First solution Variables:

- \( st_i \): number of skilled technicians at the beginning of month \( i \).
- \( tt_i \): number of trainee technicians hired at the beginning of month \( i \).
- \( tt2_i \): number of trainee technicians who are starting their second month of training at the beginning of month \( i \).

Objective:

- \( \min \sum_{i=1}^{5} (2000st_i + 1000tt_i + 1000tt2_i) \).

Constraints:

- \( 160st_i \geq 50tt_i + 10tt2_i + d_i \) (the 160 hours per each skilled technician must be enough for: the training of the first month trainees, the training of the second month trainees, and to satisfy the work demand).
- \( st_1 = 50, \, tt2_1 = 0. \) We start with 50 STs, and no TTs who are starting their 2nd month of training.
- \( st_{i+1} = 0.95st_i + tt2_i \) (\( i = 1, \ldots, 4 \)). STs are made from the previous months’ STs minus the quitting 5%, plus the trainees who were in their second month the month before.
- \( tt2_{i+1} = tt_i \) (\( i = 1, \ldots, 4 \)).

Second solution

We can simply substitute \( tt_{i-1} \) for \( tt2_i \) to get the formulation

Objective:

- \( \min \sum_{i=1}^{5} (2000st_i + 1000(tt_i + tt_{i-1})) \).

Constraints:

- \( 160st_i \geq 50tt_i + 10tt_{i-1} + d_i \) (\( i = 1, \ldots, 5 \));
- \( st_1 = 50, \, tt_0 = 0. \)
- \( st_{i+1} = 0.95st_i + tt_{i-1} \) (\( i = 1, \ldots, 4 \)). STs are made from the previous months’ STs minus the quitting 5%, plus the trainees who were in their second month the month before.
Wrong solution Same as the second solution, but have

- \( \min \sum_{i=1}^{5} (2000s_i + 1000t_i) \)

in the objective. This is incorrect, since if a trainee enters, then he/she will be paid the 1000 dollars for 2 months, not one!