Let

- \( ET_t \): number of ETs at the beginning of month \( t \).
- \( AT_t \): number of ATs at the beginning of month \( t \).
- \( SW_t \): number of SWs at the beginning of month \( t \).

The important constraints are the ones that express workforce availability for training and satisfying demand can be expressed in 2 ways.

**Solution 1** Define two new variables:

- \( T_{s,t} \): total number of hours spent by SWs on supervision in month \( t \).
- \( T_{d,t} \): total number of hours spent by SWs on satisfying demand in month \( t \).

Use the constraints

1. \( 200SW_t \geq T_{s,t} + T_{d,t} \)
2. \( T_{s,t} \geq 40AT_t + 150ET_t \)
3. \( 120AT_t + T_{d,t} \geq d_t \)

**Solution 2**

Use the constraints

- \( A \) \( 200SW_t \geq 40AT_t + 150ET_t \)
- \( B \) \( 120AT_t + 200SW_t \geq d_t + 40AT_t + 150ET_t \)

The two solutions are actually equivalent.

First, we show that (1), (2), (3) imply (A), (B):

- Obviously, (1) and (2) imply (A).
- Adding (2) and (3) gives
  \[ 120AT_t + T_{d,t} + T_{s,t} \geq d_t + 40AT_t + 150ET_t \]

Using (1) with this latest inequality gives (B)

Second, we show that (A), (B) imply (1), (2), (3), if we set

\[
T_{s,t} := 40AT_t + 150ET_t \\
T_{d,t} := 200SW_t - T_{s,t}
\]
• Then (1) and (2) are obviously satisfied.

• (3) is satisfied, this can be seen by plugging in the definition of $T_{d,t}$.

Wrong solution

Use only the constraints (B).

This is wrong; for some data it may give a correct solution, but in general it will not. Observe that the constraint (B) can be satisfied even if $SW_t = 0$, as long as

$$80AT_t \geq d_t + 150ET_t.$$ 

However, this does not reflect reality: the model setup implies that as long as we have ETs, we have to have SWs too, ATs alone are not enough.