**First solution**

We assume that we are at the first variant, i.e., employees overwork on their day after their 5 day shift.

- **Variables:**
  - $x_i$: number of employees who start work on day $i$, and work exactly 5 consecutive days.
  - $y_i$: number of employees who start work on day $i$, and work exactly 6 consecutive days.

- **Objective:**
  - Minimize $\sum_{i=1}^{7} (250x_i + (250 + 62)y_i)$.

To get the constraints right, we draw a table to indicate, on which days the employees are working.

<table>
<thead>
<tr>
<th>day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>employees $x_1$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_1$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x_2$</td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_2$</td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$x_3$</td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$y_3$</td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$x_4$</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$y_4$</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$x_5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$y_5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$x_6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$y_6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$x_7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$y_7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

- So the constraints are, assuming that the demand for employees on day $i$ is $d_i$:
  - $x_1 + y_1 + x_3 + x_4 + y_4 + x_5 + y_5 + x_6 + y_6 + x_7 + y_7 \geq d_1$.
  - $x_1 + y_1 + x_2 + y_2 + y_4 + x_5 + y_5 + x_6 + y_6 + x_7 + y_7 \geq d_2$.
  - etc.
  - $y_2 + x_3 + y_3 + x_4 + y_4 + x_5 + y_5 + x_6 + y_6 + x_7 + y_7 \geq d_7$.

Note that this is actually the solution for the second variant as well! Reason: we can look at the $y_1$ group as employees who work day 1 through day 6: for some of these
the extra day is day 1, and for some of these the extra day is day 6. The cost is the same for both types.

This argument proves that variants 1 and 2 have the same optimal solution value.

**Second solution**

This is formulated for the first variant (which has the same value as the second variant, as we have just seen).

- Variables:
  - $x_i$: number of employees who start work on day $i$, and work 5 consecutive days, including those who work one day more.
  - $y_i$: number of employees who are forced to work overtime on day $i$, after completing their 5 day long schedule on day $i - 1$. (day 7 − 1 is considered equal to day 1)

- Objective:
  - Minimize $\sum_{i=1}^{7} (250x_i + 62y_i)$.

- The first group of constraints arise by simply adding $y_i$ to the left hand side of the inequality in the original postoffice problem:
  - (*) $x_1 + x_4 + x_5 + x_6 + x_7 + y_1 \geq d_1$.
  - $x_1 + x_2 + x_5 + x_6 + x_7 + y_2 \geq d_2$.
  - etc.
  - (***) $x_3 + x_4 + x_5 + x_6 + x_7 + y_7 \geq d_7$.

- The second group of constraints force that the $y_1$ workers are drawn from the $x_3$ workers; the $y_2$ workers are drawn from the $x_4$ workers, etc.
  - $y_1 \leq x_3; y_2 \leq x_4; \ldots; y_7 \leq x_2$.

**Third, wrong solution**

This is for the second variant, but incorrect!

- Variables:
  - $x_i$: number of employees who start work on day $i$, and work 5 consecutive days, including those who work possibly a day before, or after this shift as well.
  - $y_i$: number of employees who are forced to work overtime on day $i$, either before, or after their regular 5 days.

- Objective:
  - Minimize $\sum_{i=1}^{7} (250x_i + 62y_i)$. 
The first group of constraints is the same as the ones between (*) and (**), and the second is

\[ y_1 \leq x_2 + x_3; \ y_2 \leq x_3 + x_4; \ldots; \ y_7 \leq x_1 + x_2. \]

This is wrong, since these constraints allow \( y_1 = x_2 \) and \( y_7 = x_2 \), that is, the \( x_2 \) types to work both on their offdays!

**Fourth solution**

This is for the second variant, and correct, though pretty complicated.

- \( x_{1,1} \): the number of \( x_1 \) types who overwork on Saturday, ie. the first day after their regular schedule; \( x_{1,2} \): the number of \( x_1 \) types who overwork on Sunday, ie. the second day after their regular schedule;
- \( \ldots; \)
- \( x_{7,1} \): the number of \( x_7 \) types who overwork on Friday, ie. the first day after their regular schedule; \( x_{7,2} \): the number of \( x_7 \) types who overwork on Saturday, ie. the second day after their regular schedule;

Then the second set of constraints is

\[ x_{1,1} + x_{1,2} \leq x_1; \ldots; x_{7,1} + x_{7,2} \leq x_7; \]

\[ y_1 = x_{2,2} + x_{3,1}; \ y_2 = x_{3,2} + x_{4,1}; \ldots; \ y_7 = x_{1,2} + x_{2,1}. \]