2}{1-\beta\varphi_w^2}$. The first order condition of the Nash bargaining provided $\widehat{\chi}_t(r) = -(1-\chi)\left(\widehat{\Sigma}_t(r) - \widehat{\Delta}_t\right)$. Averaging this accross all firms will result in

$\widehat{\chi}_t(r) - \widehat{\chi}_t = -(1-\chi)\left(\widehat{\Sigma}_t(r) - \widehat{\Sigma}_t\right)$

$\Rightarrow$

$\widehat{\chi}_t(r) = \widehat{\chi}_t - (1-\chi)\,\rho w \Sigma \epsilon \Psi\left(\widehat{w}_t - \widehat{w}_t^*\right)$

$\Rightarrow$

$E_t\widehat{\chi}_{t+1}(r) = E_t\widehat{\chi}_{t+1} - (1-\chi)\,\rho w \Sigma \epsilon \Psi E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$

Given these findings, we can derive the percentage deviaiton of the bargaining firm's surplus from hiring another worker. That surplus is given by

$$H_t(r) = w_t(r) - b - \beta E_t \Lambda_{t,t+1}\left[s_{t+1}H_{x,t+1} - \rho_{t+1}^{in}\Upsilon\right] + \beta E_t \Lambda_{t,t+1}\left(1-\rho_{t+1}\right)H_{t+1}(r)$$

Log-linearizing it around the steady state and canceling the constant terms result in

$$H\widehat{H}_t(r) = w\widehat{w}_t(r) + \left[\rho^{in}\Upsilon H^{-1} - s + 1 - \rho\right]\beta E_t\widehat{\Lambda}_{t,t+1} + \beta\rho^{in}\Upsilon H^{-1}E_t\widehat{\rho}_{t+1}^{in} - \rho\beta E_t\widehat{\rho}_{t+1}$$

$$-s\beta H E_t\left[\widehat{s}_{t+1} + \widehat{H}_{t+1}\right] + \beta(1-\rho)E_t\widehat{H}_{t+1}(r)$$

where $H^{-1} = \frac{1-\chi}{\chi\kappa x}$. Then,

$$\widehat{H}_t(r) - \widehat{H}_t = wH^{-1}\left[\widehat{w}_t(r) - \widehat{w}_t\right] + \beta(1-\rho)E_t\left[\widehat{H}_{t+1}(r) - \widehat{H}_{t+1}\right]$$

We can iterate this equation to derive

$$E_t\left[\widehat{H}_{t+1}(r) - \widehat{H}_{t+1}\right] = wH^{-1}E_t\left[\widehat{w}_{t+1}(r) - \widehat{w}_{t+1}\right] + \beta(1-\rho)E_t\left[\widehat{H}_{t+2}(r) - \widehat{H}_{t+2}\right]$$

$$E_t\left[\widehat{H}_{t+1}(r) - \widehat{H}_{t+1}\right] = wH^{-1}\varphi_w\left[\widehat{w}_t^* - \widehat{w}_t\right] + \beta(1-\rho)E_t\left[\widehat{H}_{t+2}(r) - \widehat{H}_{t+2}\right]$$

$\Rightarrow$

$$E_t\left[\widehat{H}_{t+2}(r) - \widehat{H}_{t+2}\right] = wH^{-1}E_t\left(\widehat{w}_{t+2}(r) - \widehat{w}_{t+2}\right) + \beta(1-\rho_x)E_t\left[\widehat{H}_{t+3}(r) - \widehat{H}_{t+3}\right]$$

$$E_t\left[\widehat{H}_{t+2}(r) - \widehat{H}_{t+2}\right] = wH^{-1}\varphi_w^2\left(\widehat{w}_t^* - \widehat{w}_t\right) + \beta(1-\rho_x)E_t\left[\widehat{H}_{t+3}(r) - \widehat{H}_{t+3}\right]$$

$\Rightarrow$

$$\widehat{H}_t(r) - \widehat{H}_t = wH^{-1}\left[\left(\widehat{w}_t^* - \widehat{w}_t\right) + \beta(1-\rho)\varphi_w\left(\widehat{w}_t^* - \widehat{w}_t\right) + \left[\beta(1-\rho)\varphi_w\right]^2\left(\widehat{w}_t^* - \widehat{w}_t\right) + ...\right]$$

$\Rightarrow$

$$\widehat{H}_t(r) = \widehat{H}_t - w\Delta H^{-1}\left(\widehat{w}_t - \widehat{w}_t^*\right)$$

We know from the Nash bargaining condition that

$$\chi_t(r)J_t(r) = (1-\chi_t(r))H_t(r)$$

$\Rightarrow$

$$\widehat{H}_t(r) = \widehat{J}_t(r) + (1-\chi)^{-1}\widehat{\chi}_t(r) \Rightarrow \widehat{H}_t(r) = \widehat{x}_t(r) + (1-\chi)^{-1}\widehat{\chi}_t(r)$$

Using $\widehat{x}_t(r) = \widehat{x}_t + w\Sigma\epsilon\left(\widehat{w}_t - \widehat{w}_t^*\right)$ and $\widehat{\chi}_t(r) = \widehat{\chi}_t - (1-\chi)\rho w\Sigma\epsilon\Psi\left(\widehat{w}_t - \widehat{w}_t^*\right)$ we can derive

$$\widehat{H}_t(r) = \widehat{x}_t + w\Sigma\epsilon\left(\widehat{w}_t - \widehat{w}_t^*\right) + (1-\chi)^{-1}\left[\widehat{\chi}_t - (1-\chi)\rho w\Sigma\epsilon\Psi\left(\widehat{w}_t - \widehat{w}_t^*\right)\right]$$

$\Rightarrow$

$$\widehat{H}_t(r) = \widehat{x}_t + (1-\chi)^{-1}\widehat{\chi}_t + (1-\rho\Psi)w\Sigma\epsilon\left(\widehat{w}_t - \widehat{w}_t^*\right)$$

If we plug this into the aggregate surplus of the firms in the economy, we will have

$$\widehat{H}_t = \widehat{x}_t + (1-\chi)^{-1}\widehat{\chi}_t + (1-\rho\Psi)\, w\Sigma\epsilon\,(\widehat{w}_t - \widehat{w}_t^*) + w\Delta H^{-1}(\widehat{w}_t - \widehat{w}_t^*)$$

$\Rightarrow$

$$\widehat{H}_t = \widehat{x}_t + (1-\chi)^{-1}\widehat{\chi}_t + \left[(1-\rho\Psi)\, w\Sigma\epsilon + w\Delta H^{-1}\right](\widehat{w}_t - \widehat{w}_t^*)$$

$\Rightarrow$

$$E_t\widehat{H}_{t+1} = E_t\widehat{x}_{t+1} + (1-\chi)^{-1} E_t\widehat{\chi}_{t+1} + \left[(1-\rho\Psi)\, w\Sigma\epsilon + w\Delta H^{-1}\right] E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

Now we can use these equations to derive the percentage deviation of the target wage from its steady state such that

$$\widehat{w}_t^o(r) = \chi p^w f_n w^{-1}\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] + \left[p^w f_n - b - \beta\tfrac{\kappa}{2}x^2 - (s+\rho)\beta H\right]\chi w^{-1}\widehat{\chi}_t(r)$$
$$+\left[\left(s+\tfrac{\rho}{2}\right)\chi\kappa x - \rho^{in}\Upsilon\right]\beta w^{-1}E_t\widehat{\Lambda}_{t,t+1} + \beta\tfrac{\chi}{(1-\chi)}\kappa x^2 w^{-1}E_t\widehat{\chi}_{t+1}(r) + \beta\chi\kappa x^2 w^{-1}E_t\widehat{x}_{t+1}(r)$$
$$-\beta\rho^{in}\Upsilon w^{-1}E_t\widehat{\rho}_{t+1}^{in} + \beta s\chi\kappa x w^{-1}E_t\left[\widehat{s}_{t+1} + \widehat{H}_{t+1}\right]$$
$$\widehat{w}_t^o(r) = \varphi_{f_n}\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] + \varphi_\chi\widehat{\chi}_t(r) + \varphi_\Lambda E_t\widehat{\Lambda}_{t,t+1} + \tfrac{\varphi_x}{(1-\chi)}E_t\widehat{\chi}_{t+1}(r)$$
$$+\varphi_x E_t\widehat{x}_{t+1}(r) - \beta\rho^{in}\Upsilon w^{-1}E_t\widehat{\rho}_{t+1}^{in} + \varphi_s E_t\left[\widehat{s}_{t+1} + \widehat{H}_{t+1}\right]$$

where $\varphi_{f_n} = \chi p^w f_n w^{-1}$, $\varphi_\chi = \left[p^w f_n - b - \beta\frac{\kappa}{2}x^2 - (s+\rho)\beta H\right]\chi w^{-1}$, $\varphi_\Lambda = \left[\left(s+\frac{\rho}{2}\right)\chi\kappa x - \rho^{in}\Upsilon\right]\beta w^{-1}$, $\varphi_x = \beta\chi\kappa x^2 w^{-1}$, and $\varphi_s = \beta s\chi\kappa x w^{-1}$. If we replace the findings above, we will have

$$\widehat{w}_t^o(r) = \varphi_{f_n}\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] + \varphi_\chi\left[\widehat{\chi}_t - (1-\chi)\rho w\Sigma\epsilon\Psi(\widehat{w}_t - \widehat{w}_t^*)\right]$$
$$+\varphi_\Lambda E_t\widehat{\Lambda}_{t,t+1} + \tfrac{\varphi_x}{(1-\chi)}E_t\left[\widehat{\chi}_{t+1} - (1-\chi)\rho w\Sigma\epsilon\Psi\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)\right]$$
$$+\varphi_x E_t\left[\widehat{x}_{t+1} + w\Sigma\epsilon\varphi_w(\widehat{w}_t - \widehat{w}_t^*)\right] - \beta\rho^{in}\Upsilon w^{-1}E_t\widehat{\rho}_{t+1}^{in} + \varphi_s E_t\widehat{s}_{t+1}$$
$$+\varphi_s E_t\left[\widehat{x}_{t+1} + (1-\chi)^{-1}\widehat{\chi}_{t+1} + \left[(1-\rho\Psi)\, w\Sigma\epsilon + w\Delta H^{-1}\right]\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)\right]$$

$\Rightarrow$

$$\widehat{w}_t^o(r) = \varphi_{f_n}\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] + \varphi_\chi\widehat{\chi}_t + \varphi_\Lambda E_t\widehat{\Lambda}_{t,t+1} + \tfrac{\varphi_x+\varphi_s}{(1-\chi)}E_t\widehat{\chi}_{t+1} + [\varphi_x + \varphi_s]E_t\widehat{x}_{t+1}$$
$$-\beta\rho^{in}\Upsilon w^{-1}E_t\widehat{\rho}_{t+1}^{in} + \varphi_s E_t\widehat{s}_{t+1} + \tau_1(\widehat{w}_t - \widehat{w}_t^*) + \tau_2 E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

where $\tau_1 = \left[\varphi_x\varphi_w - (1-\chi)\varphi_\chi\rho\Psi\right]w\Sigma\epsilon$ and $\tau_2 = \left[(\varphi_x - \varphi_s)\rho\Psi\Sigma\epsilon + \varphi_s\left(\Sigma\epsilon + \Delta H^{-1}\right)\right]w$.

Finally we can derive the percentage deviation of the bargained wage from its steady state such that

$$\Sigma_t(r)\, w_t^* = w_t^o(r) + \beta\varphi_w E_t\Lambda_{t,t+1}\tfrac{n_{t+1}}{n_t}(r)\,\Sigma_{t+1}(r)\, w_{t+1}^*$$

$\Rightarrow$

$\Sigma w\left[\widehat{\Sigma}_t\left(r\right)+\widehat{w}_t^*\right]=w\widehat{w}_t^o\left(r\right)+\beta\varphi_w\Sigma wE_t\left[\widehat{\Lambda}_{t,t+1}+\widehat{n}_{t+1}\left(r\right)-\widehat{n}_t\left(r\right)+\widehat{\Sigma}_{t+1}\left(r\right)+\widehat{w}_{t+1}^*\right]$

$\Rightarrow$

$\widehat{w}_t^*=\Sigma^{-1}\widehat{w}_t^o\left(r\right)+\beta\varphi_wE_t\widehat{\Lambda}_{t,t+1}+\beta\varphi_wE_t\left[\widehat{n}_{t+1}\left(r\right)-\widehat{n}_t\left(r\right)\right]-\widehat{\Sigma}_t\left(r\right)+\beta\varphi_wE_t\widehat{\Sigma}_{t+1}\left(r\right)+\beta\varphi_wE_t\widehat{w}_{t+1}^*$

$\Rightarrow$

$\widehat{w}_t^*=\Sigma^{-1}\widehat{w}_t^o\left(r\right)+\beta\varphi_wE_t\widehat{\Lambda}_{t,t+1}+\rho\beta\varphi_wE_t\left[\widehat{x}_{t+1}\left(r\right)-\widehat{\rho}_{t+1}\right]-\widehat{\Sigma}_t\left(r\right)+\beta\varphi_wE_t\widehat{\Sigma}_{t+1}\left(r\right)+\beta\varphi_wE_t\widehat{w}_{t+1}^*$

$\Rightarrow$

$\widehat{w}_t^*=\Sigma^{-1}\widehat{w}_t^o\left(r\right)+\beta\varphi_wE_t\widehat{\Lambda}_{t,t+1}-\rho\beta\varphi_wE_t\widehat{\rho}_{t+1}+\rho\beta\varphi_wE_t\widehat{x}_{t+1}\left(r\right)-\widehat{\Sigma}_t\left(r\right)+\beta\varphi_wE_t\widehat{\Sigma}_{t+1}\left(r\right)+$
$\beta\varphi_wE_t\widehat{w}_{t+1}^*$

$\Rightarrow$

$\widehat{w}_t^*=\Sigma^{-1}\widehat{w}_t^o\left(r\right)+\beta\varphi_wE_t\widehat{\Lambda}_{t,t+1}-\rho\beta\varphi_wE_t\widehat{\rho}_{t+1}+\rho\beta\varphi_wE_t\left[\widehat{x}_{t+1}+w\Sigma\epsilon\varphi_w\left(\widehat{w}_t-\widehat{w}_t^*\right)\right]-\widehat{\Sigma}_t-\rho w\Sigma\epsilon\Psi\left(\widehat{w}_t-\widehat{w}_t^*\right)$
$+\beta\varphi_wE_t\left[\widehat{\Sigma}_{t+1}+\rho w\Sigma\epsilon\Psi\varphi_w\left(\widehat{w}_t-\widehat{w}_t^*\right)\right]+\beta\varphi_wE_t\widehat{w}_{t+1}^*$

$\Rightarrow$

$\widehat{\Sigma}_t+\widehat{w}_t^*=\Sigma^{-1}\widehat{w}_t^o\left(r\right)+\rho\beta\varphi_wE_t\left[\widehat{x}_{t+1}-\widehat{\rho}_{t+1}\right]+\left[\rho\beta\varphi_w^2w\Sigma\epsilon+\rho\beta\varphi_w^2\Psi w\Sigma\epsilon-\rho\Psi\epsilon\Sigma w\right]\left(\widehat{w}_t-\widehat{w}_t^*\right)+$
$\beta\varphi_wE_t\left[\widehat{\Lambda}_{t,t+1}+\widehat{\Sigma}_{t+1}+\widehat{w}_{t+1}^*\right]$

where $\rho\beta\varphi_w^2w\Sigma\epsilon+\rho\beta\varphi_w^2\Psi w\Sigma\epsilon-\rho\Psi\epsilon\Sigma w=\left[\beta\varphi_w^2\left(1+\Psi\right)-\Psi\right]\rho w\Sigma\epsilon=\left[\beta\varphi_w^2\left(1+\Psi\right)-\Psi\right]\rho w\Sigma\epsilon=$ 0. Therefore

$\widehat{\Sigma}_t+\widehat{w}_t^*=\Sigma^{-1}\widehat{w}_t^o\left(r\right)+\rho\beta\varphi_wE_t\left[\widehat{x}_{t+1}-\widehat{\rho}_{t+1}\right]+\beta\varphi_wE_t\left[\widehat{\Lambda}_{t,t+1}+\widehat{\Sigma}_{t+1}+\widehat{w}_{t+1}^*\right]$

7.7 Complete log-linearized model

$$\widehat{m}_t=\sigma\widehat{u}_t+\left(1-\sigma\right)\widehat{v}_t \tag{47}$$

$$\widehat{q}_t=\widehat{m}_t-\widehat{v}_t \tag{48}$$

$$\widehat{s}_t=\widehat{m}_t-\widehat{u}_t \tag{49}$$

$$\widehat{y}_t=a_t+\alpha\widehat{k}_t+\left(1-\alpha\right)\widehat{n}_t \tag{50}$$

$$a_{t+1}=\rho_aa_t+\varepsilon_{t+1}^a \tag{51}$$

$$\widehat{x}_t=\widehat{q}_t+\widehat{v}_t-\widehat{n}_{t-1} \tag{52}$$

$$\widehat{n}_t(i) = (1-\rho)\,\widehat{n}_{t-1}(i) + \rho\left(\widehat{q}_t + \widehat{v}_t(i) - \widehat{\rho}_t\right) \tag{53}$$

$$\widehat{\rho}_t = \frac{\rho^{in}}{\rho^{\rho}}\widehat{\rho}_t^{in} \tag{54}$$

$$\widehat{\rho}_{t+1}^{in} = \rho_\rho \widehat{\rho}_t^{in} \tag{55}$$

$$\widehat{z}_t = \widehat{p}_t^w + \widehat{y}_t - \widehat{k}_t \tag{56}$$

$$\begin{aligned}\widehat{x}_t \quad = \quad & p^w f_n \epsilon\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] - w\epsilon\widehat{w}_t + \left(1 - \frac{\rho}{2} - \rho^{in}\Upsilon\epsilon\right)\beta E_t \widehat{\Lambda}_{t,t+1} + \beta E_t \widehat{x}_{t+1} - \rho\beta E_t \widehat{\rho}_{t+1} \\ & -\beta\rho^{in}\Upsilon\epsilon E_t \widehat{\rho}_{t+1}^{in}\end{aligned} \tag{57}$$

$$\widehat{f}_{nt} = \widehat{y}_t - \widehat{n}_t \tag{58}$$

$$\widehat{u}_t = -\frac{n}{u}\widehat{n}_{t-1} \tag{59}$$

$$\widehat{\Delta}_t = (1-\rho)\,\beta\varphi_w E_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{\Delta}_{t+1}\right] - \rho\beta\varphi_w E_t \widehat{\rho}_{t+1} \tag{60}$$

$$\widehat{\Sigma}_t = \beta\varphi_w E_t\left[\widehat{\Lambda}_{t,t+1} + \rho\left(\widehat{x}_{t+1} - \widehat{\rho}_{t+1}\right) + \widehat{\Sigma}_{t+1}\right] \tag{61}$$

$$\widehat{\chi}_t = -(1-\chi)\left[\widehat{\Sigma}_t - \widehat{\Delta}_t\right] \tag{62}$$

$$\widehat{\Sigma}_t + \widehat{w}_t^* = \Sigma^{-1}\widehat{w}_t^o(r) + \rho\beta\varphi_w E_t\left[\widehat{x}_{t+1} - \widehat{\rho}_{t+1}\right] + \beta\varphi_w E_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{\Sigma}_{t+1} + \widehat{w}_{t+1}^*\right] \tag{63}$$

$$\begin{aligned}\widehat{w}_t^o(r) \quad = \quad & \varphi_{f_n}\left[\widehat{p}_t^w + \widehat{f}_{nt}\right] + \varphi_\chi \widehat{\chi}_t + \varphi_\Lambda E_t \widehat{\Lambda}_{t,t+1} + \frac{\varphi_x + \varphi_s}{(1-\chi)} E_t \widehat{\chi}_{t+1} + \left[\varphi_x + \varphi_s\right] E_t \widehat{x}_{t+1} \\ & + \varphi_s E_t \widehat{s}_{t+1} - \beta\rho^{in}\Upsilon w^{-1} E_t \widehat{\rho}_{t+1}^{in} + \tau_1\left(\widehat{w}_t - \widehat{w}_t^*\right) + \tau_2 E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)\end{aligned} \tag{64}$$

$$\widehat{w}_t = (1-\varphi_w)\,\widehat{w}_t^* + \varphi_w \widehat{w}_{t-1} \tag{65}$$

$$\widehat{\lambda}_t = E_t\left(\widehat{\lambda}_{t+1} - \widehat{\pi}_{t+1}\right) + \widehat{r}_t^n \tag{66}$$

$$\widehat{\lambda}_t = E_t\left(\widehat{\lambda}_{t+1} + \beta z \widehat{z}_{t+1}\right) \tag{67}$$

$$\widehat{\lambda}_t = -\widehat{c}_t \tag{68}$$

$$E_t \widehat{\Lambda}_{t,t+1} = E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t \tag{69}$$

$$\widehat{r}_t^n = E_t \widehat{\pi}_{t+1} + (1-\beta)\,\widehat{r}_t \tag{70}$$

$$\widehat{r}_t^n = \rho_m \widehat{r}_{t-1}^n + \gamma_\pi \left(1 - \rho_m\right) E_t \widehat{\pi}_{t+1} + \gamma_y \left(1 - \rho_m\right) \widehat{y}_t + \varepsilon_{t+1}^m \tag{71}$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\left[\widehat{k}_{t+1} - (1-\delta)\widehat{k}_t\right] + \frac{\kappa x^2 n}{2y}\left[2\widehat{x}_t + \widehat{n}_t\right] \tag{72}$$

$$\widehat{\pi}_t = \theta \widehat{p}_t^w + \beta E_t \widehat{\pi}_{t+1} \tag{73}$$

$$\widehat{\theta}_t = \widehat{v}_t - \widehat{h}_t \tag{74}$$