We want to write down a solution as *clear* as possible, not one with the fewest variables.

1. **Variables:**
   
   (a) \(x_1, x_2\) = barrels of crude 1, crude 2 purchased (and distilled, of course), resp.
   
   (b) \(n, d_1, d_2\) = barrels of naphta, distilled 1, distilled 2 made, resp.
   
   (c) \(c_1, c_2, g, j, h\) = barrels of cracked 1, cracked 2, gasoline, jet fuel, heating oil made, resp.

   (a) In general, variable \(xy\) will denote the amount of \(x\) turned into \(y\); eg.
   
   (b) \(d_1 h\) = barrels of distilled 1 turned into heating oil.

   (c) \(d_1 c_1\) = barrels of distilled 1 turned into cracked 1.

2. **Constraints about the flow of material:**

   (a) \(x_1n = 0.6x_1, x_1d_1 = 0.3x_1, x_1d_2 = 0.1x_1\): What we make out of crude 1.

   (b) \(x_2n = 0.4x_2, x_2d_1 = 0.2x_2, x_2d_2 = 0.4x_2\): What we make out of crude 2.

   (c) \(n = x_1n + x_2n, d_1 = x_1d_1 + x_2d_1, d_2 = x_1d_2 + x_2d_2\): Origin of naphta, distilled 1, distilled 2.

   (d) \(n = ng + nj, d_1 = d_1c_1 + d_1c_2 + d_1h, d_2 = d_2c_1 + d_2c_2 + d_2h\): What we make out of naphta, distilled 1, distilled 2.

   (e) \(d_1c_1 = 0.8(d_1c_1 + d_1c_2), d_1c_2 = 0.2(d_1c_1 + d_1c_2)\): Each barrel of distilled 1 sent through the cracker yields 0.8 barrels of cracked 1, and 0.2 barrels of cracked 2.

   (f) \(d_2c_1 = 0.7(d_2c_1 + d_2c_2), d_2c_2 = 0.3(d_2c_1 + d_2c_2)\): Each barrel of distilled 2 sent through the cracker yields 0.7 barrels of cracked 1, and 0.3 barrels of cracked 2.

   (g) \(c_1 = d_1c_1 + d_2c_1, c_2 = d_1c_2 + d_2c_2\): Origin of cracked 1, cracked 2.

   (h) \(c_1 = c_1g + c_1j, c_2 = c_2g + c_2j\): What we make out of cracked 1, cracked 2.

   (i) \(g = ng + c_1g + c_2g, j = nj + c_1j + c_2j, h = d_1h + d_2h\): Origin of gasoline, jet fuel, heating oil.

3. **Constraints about availability, requirement and capacity:**

   (a) \(x_1 \leq 10,000, x_2 \leq 10,000\): at most 10,000 barrels of each type of crude can be purchased daily.

   (b) \(x_1 + x_2 \leq 15,000\): at most 15,000 barrels of crude 1 and crude 2 can be distilled each day.

   (c) \(d_1c_1 + d_1c_2 + d_2c_1 + d_2c_2 \leq 5,000\): Each day, at most 5,000 barrels of distilled oil can be sent through the cracker.

   (d) \(g \geq 3,000, j \geq 3,000, h \geq 3,000\): at least 3,000 barrels of gasoline, jet fuel, and heating oil must be made daily.
4. **Blending constraints:**
   
   (a) \(8n_g + 9c_1g + 6c_2g \geq 8.5g\): Octane level of gasoline must be at least 8.5.
   
   (b) \(8n_j + 9c_1j + 6c_2j \geq 7.0j\): Octane level of jet fuel must be at least 7.0.
   
   (c) \(4d_1h + 5d_2h \geq 4.5h\): Octane level of heating oil must be at least 4.5.

5. **Nonnegativity constraints:**
   
   (a) All variables nonnegative.

6. **Objective:** Maximize
   
   (a) \(18g + 16j + 14h\): Sales price of gasoline, jet fuel, heating oil.
   
   (b) \(-(12x_1 + 10x_2)\): Purchase price of crude 1, crude 2.
   
   (c) \(-(0.1(x_1 + x_2))\): Cost of distilling crude 1, crude 2.
   
   (d) \(-(0.15(d_1c_1 + d_1c_2 + d_2c_1 + d_2c_2))\): Cost of sending oil through the cracker.

**Remarks**

1. The constraints in (2e) are each equivalent to \(0.8d_1c_2 = 0.2d_1c_1\).

2. If say we get only 0.5 barrels of cracked 1, and 0.2 barrels of cracked 2 from each barrel of distilled 1, then constraint (1) is harder to figure out. Then
   
   • Let \(d_1c\): barrels of distilled 1 sent through the cracker.
   
   • Replace \(d_1 = d_1h + d_1c_1 + d_1c_2\) with \(d_1 = d_1h + d_1c\).
   
   • Add the constraints \(d_1c_1 = 0.5d_1c, d_1c_2 = 0.2d_1c\).
   
   • From this, we can again eliminate \(d_1c\), to get \(0.2d_1c_1 = 0.5d_1c_2, d_1 = d_1h + 2d_1c_1\).
Figure 1: